Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

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2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

SUGGESTED ANSWERS

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

Setter: Mr Tan Lip Sing
Vetter: Ms Sabrina Tan

This question paper consists of **23** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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1	**(a)**	Differentiate $\ln\left(\frac{3x}{x^2+1}\right)$ with respect to x.	[3]
	(b)	Hence find $\int \frac{2x}{x^2+1}\,\mathrm{d}x$.	[2]

1	(a)	$Let\ y = \ln\left(\frac{3x}{x^2+1}\right) = \ln(3x) - \ln\left(x^2+1\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x} - \frac{2x}{x^2+1}$ OR $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{3\left(x^2+1\right)-3x(2x)}{\left(x^2+1\right)^2}}{\frac{3x}{x^2+1}}$ $= \frac{3x^2+3-6x^2}{\left(x^2+1\right)^2} \times \frac{x^2+1}{3x}$ $= \frac{1-x^2}{x\left(x^2+1\right)}$	
	(b)	$\int\left(\frac{1}{x} - \frac{2x}{x^2+1}\right)\mathrm{d}x = \ln\left(\frac{3x}{x^2+1}\right) + c$ $\int \frac{2x}{x^2+1}\,dx = \int \frac{1}{x}\,\mathrm{d}x - \ln\left(\frac{3x}{x^2+1}\right) + c$ $= \ln x - \ln\left(\frac{3x}{x^2+1}\right) + c$ OR $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-x^2}{x\left(x^2+1\right)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$	

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		Solve for A, B & C. $A = 1,\ B = -2,\ C = 0$ $\frac{1-x^2}{x\left(x^2+1\right)} = \frac{1}{x} - \frac{2x}{x^2+1}$ $\int\left(\frac{1}{x} - \frac{2x}{x^2+1}\right)\mathrm{d}x = \ln\left(\frac{3x}{x^2+1}\right) + c$ $\int \frac{2x}{x^2+1}\,dx = \int \frac{1}{x}\,\mathrm{d}x - \ln\left(\frac{3x}{x^2+1}\right) + c$ $= \ln x - \ln\left(\frac{3x}{x^2+1}\right) + c$	

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2	It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$. Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers.	[4]

2	$\cos A = \frac{1}{2}$, $\sin A = \frac{\sqrt{3}}{2}$ $\sin B = -\frac{1}{\sqrt{2}}$, $\cos B = -\frac{1}{\sqrt{2}}$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$ $= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ $= -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$	

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3	Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. $\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$	[5]

	Volume of cone, $V = \frac{1}{3}\pi r^2 (18r) = 6\pi r^3$ $\frac{dV}{dr} = 18\pi r^2$ After 3s, $V = 2\pi \times 3 = 6\pi$ $6\pi r^3 = 6\pi$ $r = 1$ When $t = 3s$, $r = 1cm$, $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $2\pi = 18\pi (1)^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{1}{9}$ cm/s Rate of change of radius of cone after 3 s is $\frac{1}{9}$ cm/s	

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4	A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.		
	(a)	Find the coordinates of A and of B.	[4]
	(b)	Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB. Is he correct? Justify your answer with workings.	[3]

4 **(a)**	$4y = 2x + 1$ .....(1) $y = \frac{2x+1}{4}$ .....(2) $3y - x = 4xy$ .....(3) Substitute (2) into (3): $3\left(\frac{2x+1}{4}\right) - x = 4x\left(\frac{2x+1}{4}\right)$ $\frac{3}{2}x + \frac{3}{4} - x = 2x^2 + x$ $6x + 3 - 4x = 8x^2 + 4x$ $8x^2 + 2x - 3 = 0$ $(2x-1)(4x+3) = 0$ $x = \frac{1}{2}$ *or* $x = -\frac{3}{4}$ $y = \frac{1}{2}$ *or* $y = -\frac{1}{8}$ $A = \left(\frac{1}{2}, \frac{1}{2}\right)$ $B = \left(-\frac{3}{4}, -\frac{1}{8}\right)$	
(b)	Gradient of $2y - 4x = 5$ is 2 Gradient of AB $= \frac{-\frac{1}{8} - \frac{1}{2}}{-\frac{3}{4} - \frac{1}{2}} = \frac{1}{2}$ Since $2 \times \frac{1}{2} \neq -1$, the 2 lines are not perpendicular to each other. Henry is not correct.	

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5	**(a)**	Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$.	[2]
	(b)	Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion.	[4]

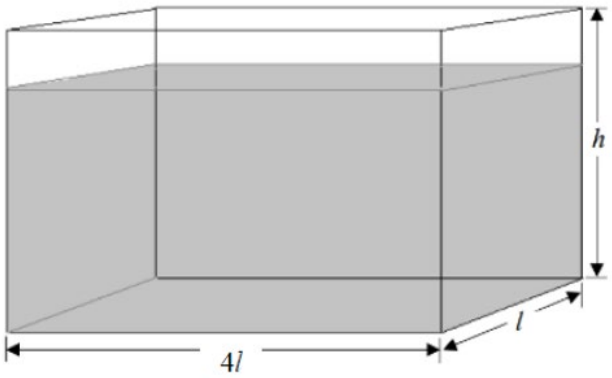
5	**(a)**	$\left(2-\frac{3}{x}\right)^8 = 2^8 + \binom{8}{1}\left(2^7\right)\left(-\frac{3}{x}\right) + \binom{8}{2}\left(2^6\right)\left(-\frac{3}{x}\right)^2 + ...$ $= 256 - \frac{3072}{x} + \frac{16128}{x^2} + ...$	
	(b)	$\left(1-2x-kx^2\right)\left(1-\frac{2}{x}\right)^8$ $= \left(1-2x-kx^2\right)\left(256 - \frac{3072}{x} + \frac{16128}{x^2} + ...\right)$ $= 256 - 2(256)x + 2(3072) + 3072kx - 16128k + ...$ $= 256 - 512x + 6144 + 3072kx - 16128k + ...$ Since there is no x term in the expansion, $3072k - 512 = 0$ $k = \frac{1}{6}$ The constant term $= 256 + 6144 - 16128k$ $= 6400 - 2688$ $= 3712$	

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6	**(a)**	Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants.	[2]
	(b)	Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions.	[2]

6	**(a)**	$-4x^2 + 4x - 3$ $= -4(x^2 - x) - 3$ $= -4\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2\right] - 3$ $= -4\left(x - \frac{1}{2}\right)^2 + 1 - 3$ $= -4\left(x - \frac{1}{2}\right)^2 - 2$	
	(b)	$4x - 4x^2 - 3 = 0$ $y = 0$ $(x - axis)$ Since the maximum value of y is -2, the graph of $y = 4x - 4x^2 - 3$ will not intersect the x-axis, $4x - 4x^2 - 3 = 0$ has no real solutions. OR $-4\left(x - \frac{1}{2}\right)^2 - 2 = 0$ $\left(x - \frac{1}{2}\right)^2 = -\frac{1}{2}$ $\left(x - \frac{1}{2}\right) = \pm\sqrt{-\frac{1}{2}}$ (*no real solutions*)	

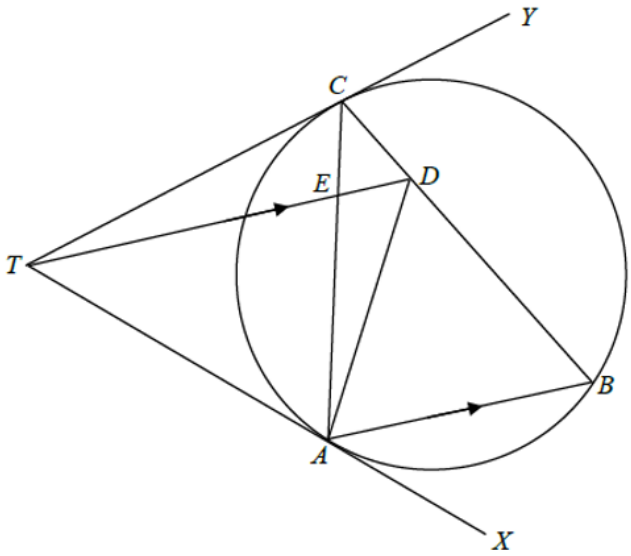
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7	Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be $5\ \text{m}^2$.  		
	(a)	Show that the volume of the tank, $V\ \text{m}^3$, is given by $V = \frac{2}{5}\left(5l - 4l^3\right)$.	[3]
	(b)	Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim.	[4]

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7	(a)	Total outer surface area $= 5\,m^2$ $2(4lh) + 2(lh) + 4l^{\,2} = 5$ $10lh + 4l^2 = 5$ $h = \dfrac{5 - 4l^2}{10l}$ $V = 4l \times l \times h$ $= 4l^2 \times \dfrac{5 - 4l^2}{10l}$ $= 2l \times \dfrac{5 - 4l^2}{5}$ $= \dfrac{2}{5}(5l - 4l^3)$ (*Shown*)	
	(b)	$V = \dfrac{2}{5}(5l - 4l^3)$ $\dfrac{dV}{dl} = \dfrac{2}{5}(5 - 12l^2)$ *or* $2 - \dfrac{24}{5}l^2$ $\dfrac{d^2V}{dl^2} = -\dfrac{48}{5}l < 0$ (maximum V) $\dfrac{dV}{dl} = 0$ $\dfrac{2}{5}(5 - 12l^2) = 0$ $l^2 = \dfrac{5}{12}$ $l = \sqrt{\dfrac{5}{12}}$ Area of rectangular base $= 4l^2 = 4\left(\dfrac{5}{12}\right) = \dfrac{5}{3}$ *or* $1.67\ m^2$	

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8	In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB. 		
	(a)	Prove that angle ACB is equal to angle ATD.	[2]
	(b)	Using the result from part **(a)**, explain whether a circle can be drawn passing through the points T, A, D and C.	[2]
	(c)	Prove that $CE \times EA = DE \times TE$.	[3]

8	(a)	$\angle ATD = \angle XAB$ (corresponding angles, $AB//TD$) $\angle XAB = \angle ACB$ (Alternate Segment Theorem) $\angle ACB = \angle ATD$ (Proved)	
	(b)	From Part (a), $\angle ACD = \angle ATD$, the properties of circles tell us that angles in the same segment of a circle are equal. Therefore, we can draw a circle passing through points T, A, D and C.	
		$\angle ATE = \angle DCE$ (from Part (a)) $\angle TEA = \angle CED$ (vertically opposite angles) Therefore ΔATE is similar to ΔDCE (AA Similarity test) $\frac{TE}{CE} = \frac{EA}{DE}$ $CE \times EA = DE \times TE$	

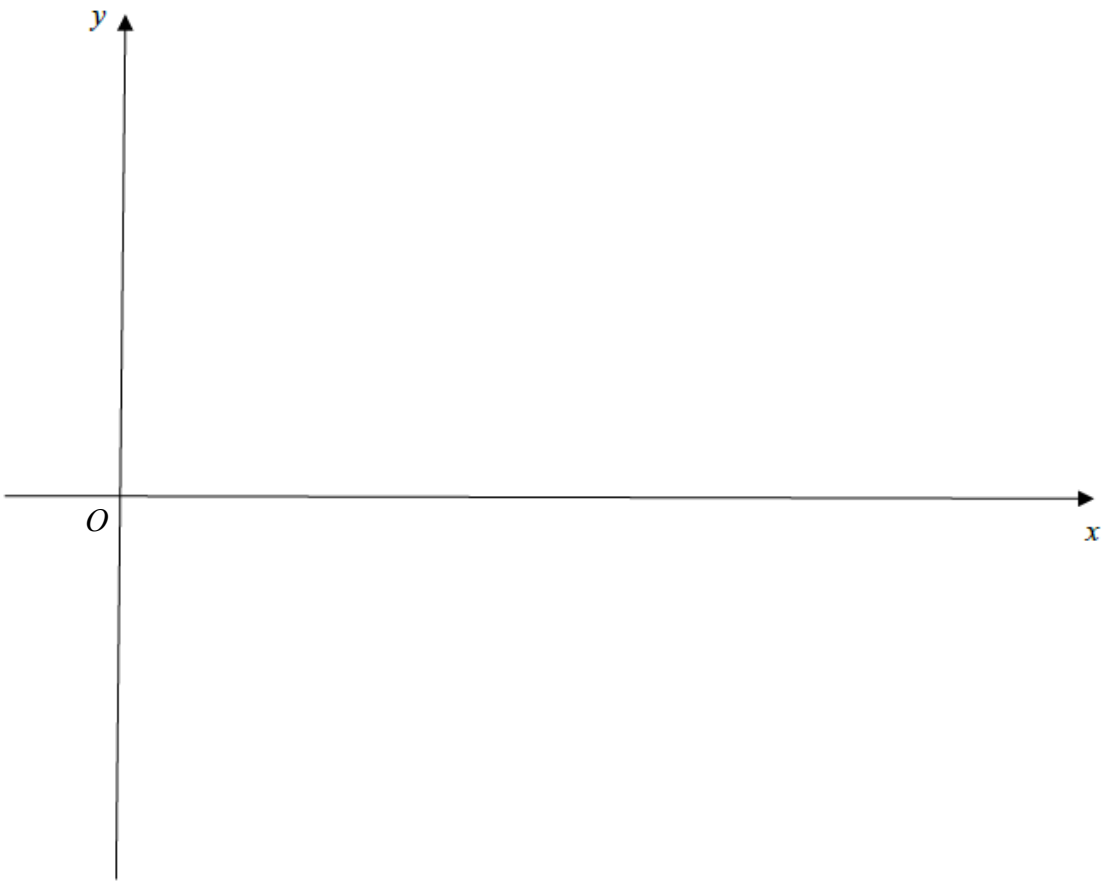
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9	**(a)**	A square has an area $\left(17-8\sqrt{2}\right)\text{cm}^2$. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)\text{cm}$, where a and b are integers. Show that $2b^4-17b^2+16=0$.	[4]
	(b)	[The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.] The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)\text{cm}$ and an area of $\left(3\sqrt{3}-\sqrt{15}\right)\text{cm}^2$. Show that $r=\frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)\text{cm}$, where p and q are integers.	[5]

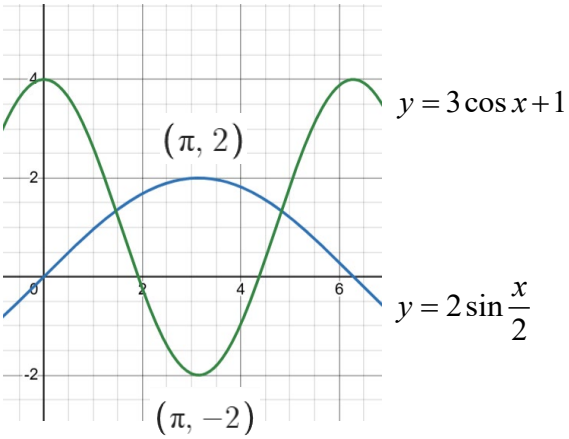
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9	(a)	$\left(a+b\sqrt{2}\right)^2 = 17-8\sqrt{2}$ $a^2+2ab\sqrt{2}+2b^2 = 17-8\sqrt{2}$ $\Rightarrow a^2+2b^2 = 17$ .....(1) $2ab = -8$ $a = -\frac{4}{b}$ .....(2) Substitute (2) into (1): $\left(-\frac{4}{b}\right)^2 + 2b^2 = 17$ $\frac{16}{b^2} + 2b^2 = 17$ $16 + 2b^4 = 17b^2$ $2b^4 - 17b^2 + 16 = 0$ (Shown)	
	(b)	$A = \frac{1}{2}r^2\theta$, $S = r\theta$ $\frac{A}{S} = \frac{r}{2}$ $r = \frac{2A}{S}$ $r = \frac{2\left(3\sqrt{3}-\sqrt{15}\right)}{\sqrt{15}-\sqrt{3}} = \frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ (*showed*) $r = \frac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}} \times \frac{\sqrt{15}+\sqrt{3}}{\sqrt{15}+\sqrt{3}}$ $= \frac{6\sqrt{45}+18-30-2\sqrt{45}}{15-3}$ $= \frac{4\sqrt{45}-12}{12}$ $= \frac{\sqrt{9}\sqrt{5}}{3} - 1$ $= \sqrt{5}-1$	

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10	It is given that $\mathrm{f}(x) = 2\sin\frac{x}{2}$ and $\mathrm{g}(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.		
	(a)	State the period of $\mathrm{f}(x)$.	[1]
	(b)	State the smallest value of $\mathrm{f}(x)$.	[1]
	(c)	State the largest value of $\mathrm{g}(x)$.	[1]
	(d)	Sketch on the same axes, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ for $0 \le x \le 2\pi$. Label your graphs clearly.	[4]
			
	(e)	The solutions to the equation $\mathrm{f}(x) = \mathrm{g}(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$. State, in terms of a and b, the range of values of x for which $\mathrm{f}(x) > \mathrm{g}(x)$.	[1]

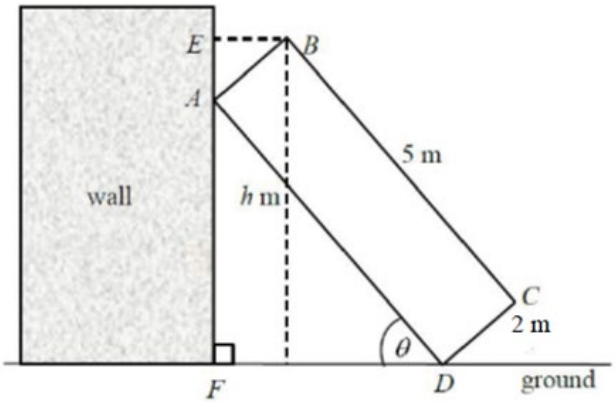
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10	**(a)**	Period of $\mathrm{f}(x) = 720°$ *or* 4π	
	(b)	Smallest value of $\mathrm{f}(x) = 0$	
	(c)	Smallest value of $\mathrm{g}(x) = 4$	
	(d)		
	(e)	$a < x < b$	

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11 The diagram shows a rectangular wooden plank $ABCD$, 5 m by 2 m, leaning against a vertical wall. AD makes an acute angle θ with the horizontal ground.

E is a point on the vertical wall such that EB is parallel to the ground and F is a point on the ground such that AF is perpendicular to FD.
The point B is h m vertically above the ground.



(a) Show that $h = 5\sin\theta + 2\cos\theta$. [2]

(b) Express h in the form $R\sin(\theta + \alpha)$, where $R > 0$, and $0° < \alpha < 90°$. [3]

(c) Find the greatest possible value of h and the value of θ at which it occurs. [2]

(d) Find the value of θ for which $h = \sqrt{15}$ m. [2]

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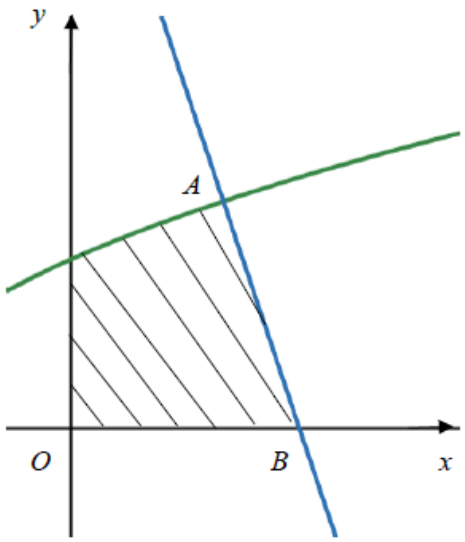
11	(a)	In ΔABE, $\angle EAB = \theta$, $AB = 2\,\text{cm}$, $EA = 2\cos\theta$ In ΔAFD, $\angle ADF = \theta$, $AD = 5\,\text{cm}$, $AF = 5\sin\theta$ $h = AF + EA = 5\sin\theta + 2\cos\theta$ (showed)	
	(b)	$h = 5\sin\theta + 2\cos\theta$ $R = \sqrt{5^2 + 2^2} = \sqrt{29}$ $\alpha = \tan^{-1}\frac{2}{5} = 21.801°$ $h = \sqrt{29}\sin(\theta + 21.8°)$	
	(c)	$h = \sqrt{29}\sin(\theta + 21.8°)$ Greatest possible value of $h = \sqrt{29}$ $\sin(\theta + 21.8°) = 1$ $\theta + 21.8° = 90°$ $\theta = 68.2°$	
	(d)	$\sqrt{29}\sin(\theta + 21.8014°) = \sqrt{15}$ $\sin(\theta + 21.8014°) = \sqrt{\frac{15}{29}}$ $\theta + 21.8014° = 45.98805°$ $\theta = 24.2°$	

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12	**(a)**	By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$.	[1]
	(b)	Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions.	[5]
	(c)	Hence, find $\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,\mathrm{d}x$.	[3]

12	(a)	$$\begin{array}{r} 4 \\ x^3+x^2 \overline{)\,4x^3+5x^2+x-1} \\ \underline{4x^3+4x^2} \\ x^2+x-1 \end{array}$$	
	(b)	$\dfrac{4x^3+5x^2+x-1}{x^2(x+1)} = 4+\dfrac{x^2+x-1}{x^2(x+1)}$ $\dfrac{x^2+x-1}{x^2(x+1)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$ $x^2+x-1 = Ax(x+1)+B(x+1)+Cx^2$ When $x=0,\ B=-1$ When $x=-1,\ C=-1$ When $x=1,\ 2A-2-1=1$ $A=2$ $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)} = 4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}$	
	(c)	$\int \dfrac{4x^3+5x^2+x-1}{x^2(x+1)}\,\mathrm{d}x$ $= \int 4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}\,dx$ $= 4x+2\ln x+\dfrac{1}{x}-\ln(x+1)+c$	

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13	The diagram shows part of the curve $y = \sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B.  		
	(a)	Find the equation of the normal to the curve at A.	[5]
	(b)	Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis.	[5]

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13	(a)	$y=\sqrt{2x+5}$ Coordinates of $A=(2,\,3)$ $\frac{dy}{dx}=\frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)=\frac{1}{\sqrt{2x+5}}$ At A, $\frac{dy}{dx}=\frac{1}{3}$ Equation of normal AB is $y-3=-3(x-2)$ $y=-3x+9$	
	(b)	At $B, y=0$ $-3x+9=0$ $x=3$ $B=(3,\,0)$ Area of shaded region $=\frac{1}{2}(3-2)(3)+\int_0^2\left(\sqrt{2x+5}\right)dx$ $=1.5+\left[\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}\times 2}\right]_0^2$ $=1.5+\left[\left(\frac{9^{\frac{3}{2}}}{3}\right)-\left(\frac{5^{\frac{3}{2}}}{3}\right)\right]$ $=1.5+(5.27322)$ $=6.77\ units^2$	

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