Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

SUGGESTED ANSWERS

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

This question paper consists of **23** printed pages and **1** blank page.

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Downloaded from SingaKids.com

1	(a)	Differentiate $\ln\left(\frac{3x}{x^2+1}\right)$ with respect to x.	[3]
	(b)	Hence find $\int \frac{2x}{x^2+1}\,dx$.	[2]

1	(a)	$Let\ y = \ln\left(\frac{3x}{x^2+1}\right) = \ln(3x) - \ln(x^2+1)$ $\frac{dy}{dx} = \frac{1}{x} - \frac{2x}{x^2+1}$ OR $\frac{dy}{dx} = \frac{\frac{3(x^2+1)-3x(2x)}{(x^2+1)^2}}{\frac{3x}{x^2+1}}$ $= \frac{3x^2+3-6x^2}{(x^2+1)^2} \times \frac{x^2+1}{3x}$ $= \frac{1-x^2}{x(x^2+1)}$	
	(b)	$\int\left(\frac{1}{x} - \frac{2x}{x^2+1}\right)dx = \ln\left(\frac{3x}{x^2+1}\right) + c$ $\int \frac{2x}{x^2+1}\,dx = \int \frac{1}{x}\,dx - \ln\left(\frac{3x}{x^2+1}\right) + c$ $= \ln x - \ln\left(\frac{3x}{x^2+1}\right) + c$ OR $\frac{dy}{dx} = \frac{1-x^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$	

Downloaded from SingaKids.com

Solve for A, B & C.

$A = 1,\ B = -2,\ C = 0$

$$\frac{1-x^2}{x\left(x^2+1\right)} = \frac{1}{x} - \frac{2x}{x^2+1}$$

$$\int\left(\frac{1}{x} - \frac{2x}{x^2+1}\right)dx = \ln\left(\frac{3x}{x^2+1}\right) + c$$

$$\int \frac{2x}{x^2+1}\,dx = \int \frac{1}{x}\,dx - \ln\left(\frac{3x}{x^2+1}\right) + c$$

$$= \ln x - \ln\left(\frac{3x}{x^2+1}\right) + c$$

Downloaded from SingaKids.com

2	It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$. Find, without using a calculator, the exact value of $\cos(A-B)$, leaving your answer in the form $p\sqrt{2}+q\sqrt{6}$, where p and q are real numbers.	[4]

2	$\cos A = \frac{1}{2}$, $\sin A = \frac{\sqrt{3}}{2}$ $\sin B = -\frac{1}{\sqrt{2}}$, $\cos B = -\frac{1}{\sqrt{2}}$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$ $= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$ $= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ $= -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$	

Downloaded from SingaKids.com

3	Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. $\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$	[5]

Volume of cone, $V = \frac{1}{3}\pi r^2 (18r) = 6\pi r^3$

$\frac{dV}{dr} = 18\pi r^2$

After 3s, $V = 2\pi \times 3 = 6\pi$

$6\pi r^3 = 6\pi$

$r = 1$

When $t = 3s$, $r = 1cm$, $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$2\pi = 18\pi (1)^2 \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{9}$ cm/s

Rate of change of radius of cone after 3 s is $\frac{1}{9}$ cm/s

Downloaded from SingaKids.com

4	A and B are the points of intersection of the line $4y = 2x+1$ and the curve $3y - x = 4xy$.		
	(a)	Find the coordinates of A and of B.	[4]
	(b)	Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB. Is he correct? Justify your answer with workings.	[3]

4 **(a)**	$4y = 2x+1$ .....(1) $y = \frac{2x+1}{4}$ .....(2) $3y - x = 4xy$ .....(3) Substitute (2) into (3): $3\left(\frac{2x+1}{4}\right) - x = 4x\left(\frac{2x+1}{4}\right)$ $\frac{3}{2}x + \frac{3}{4} - x = 2x^2 + x$ $6x + 3 - 4x = 8x^2 + 4x$ $8x^2 + 2x - 3 = 0$ $(2x-1)(4x+3) = 0$ $x = \frac{1}{2}$ *or* $x = -\frac{3}{4}$ $y = \frac{1}{2}$ *or* $y = -\frac{1}{8}$ $A = \left(\frac{1}{2}, \frac{1}{2}\right)$ $B = \left(-\frac{3}{4}, -\frac{1}{8}\right)$	
(b)	Gradient of $2y - 4x = 5$ is 2 Gradient of AB $= \frac{-\frac{1}{8} - \frac{1}{2}}{-\frac{3}{4} - \frac{1}{2}} = \frac{1}{2}$ Since $2 \times \frac{1}{2} \neq -1$, the 2 lines are not perpendicular to each other. Henry is not correct.	

Downloaded from SingaKids.com

5	(a)	Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$.	[2]
	(b)	Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion.	[4]

5	(a)	$\left(2-\frac{3}{x}\right)^8 = 2^8 + \binom{8}{1}(2^7)\left(-\frac{3}{x}\right) + \binom{8}{2}(2^6)\left(-\frac{3}{x}\right)^2 + \ldots\ldots$ $= 256 - \frac{3072}{x} + \frac{16128}{x^2} + \ldots\ldots$	
	(b)	$\left(1-2x-kx^2\right)\left(1-\frac{2}{x}\right)^8$ $= \left(1-2x-kx^2\right)\left(256 - \frac{3072}{x} + \frac{16128}{x^2} + \ldots\ldots\right)$ $= 256 - 2(256)x + 2(3072) + 3072kx - 16128k + \ldots\ldots$ $= 256 - 512x + 6144 + 3072kx - 16128k + \ldots\ldots$ Since there is no x term in the expansion, $3072k - 512 = 0$ $k = \frac{1}{6}$ The constant term $= 256 + 6144 - 16128k$ $= 6400 - 2688$ $= 3712$	

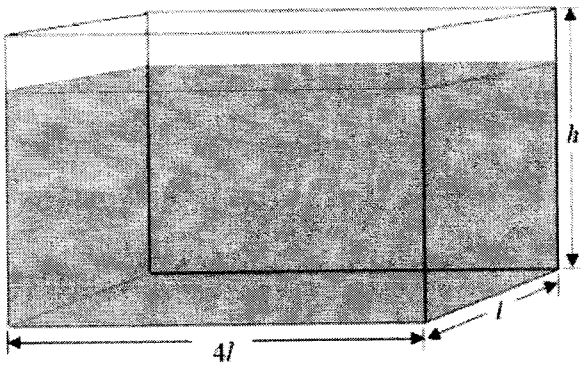
Downloaded from SingaKids.com

6	(a)	Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants.	[2]
	(b)	Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions.	[2]

6	(a)	$-4x^2 + 4x - 3$ $= -4(x^2 - x) - 3$ $= -4\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2\right] - 3$ $= -4\left(x - \frac{1}{2}\right)^2 + 1 - 3$ $= -4\left(x - \frac{1}{2}\right)^2 - 2$	
	(b)	$4x - 4x^2 - 3 = 0$ $y = 0$ ($x-axis$) Since the maximum value of y is -2, the graph of $y = 4x - 4x^2 - 3$ will not intersect the x-axis, $4x - 4x^2 - 3 = 0$ has no real solutions. OR $-4\left(x - \frac{1}{2}\right)^2 - 2 = 0$ $\left(x - \frac{1}{2}\right)^2 = -\frac{1}{2}$ $\left(x - \frac{1}{2}\right) = \pm\sqrt{-\frac{1}{2}}$ (*no real solutions*)	

Downloaded from SingaKids.com

7 Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be 5 m^2.



(a) Show that the volume of the tank, V m^3, is given by $V = \frac{2}{5}(5l - 4l^3)$. [3]

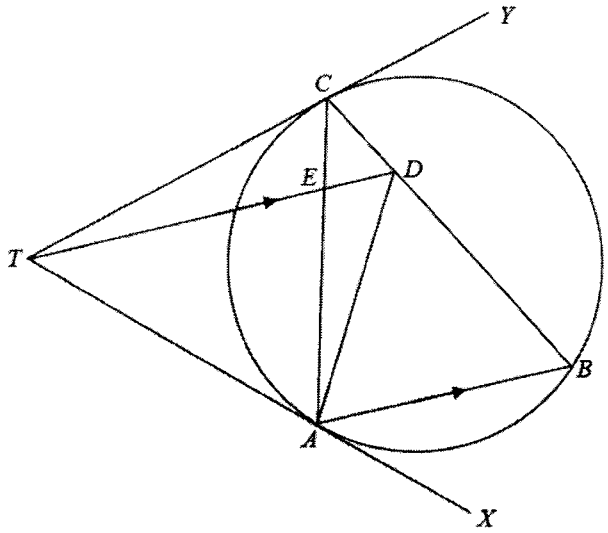
(b) Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim. [4]

Downloaded from SingaKids.com

7	(a)	Total outer surface area $= 5m^2$ $2(4lh)+2(lh)+4l^2=5$ $10lh+4l^2=5$ $h=\frac{5-4l^2}{10l}$ $V=4l\times l\times h$ $=4l^2\times\frac{5-4l^2}{10l}$ $=2l\times\frac{5-4l^2}{5}$ $=\frac{2}{5}(5l-4l^3)$ *(Shown)*	
	(b)	$V=\frac{2}{5}(5l-4l^3)$ $\frac{dV}{dl}=\frac{2}{5}(5-12l^2)$ *or* $2-\frac{24}{5}l^2$ $\frac{d^2V}{dl^2}=-\frac{48}{5}l<0$ (maximum V) $\frac{dV}{dl}=0$ $\frac{2}{5}(5-12l^2)=0$ $l^2=\frac{5}{12}$ $l=\sqrt{\frac{5}{12}}$ Area of rectangular base $=4l^2=4\left(\frac{5}{12}\right)=\frac{5}{3}$ *or* $1.67\ m^2$	

Downloaded from SingaKids.com

8 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB.



(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Using the result from part **(a)**, explain whether a circle can be drawn passing through the points T, A, D and C. [2]

(c) Prove that $CE \times EA = DE \times TE$. [3]

8	(a)	$\angle ATD = \angle XAB$ (corresponding angles, $AB//TD$) $\angle XAB = \angle ACB$ (Alternate Segment Theorem) $\angle ACB = \angle ATD$ (Proved)	
	(b)	From Part (a), $\angle ACD = \angle ATD$, the properties of circles tell us that angles in the same segment of a circle are equal. Therefore, we can draw a circle passing through points T, A, D and C.	
		$\angle ATE = \angle DCE$ (from Part (a)) $\angle TEA = \angle CED$ (vertically opposite angles) Therefore ΔATE is similar to ΔDCE (AA Similarity test) $\frac{TE}{CE} = \frac{EA}{DE}$ $CE \times EA = DE \times TE$	

Downloaded from SingaKids.com

9	(a)	A square has an area $\left(17-8\sqrt{2}\right)\text{cm}^2$. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)\text{cm}$, where a and b are integers. Show that $2b^4-17b^2+16=0$.	[4]
	(b)	[The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.] The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)\text{cm}$ and an area of $\left(3\sqrt{3}-\sqrt{15}\right)\text{cm}^2$. Show that $r=\frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)\text{cm}$, where p and q are integers.	[5]

Downloaded from SingaKids.com

9	(a)	$\left(a+b\sqrt{2}\right)^2 = 17-8\sqrt{2}$ $a^2+2ab\sqrt{2}+2b^2 = 17-8\sqrt{2}$ $\Rightarrow a^2+2b^2 = 17$ .....(1) $2ab = -8$ $a = -\frac{4}{b}$ .....(2) Substitute (2) into (1): $\left(-\frac{4}{b}\right)^2+2b^2 = 17$ $\frac{16}{b^2}+2b^2 = 17$ $16+2b^4 = 17b^2$ $2b^4-17b^2+16 = 0$ (Shown)	
	(b)	$A = \frac{1}{2}r^2\theta$, $S = r\theta$ $\frac{A}{S} = \frac{r}{2}$ $r = \frac{2A}{S}$ $r = \frac{2\left(3\sqrt{3}-\sqrt{15}\right)}{\sqrt{15}-\sqrt{3}} = \frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ *(showed)* $r = \frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}} \times \frac{\sqrt{15}+\sqrt{3}}{\sqrt{15}+\sqrt{3}}$ $= \frac{6\sqrt{45}+18-30-2\sqrt{45}}{15-3}$ $= \frac{4\sqrt{45}-12}{12}$ $= \frac{\sqrt{9}\sqrt{5}}{3}-1$ $= \sqrt{5}-1$	

Downloaded from SingaKids.com

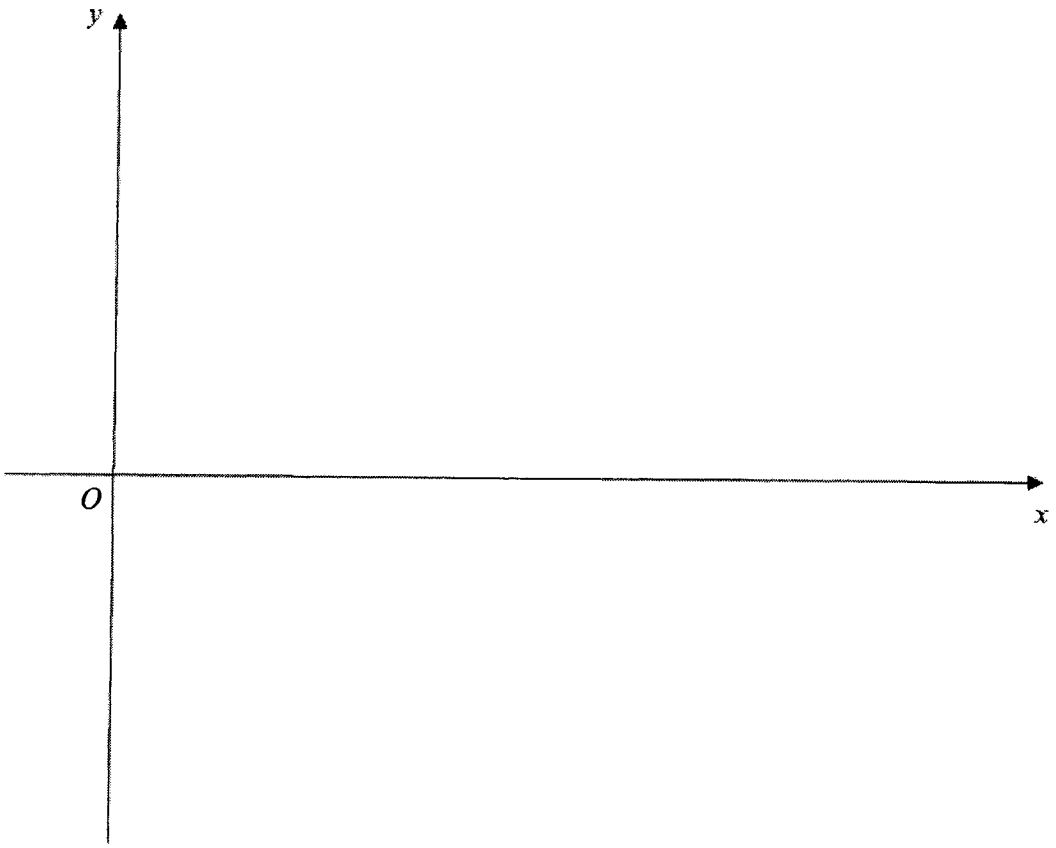
10 It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.

(a) State the period of $f(x)$. [1]

(b) State the smallest value of $f(x)$. [1]

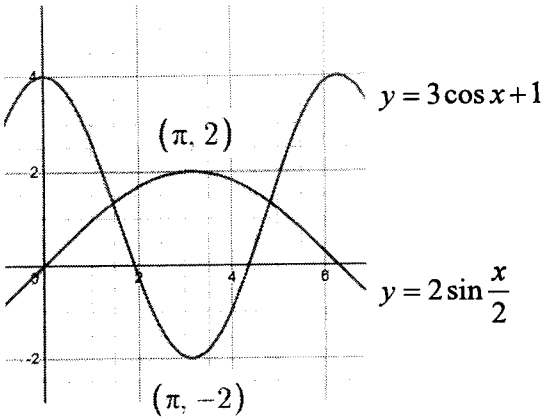
(c) State the largest value of $g(x)$. [1]

(d) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$.
Label your graphs clearly. [4]



(e) The solutions to the equation $f(x) = g(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$.
State, in terms of a and b, the range of values of x for which $f(x) > g(x)$. [1]

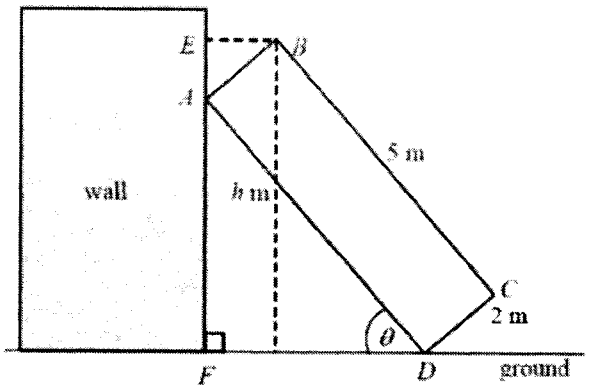
Downloaded from SingaKids.com

10	**(a)**	Period of $f(x) = 720°$ *or* 4π	
	(b)	Smallest value of $f(x) = 0$	
	(c)	Smallest value of $g(x) = 4$	
	(d)		
	(e)	$a < x < b$	

Downloaded from SingaKids.com

11 The diagram shows a rectangular wooden plank $ABCD$, 5 m by 2 m, leaning against a vertical wall. AD makes an acute angle θ with the horizontal ground.

E is a point on the vertical wall such that EB is parallel to the ground and F is a point on the ground such that AF is perpendicular to FD.
The point B is h m vertically above the ground.



(a) Show that $h = 5\sin\theta + 2\cos\theta$. [2]

(b) Express h in the form $R\sin(\theta+\alpha)$, where $R > 0$, and $0° < \alpha < 90°$. [3]

(c) Find the greatest possible value of h and the value of θ at which it occurs. [2]

(d) Find the value of θ for which $h = \sqrt{15}$ m. [2]

Downloaded from SingaKids.com

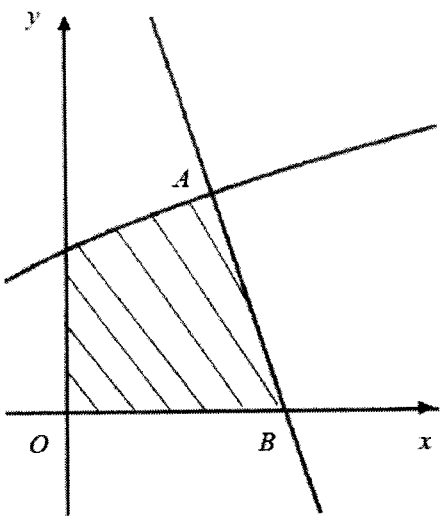
11	(a)	In ΔABE, $\angle EAB = \theta$, $AB = 2\,\text{cm}$, $EA = 2\cos\theta$ In ΔAFD, $\angle ADF = \theta$, $AD = 5\,\text{cm}$, $AF = 5\sin\theta$ $h = AF + EA = 5\sin\theta + 2\cos\theta$ (showed)	
	(b)	$h = 5\sin\theta + 2\cos\theta$ $R = \sqrt{5^2 + 2^2} = \sqrt{29}$ $\alpha = \tan^{-1}\frac{2}{5} = 21.801°$ $h = \sqrt{29}\sin(\theta + 21.8°)$	
	(c)	$h = \sqrt{29}\sin(\theta + 21.8°)$ Greatest possible value of $h = \sqrt{29}$ $\sin(\theta + 21.8°) = 1$ $\theta + 21.8° = 90°$ $\theta = 68.2°$	
	(d)	$\sqrt{29}\sin(\theta + 21.8014°) = \sqrt{15}$ $\sin(\theta + 21.8014°) = \sqrt{\frac{15}{29}}$ $\theta + 21.8014° = 45.98805°$ $\theta = 24.2°$	

Downloaded from SingaKids.com

12	**(a)**	By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$.	[1]
	(b)	Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions.	[5]
	(c)	Hence, find $\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,dx$.	[3]

12	(a)	$$\begin{array}{r} 4 \\ x^3+x^2 \overline{)\,4x^3+5x^2+x-1} \\ \underline{4x^3+4x^2} \\ x^2+x-1 \end{array}$$	
	(b)	$\dfrac{4x^3+5x^2+x-1}{x^2(x+1)} = 4+\dfrac{x^2+x-1}{x^2(x+1)}$ $\dfrac{x^2+x-1}{x^2(x+1)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$ $x^2+x-1 = Ax(x+1)+B(x+1)+Cx^2$ When $x=0$, $B=-1$ When $x=-1$, $C=-1$ When $x=1$, $2A-2-1=1$ $A=2$ $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)} = 4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}$	
	(c)	$\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,dx$ $= \int 4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}\,dx$ $= 4x+2\ln x+\dfrac{1}{x}-\ln(x+1)+c$	

Downloaded from SingaKids.com

13	The diagram shows part of the curve $y=\sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B. 		
	(a)	Find the equation of the normal to the curve at A.	[5]
	(b)	Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis.	[5]

Downloaded from SingaKids.com

13	(a)	$y=\sqrt{2x+5}$ Coordinates of $A=(2,3)$ $\frac{dy}{dx}=\frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)=\frac{1}{\sqrt{2x+5}}$ At A, $\frac{dy}{dx}=\frac{1}{3}$ Equation of normal AB is $y-3=-3(x-2)$ $y=-3x+9$	
	(b)	At $B, y=0$ $-3x+9=0$ $x=3$ $B=(3,0)$ Area of shaded region $=\frac{1}{2}(3-2)(3)+\int_0^2\left(\sqrt{2x+5}\right)dx$ $=1.5+\left[\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}\times 2}\right]_0^2$ $=1.5+\left[\left(\frac{9^{\frac{3}{2}}}{3}\right)-\left(\frac{5^{\frac{3}{2}}}{3}\right)\right]$ $=1.5+(5.27322)$ $=6.77\ units^2$	

Downloaded from SingaKids.com

Downloaded from SingaKids.com

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2025 Tuesday 2 hours 15 min

PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL
PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL PRESBYTERIAN HIGH SCHOOL

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

WORKED SOLUTIONS

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

Downloaded from SingaKids.com

1 The equation of a curve is $y = \frac{5x}{6-2x}$. Find the x-coordinates of the points at which the gradient of the curve is 7.5. [4]

$$y = \frac{5x}{6-2x}$$

$$\frac{dy}{dx} = \frac{(6-2x)(5) - 5x(-2)}{(6-2x)^2}$$

$$= \frac{30}{(6-2x)^2}$$

$$\frac{30}{(6-2x)^2} = 7.5$$

$$(6-2x)^2 = 4$$

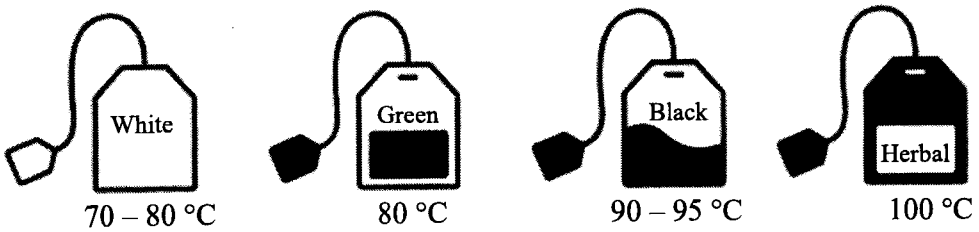
$$6-2x = \pm 2$$

$$-2x = -4 \quad or \quad -8$$

$$x = 2 \quad or \quad 4$$

Downloaded from SingaKids.com

2 The temperature of water in a tea cup, T °C, t minutes after boiling water is poured into the empty cup can be modelled by the formula $T = 25 + 75e^{-kt}$.



The diagram above shows the ideal brewing temperature for various teas at a particular teahouse. At this teahouse, green tea leaves are placed into the tea cup for brewing at the ideal temperature when $t = 6$. Determine whether the teahouse's standards for brewing tea would be adhered to if black tea leaves are placed into the teacup when $t = 2$. [4]

When $t = 6$, $T = 80$,

$$80 = 25 + 75e^{-6k}$$

$$\frac{55}{75} = e^{-6k}$$

$$\ln\frac{55}{75} = \ln e^{-6k}$$

$$k = -\frac{1}{6}\ln\frac{55}{75} = 0.0516925$$

Alternative:

$$\ln 55 = \ln\left(75e^{-6k}\right)$$

$$\ln 55 = \ln 75 - 6k$$

$$k = \frac{1}{6}(\ln 75 - \ln 55)$$

When $t = 2$, $T = 92.6$ °C

Since the temperature of the water would be within the ideal range of 90 – 95 °C, the standards for brewing black tea **would** be adhered to.

Downloaded from SingaKids.com

3 It is given that $f'(x) = 6\cos 3x - 12\sin 3x$ and $f\left(\frac{\pi}{3}\right) = 0$.

(a) Find f(x). [4]

$f'(x) = 6\cos 3x - 12\sin 3x$

$f(x) = \frac{6\sin 3x}{3} + \frac{12\cos 3x}{3} + c$

$= 2\sin 3x + 4\cos 3x + c$

$f\left(\frac{\pi}{3}\right) = 0$

$2\sin \pi + 4\cos \pi + c = 0$

$c = 4$

$\therefore f(x) = 2\sin 3x + 4\cos 3x + 4$

(b) Find the exact value of $f''\left(\frac{\pi}{12}\right)$. [2]

$f''(x) = -18\sin 3x - 36\cos 3x$

$f''\left(\frac{\pi}{12}\right) = -18\sin\left(\frac{\pi}{4}\right) - 36\cos\left(\frac{\pi}{4}\right)$

$= -18\left(\frac{\sqrt{2}}{2}\right) - 36\left(\frac{\sqrt{2}}{2}\right)$

$= -27\sqrt{2}$

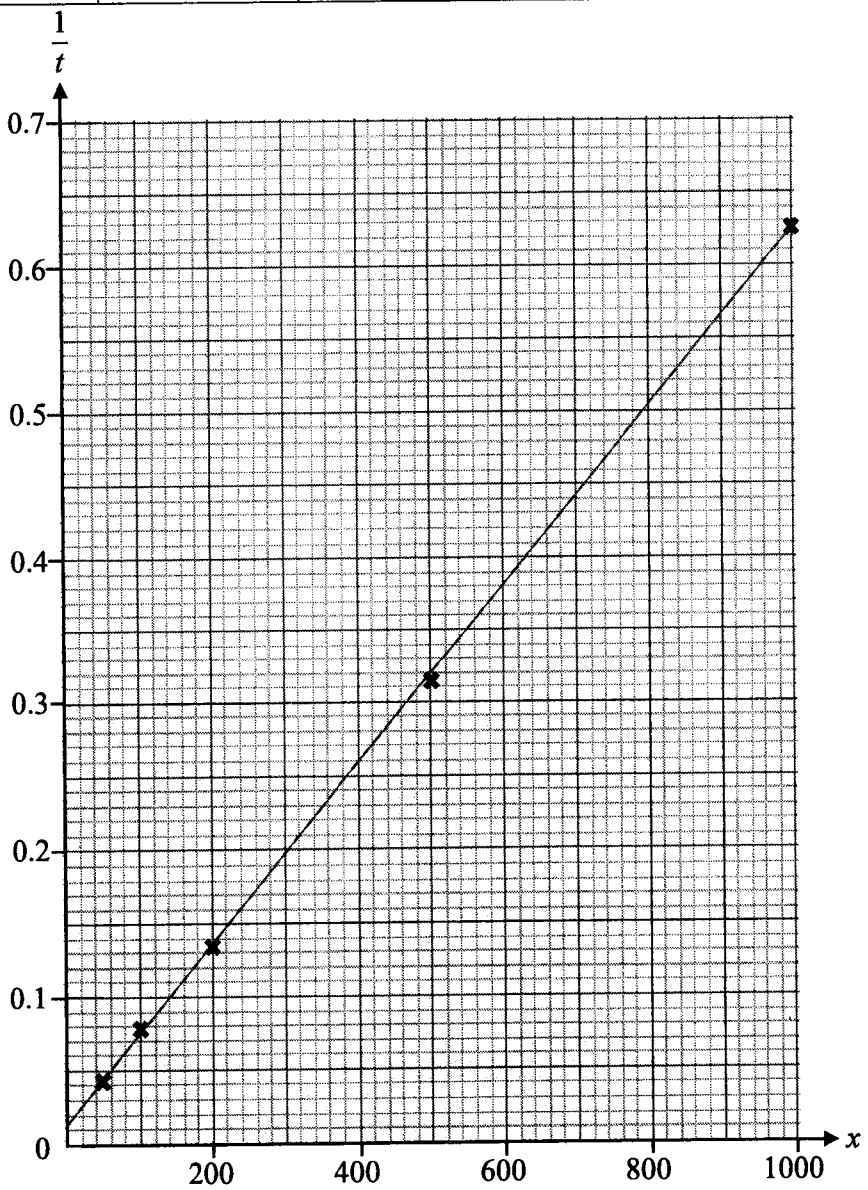
Downloaded from SingaKids.com

4 **(a)** The time taken, t in minutes, to download a particular gaming software is related to internet speed, x Mbps. The variables x and t are related by the formula $t = \frac{a}{b+x}$, where a and b are positive constants. The data below shows some measured values of x and t.

x (Mbps)	50	100	200	500	1000
t (minutes)	23	13	7.3	3.2	1.6

(i) Plot $\frac{1}{t}$ against x and draw a straight line graph. [2]

$\frac{1}{t}$	0.043	0.077	0.137	0.313	0.625



Downloaded from SingaKids.com

(ii) Use the graph to estimate the value of a and of b, correct to 2 significant figures. [4]

$\frac{1}{t} = \frac{b}{a} + \frac{1}{a}x$

Gradient $= \frac{0.62 - 0.02}{1000} = 0.0006$

$\frac{1}{a} = 0.0006$

$a = \frac{1}{0.0006} = 1700 \ (2\ s.f.)$

$\frac{b}{a} = 0.02$

$b = \frac{0.02}{0.0006} = 33 \ (2\ s.f.)$

(iii) Use the graph to estimate the time taken for John to download the gaming software if the speed of his internet is 240 Mbps. [2]

From the graph,

$\frac{1}{t} = 0.16$

$t = 6.25$ min

(iv) Suggest an alternative set of variables to be plotted on each axis to draw a straight line to represent the formula. [1]

$tb + xt = a$

$xt = -bt + a$

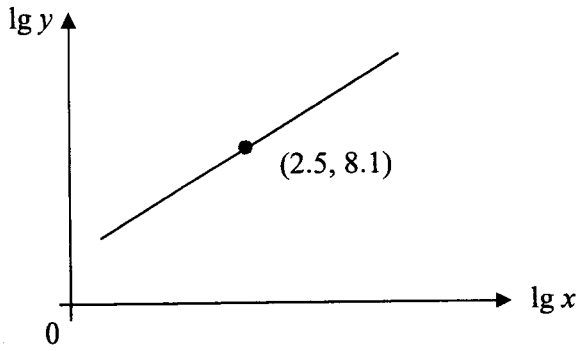
An alternative would be to plot

xt against t **OR** t against xt

Downloaded from SingaKids.com

(b) The diagram shows part of a straight line graph drawn to represent the equation $y = px^q$.

Given that the line passes through the point (2.5, 8.1) and has a gradient of 3, find the value of p correct to the nearest integer. [3]



$y = px^q$

$\lg y = \lg p + q \lg x$

When $\lg y = 8.1$, $\lg x = 2.5$,

$8.1 = \lg p + 3(2.5)$

$\lg p = 0.6$

$p = 10^{0.6} = 4$ (to nearest integer)

Downloaded from SingaKids.com

5 **(a)** Find the range of values of the constant k for which $k(x^2+x)+2(x^2+x+1)=0$ has no real roots. [4]

$(k+2)x^2+(k+2)x+2=0$

Since there are no x-intercepts,

$(k+2)^2-4(k+2)(2)<0$

$k^2+4k+4-8k-16<0$

$k^2-4k-12<0$

$(k-6)(k+2)<0$

$\therefore\ -2<k<6$

(b) Hence, explain why $-(x^2+x)+2(x^2+x+1)$ is always positive. [2]

Since the **discriminant < 0 when k = –1** and the **coefficient of x^2** , i.e. 1, is **positive**,

$-(x^2+x)+2(x^2+x+1)$ is always positive

Downloaded from SingaKids.com

6 **(a)** By using a suitable substitution, or otherwise, solve the equation

$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25$. [4]

$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25$

$3\log_6 x = \log_x 6 + 2$

$3\log_6 x = \dfrac{1}{\log_6 x} + 2$

Let $y = \log_6 x$

$3y = \dfrac{1}{y} + 2$

$3y^2 - 2y - 1 = 0$

$(3y + 1)(y - 1) = 0$

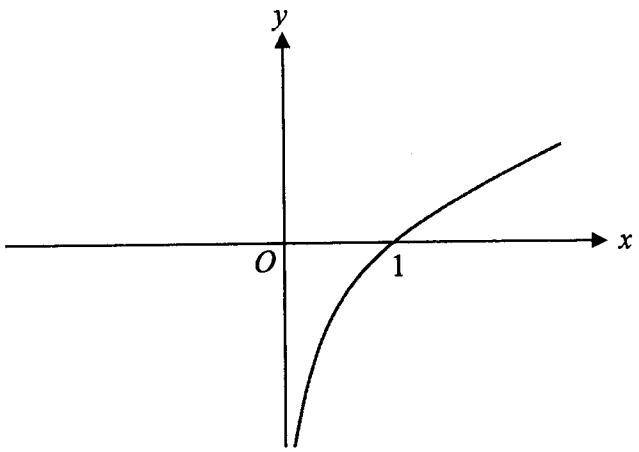
$y = -\dfrac{1}{3}$ *or* $y = 1$

$\log_6 x = -\dfrac{1}{3}$ *or* $\log_6 x = 1$

$x = 6^{-\frac{1}{3}}$ *or* $x = 6$

$= 0.550$ (3*s.f.*)

(b) Sketch the graph of $y = \log_6 x$ on the given axes. Label any axial intercepts. [1]



Downloaded from SingaKids.com

(c) **(i)** It is given that the equation of a curve is $y = e^{2x+1} + 2e^{x+1} - 3e$.

Explain why the curve has no stationary point. [3]

$\frac{dy}{dx} = 2e^{2x+1} + 2e^{x+1}$

Since $e^{2x+1} > 0$ and $e^{x+1} > 0$ for all real values of x,

$2e^{2x+1} + 2e^{x+1} > 0$, $\frac{dy}{dx} \neq 0$

Therefore the curve has no stationary points

(ii) Find the x-intercept of the curve $y = e^{2x+1} + 2e^{x+1} - 3e$. [3]

$e^{2x} \cdot e + 2e^{x} \cdot e - 3e = 0$

$e^{2x} + 2e^{x} - 3 = 0$

Let $y = e^{x}$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$e^{x} = -3$ *(rej)* *or* $e^{x} = 1$

$x = 0$

Downloaded from SingaKids.com

7 **(a)** Factorise $x^3 - 125$. [1]

$x^3 - 125 = (x-5)(x^2 + 5x + 25)$

(b) The expression $-2x^3 + 7x^2 + ax + b$, where a and b are integers, has a factor of $x - 5$ and a remainder of -11 when divided by $2x + 1$. Find the value of a and of b. [4]

Let $f(x) = -2x^3 + 7x^2 + ax + b$.

$f(5) = 0$

$-2(5)^3 + 7(5)^2 + a(5) + b = 0$

$5a + b = 75$ ----- (1)

$f\left(-\frac{1}{2}\right) = -11$

$-2\left(-\frac{1}{2}\right)^3 + 7\left(-\frac{1}{2}\right)^2 + a\left(-\frac{1}{2}\right) + b = -11$

$2 - \frac{1}{2}a + b = -11$

$a = 2b + 26$ ----- (2)

Subst (2) into (1):

$5(2b + 26) + b = 75$

$11b = -55$

$b = -5,\ a = 16$

Downloaded from SingaKids.com

(c) Hence, or otherwise, show that the equation $x^3 - 125 = -2x^3 + 7x^2 + ax + b$ has only one real root. [4]

$x^3 - 125 = -2x^3 + 7x^2 + 16x - 5$

$(x-5)(x^2 + 5x + 25) = (x-5)(-2x^2 - 3x + 1)$

$(x-5)(3x^2 + 8x + 24) = 0$

$x - 5 = 0$ *or* $3x^2 + 8x + 24 = 0$

$x = 5$ $b^2 - 4ac = 8^2 - 4(3)(24)$

$= -224$

< 0

There is no real root for $3x^2 + 8x + 24 = 0$.

Therefore there is only one real root $x = 5$.

Alternative

$x^3 - 125 = -2x^3 + 7x^2 + 16x - 5$

$3x^3 - 7x^2 - 16x - 120 = 0$

$(x-5)(3x^2 + 8x + 24) = 0$

Downloaded from SingaKids.com

8 The equation of a curve is $y = e^{-2x}\tan x$.

(a) Show that $\frac{dy}{dx} = e^{-2x}(1 - \tan x)^2$. [3]

$$\frac{dy}{dx} = e^{-2x}\sec^2 x - 2e^{-2x}\tan x$$
$$= e^{-2x}(\tan^2 x + 1 - 2\tan x)$$
$$= e^{-2x}(\tan x - 1)^2$$

(b) Find, in terms of π, the x-coordinate of the stationary point for $0 < x < \frac{\pi}{2}$. [2]

$e^{-2x}(\tan x - 1)^2 = 0$

Since $e^{-2x} > 0$ for all real values of x,

$(\tan x - 1)^2 = 0$

$\tan x = 1$

$x = \frac{\pi}{4}$

Downloaded from SingaKids.com

(c) Explain why y is never decreasing. [2]

Since $e^{-2x} > 0$ and $(\tan x - 1)^2 \geq 0$ for all real values of x,

$\frac{dy}{dx} \geqslant 0$ therefore y is never decreasing.

(d) What does your answer to **part (c)** imply about the stationary point found in **part (b)**? Explain your answer. [2]

The stationary point at $x = \frac{\pi}{4}$ is a **point of inflexion.**

Since y is never decreasing, the **gradient of the curve must be positive (or $dy/dx > 0$) slightly to the left and right** of the stationary point.

Downloaded from SingaKids.com

9 **(a)** State

(i) the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

$-60°$ or $-\frac{\pi}{3}$

(ii) the values between which the principal value of $\tan^{-1}x$ must lie, [1]

$-90° < \tan^{-1}x < 90°$

(iii) the range of values of k for which the equation $4\sin^2 x = k$ has a solution. [1]

$0 \leq k \leq 4$

Downloaded from SingaKids.com

(b) **(i)** Prove the identity $\cot x - \cot x \tan^2 x + \tan x = \dfrac{1+\cos 2x}{\sin 2x}$. [4]

$$\cot x - \cot x \tan^2 x + \tan x$$

$$= \cot x - \frac{1}{\tan x}\tan^2 x + \tan x$$

$$= \frac{\cos x}{\sin x} - \tan x + \tan x$$

$$= \frac{2\cos^2 x}{2\sin x \cos x}$$

$$= \frac{1+\cos 2x}{\sin 2x}$$

(ii) Hence solve the equation $\cot x - \cot x \tan^2 x + \tan x = \operatorname{cosec} 2x - 3$ for $0° \leq x \leq 180°$. [5]

$$\frac{1+\cos 2x}{\sin 2x} = \operatorname{cosec} 2x - 3$$

$$\frac{1+\cos 2x}{\sin 2x} = \frac{1}{\sin 2x} - 3$$

$$1 + \cos 2x = 1 - 3\sin 2x$$

$$\tan 2x = -\frac{1}{3}$$

$$\tan \alpha = \frac{1}{3}$$

$$\alpha = \tan^{-1}\left(\frac{1}{3}\right) = 18.434°$$

$$2x = 180° - 18.434°,\ 360° - 18.434°$$

$$2x = 161.566°,\ 341.566°$$

$$x = 80.8°,\ 170.8°$$

Downloaded from SingaKids.com

10 **(a)** Points $P(-2, 0)$, $Q(5, 1)$, $R(6, -6)$ and $S(-1, -7)$ lie on a circle.

PQ and SR are parallel chords of the same length.

Show that the centre of the circle is at $(2, -3)$. [2]

Centre of circle

$$= \left(\frac{-2+6}{2}, \frac{0-6}{2}\right)$$

$$= (2, -3)$$

OR

$$\text{Midpoint of } PQ = \left(\frac{-2+5}{2}, \frac{0+1}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$\text{Midpoint of } SR = \left(\frac{6-1}{2}, \frac{-6-7}{2}\right)$$

$$= \left(\frac{5}{2}, -\frac{13}{2}\right)$$

$$\text{Centre of circle} = \left(\frac{\frac{3}{2}+\frac{5}{2}}{2}, \frac{\frac{1}{2}-\frac{13}{2}}{2}\right)$$

$$= (2, -3)$$

(b) Find the equation of the circle. [3]

Radius

$$= \sqrt{(5-2)^2 + (1+3)^2}$$

$$= 5 \text{ units}$$

$$(x-2)^2 + (y+3)^2 = 25$$

OR

$$x^2 - 4x + y^2 + 6y - 12 = 0$$

Downloaded from SingaKids.com

(c) A second circle has centre C. It is given that the y-axis is the perpendicular bisector of CR and is also a tangent to the second circle.

(i) State the coordinates of C. [1]

Centre of second circle = (–6, –6)

(ii) Hence explain why the x-axis is a tangent to the second circle. [2]

Since the y-axis is a tangent to the second circle, the radius of the circle is 6 units. The highest point, H, of the second circle is (–6, 0).

Since the circle touches the x-axis at exactly one point at (–6, 0), the x-axis is a tangent to the second circle.

Or

Since CH is a vertical radius of the circle and H lies on the x-axis, the x-axis, which is horizontal, must be a tangent to the second circle (rad⊥tan)

Downloaded from SingaKids.com

11 A particle moves from point P towards point O. The speed of the particle, t seconds after passing point P, can be modelled by the formula $v = (k - 2t)^n$, where k and n are constants.

(a) Find an expression for the acceleration of the particle in terms of k, t and n. [1]

$a = -2n(k-2t)^{n-1}$

The particle has a speed of 27 m/s when $t = 3.5$ and comes to instantaneous rest 4.8 metres to the right of point O when $t = 8$.

(b) Find the value of k and of n. [3]

When $t = 8$, $v = 0$,

$0 = (k-16)^n$

$k = 16$

When $t = 3.5$, $v = 27$,

$27 = (16-7)^n$

$27 = 9^n$

$3^3 = 3^{2n}$

$n = 1.5$

Downloaded from SingaKids.com

(c) Using the values of k and n found in **part (b)**, find the total distance travelled by the particle in the first 8 seconds. [4]

$v = (16 - 2t)^{1.5}$

$s = \dfrac{(16 - 2t)^{2.5}}{(2.5)(-2)} + c$

When $t = 8$, $s = 4.8$

$4.8 = c$

$s = -\dfrac{1}{5}(16 - 2t)^{2.5} + 4.8$

When $t = 0$, $s = -200$

Total distance = 200 + 4.8 = 204.8 m

(d) Explain whether the given model can continue to describe the motion of the particle after 8 seconds. [1]

No since when $t > 8$, v and s are undefined.

END OF PAPER

Downloaded from SingaKids.com

Downloaded from SingaKids.com