Class	Index Number

Name : _____

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

Tuesday
26 August 2025

ADDITIONAL MATHEMATICS
Paper 1

4049/01
2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

This question paper consists of 23 printed pages and 1 blank page.

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Downloaded from SingaKids.com

1 A home-based sticker company that prints stickers has calculated its profit, \$$P$, for each order using the equation $P = 120x - 40 - 30x^2$, where x is the number of stickers produced in **hundreds**.

(a) Express $P = 120x - 40 - 30x^2$ in the form $P = a(x-b)^2 + c$, where a, b and c are constants. [2]

$$
\begin{aligned}
P &= 120x - 40 - 30x^2 \\
&= -30(x^2 - 4x) - 40 \\
&= -30[x^2 - 4x + (-2)^2 - (-2)^2] - 40 \\
&= -30[x - 2]^2 + 80
\end{aligned}
$$

(b) Using your answer in **(a)**, explain clearly if the company should accept an order for printing 400 stickers. [2]

No. As he will be making a loss of \$40 if he accept the order.

Downloaded from SingaKids.com

2 Find the exact coordinates of the stationary point on the curve $y = \frac{e^{2x}}{3x^2}$, $x \neq 0$. [6]

$$\frac{dy}{dx} = \frac{(3x^2)(e^{2x})(2) - (e^{2x})(6x)}{(3x^2)^2}$$

$$= \frac{6xe^{2x}(x-1)}{9x^4}$$

$$= \frac{2e^{2x}(x-1)}{3x^3}$$

$$0 = \frac{2e^{2x}(x-1)}{3x^3}$$

$2e^{2x} > 0, x \neq 0$

$x - 1 = 0$

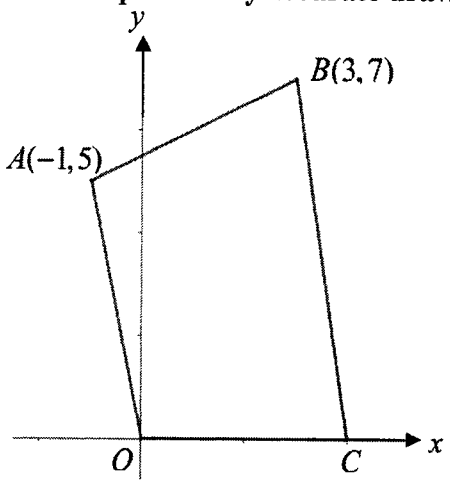
$x = 1$

$y = \frac{e^2}{3}$

Stationary point = $(1, \frac{e^2}{3})$

Downloaded from SingaKids.com

3 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral $OABC$ where O is the origin. The point A is $(-1,5)$ and the point B is (3, 7). The perpendicular bisector of AB meets the x-axis at C.

(a) Find the coordinates of C. [4]

Gradient of $AB = \frac{7-5}{3+1} = \frac{1}{2}$

Gradient of Perpendicular to AB = -2

Midpoint of AB $= \left(\frac{-1+3}{2}, \frac{5+7}{2}\right) = (1,6)$

Equation of perpendicular bisector of AB:

$y-6=-2(x-1)$

$y=-2x+8$ [M1]

At x-axis, $y=0$

$0=-2x+8$

$x=4$

Coordinates of $C = (4, 0)$

(b) Find the area of the quadrilateral $OABC$. [2]

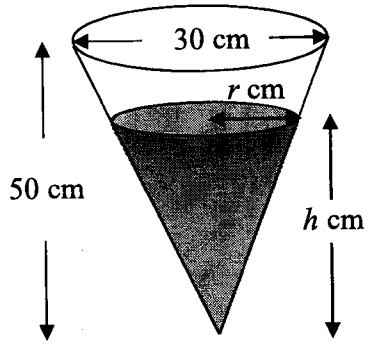
$$\text{Area} = \frac{1}{2}\left|\begin{matrix} 0 & 4 & 3 & -1 & 0 \\ 0 & 0 & 7 & 5 & 0 \end{matrix}\right\|$$

$$= \frac{1}{2}[(0+28+15+0)-(0-7+0+)]$$

$= 25$ units²

Downloaded from SingaKids.com

4 The diagram shows an inverted right circular cone with a diameter, 30 cm and height, 50 cm.
The cone was initially empty. It is being filled with water at a constant rate of 100 cm³/s. The surface of the water remains horizontal as it fills. At time t seconds, the radius of the horizontal water surface area is r cm and the depth of the water is h cm.



(a) Express r in terms of h. [2]

$$\frac{r}{15} = \frac{h}{50}$$
$$r = \frac{3}{10}h$$

(b) Show that the volume of water, V cm³, in the cone is $V = \frac{3\pi h^3}{100}$. [1]

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{3}{10}h\right)^2 h$$
$$V = \frac{3\pi h^3}{100}$$

Downloaded from SingaKids.com

(c) Find the rate of change of the depth of the water when $h = 35$ cm. [3]

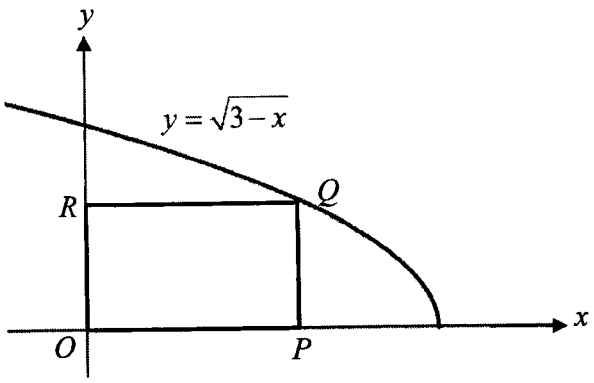
$$\frac{dV}{dh} = \frac{9}{100}\pi h^2$$
$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

When $h = 35$ cm,

$$100 = \frac{9}{100}\pi(35)^2 \times \frac{dh}{dt}$$
$$\frac{dh}{dt} = 0.288716 = 0.289 \ cm/s (3\text{sf})$$

Downloaded from SingaKids.com

5 The diagram shows part of the graph of $y = \sqrt{3-x}$, $x < 3$. O is the origin and points P and R lie on the x and y axes respectively. Point Q is a point on the curve such that $OPQR$ forms a rectangle.



(a) Given that the x-coordinate of P is h, write down an expression for the area of the rectangle, A units^2, in terms of h and show that $\frac{dA}{dh} = \frac{6-3h}{2\sqrt{3-h}}$. [3]

When $x = h$, $y = \sqrt{3-h}$

$A = h\sqrt{3-h}$

$$\frac{dA}{dh} = (h)(\frac{1}{2})(3-h)^{-\frac{1}{2}}(-1) + (3-h)^{\frac{1}{2}}(1)$$

$$= \frac{-h}{2\sqrt{3-h}} + \sqrt{3-h}$$

$$= = \frac{-h+2(3-h)}{2\sqrt{3-h}}$$

$$\frac{dA}{dh} = \frac{6-3h}{2\sqrt{3-h}}$$

Downloaded from SingaKids.com

(b) Find the stationary value of A and determine the nature of this stationary value. [4]

$$\frac{dA}{dh} = \frac{6-3h}{2\sqrt{3-h}} = 0$$

$$6-3h = 0$$
$$h = 2$$

$$A = 2\sqrt{3-2} = 2 \text{ units}^2$$

h	Slightly less than 2	2	Slightly more than 2
$\frac{dA}{dh}$	+	0	-
Shape	/	-	\

Or

$$\frac{d^2A}{dh^2} = \frac{2\sqrt{3-h}(-3) - (6-3h)(\frac{1}{2})(3-h)^{-1/2}(-1)}{4(3-h)} = -\frac{3}{2}$$

At $h = 2$, $\frac{d^2A}{dh^2} < 0$

Therefore Stationary value $A = 2$ is a maximum value.

Downloaded from SingaKids.com

6 **(a)** The expansion of $(1+kx)^n$ in ascending powers of x is $1+\frac{7}{3}x+\frac{7}{3}x^2+....$

Show that $k=\frac{1}{3}$ and calculate the value of n. [4]

$$(1+kx)^n = 1+\binom{n}{1}(kx)+\binom{n}{2}(kx)^2+...$$

$$\binom{n}{1}k=\frac{7}{3} \qquad \binom{n}{2}k^2=\frac{7}{3}$$

$$nk=\frac{7}{3} \qquad \frac{n(n-1)}{2}k^2=\frac{7}{3}$$

$$k=\frac{7}{3n}$$

$$\frac{n(n-1)}{2}\left(\frac{7}{3n}\right)^2=\frac{7}{3}$$

$$\frac{n(n-1)}{2}\left(\frac{7}{3n^2}\right)=1$$

$$\frac{7(n-1)}{6n}=1$$

$$7n-7=6n$$

$$n=7$$

$$k=\frac{7}{3(7)}=\frac{1}{3} \text{ (shown)}$$

Downloaded from SingaKids.com

(b) Explain why all terms in the expansion of $\left(px^2-\frac{1}{x^4}\right)^{12}$ contain only even powers of x. [4]

General Term $=\binom{12}{r}\left(px^2\right)^{12-r}\left(-\frac{1}{x^4}\right)^r$

$=\binom{12}{r}p^{12-r}x^{24-2r}(-1)^r\left(x^{-4}\right)^r$

$=\binom{12}{r}p^{12-r}(-1)^r x^{24-6r}$

Power of x: $24-6r=2(12-3r)$

Therefore all powers of x is 2(12-3r), all terms contain only even powers of x.

Downloaded from SingaKids.com

7 **(a)** Find all the angles between $-\pi$ and π that satisfy the equation $2\cos 2x = 4\sin x + 3$. [3]

$2\cos 2x = 4\sin x + 3$

$2(1-2\sin^2 x) = 4\sin x + 3$

$2 - 4\sin^2 x - 4\sin x - 3 = 0$

$4\sin^2 x + 4\sin x + 1 = 0$

$(2\sin x + 1)^2 = 0$

$\sin x = -\frac{1}{2}$

$\alpha = \frac{\pi}{6}$ $x = -\frac{\pi}{6}, -\frac{5\pi}{6}$ [Accept -0.524 and -2.62 rad]

Downloaded from SingaKids.com

(b) Solve the equation $3\text{cosec}^2 x - \text{cosec}\, x = 2 - \cot^2 x$ for $0° \le x \le 360°$. [5]

$3\text{cosec}^2 x - \text{cosec}\, x = 2 - \cot^2 x$

$3\text{cosec}^2 x - \text{cosec}\, x = 2 - (\text{cosec}^2 x - 1)$

$4\text{cosec}^2 x - \text{cosec}\, x - 3 = 0$

$(4\text{cosec}x + 3)(\text{cosec}x - 1) = 0$

$4\text{cosec}x + 3 = 0$ $\quad$ $\text{cosec}x - 1 = 0$

$\text{cosec}x = -\frac{3}{4}$ $\quad$ $\text{cosec}x = 1$

$\sin x = -\frac{4}{3}$ (rej) $\quad$ $\sin x = 1$

$x = 90°$

Alternate Solution:

$$\frac{3}{\sin^2 x} - \frac{1}{\sin x} = 2 - \frac{\cos^2 x}{\sin^2 x}$$

$3 - \sin x = 2\sin^2 x - \cos^2 x$

$3 - \sin x = 2\sin^2 x - (1 - \sin^2 x)$

$3\sin^2 x + \sin x - 4 = 0$

$(3\sin x + 4)(\sin x - 1) = 0$

$\sin x = -\frac{4}{3}$ *(NA)* *or* $\sin x = 1$

$x = 90°$

Downloaded from SingaKids.com

8 **(a)** Express $\dfrac{1-6x}{(x+2)(3x^2+1)}$ in the form of $\dfrac{a}{x+2}+\dfrac{bx+c}{3x^2+1}$, where a, b and c are integers. [4]

$$\frac{1-6x}{(x+2)(3x^2+1)}=\frac{a}{x+2}+\frac{bx+c}{3x^2+1}$$

$$1-6x=a(3x^2+1)+(bx+c)(x+2)$$

When $x=-2$, $a=1$

When $x=0$,

$1=1(1)+(c)(3)$

$c=0$

When $x=1$,

$1-6=1(3+1)+(b)(1+2)$

$3b=-9$

$b=-3$

$$\frac{1-6x}{(x+2)(3x^2+1)}=\frac{1}{x+2}-\frac{3x}{3x^2+1}$$

(b) Differentiate $\ln(3x^2+1)$ with respect to x. [2]

$$\frac{d}{dx}\ln(3x^2+1)=\frac{6x}{3x^2+1}$$

Downloaded from SingaKids.com

(c) Using your answer to **parts (a)** and **(b)**, show that

$$\int_0^2 \frac{1-6x}{(x+2)(3x^2+1)}\,dx = \ln 2 - \frac{1}{2}\ln 13.$$ [4]

$$\int_0^2 \frac{1-6x}{(x+2)(3x^2+1)}\,dx = \int_0^2 \frac{1}{x+2} - \frac{3x}{3x^2+1}\,dx$$

$$= \int_0^2 \frac{1}{x+2}dx - \int_0^2 \frac{3x}{3x^2+1}\,dx$$

$$= \left[\ln(x+2)\right]_0^2 - \frac{1}{2}\int_0^2 \frac{6x}{3x^2+1}\,dx$$

$$= \ln(4) - \ln(2) - \frac{1}{2}\left[\ln(3x^2+1)\right]_0^2$$

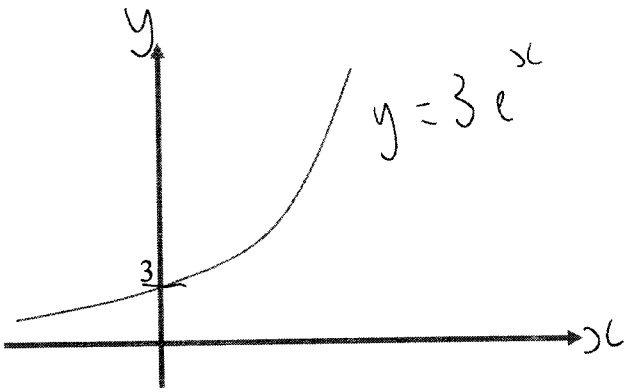
$$= \ln(4) - \ln(2) - \frac{1}{2}\left[\ln 13 - \ln 1\right]$$

$$= \ln\left(\frac{4}{2}\right) - \frac{1}{2}\ln 13$$

$$= \ln 2 - \frac{1}{2}\ln 13 \text{ (shown)}$$

Downloaded from SingaKids.com

9 **(a)** **(i)** Sketch the graph of $y = 3e^x$. [1]



(ii) Find the equation of a suitable straight line that can be inserted in **part (a)(i)** to solve the equation $6e^x - x = 6$. [2]

$6e^x = 6 + x$

$3e^x = 3 + \frac{1}{2}x$

Insert: $y = 3 + \frac{1}{2}x$

Downloaded from SingaKids.com

(b) **(i)** Without using a calculator, show that $\tan 75° = a + b\sqrt{3}$, where a and b are integers. [3]

$$\begin{aligned}
\tan 75° &= \tan(30° + 45°) \\
&= \frac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°} \\
&= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \\
&= \left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) \div \left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \\
&= \left(\frac{1+\sqrt{3}}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{3}-1}\right) \\
&= \frac{1+\sqrt{3}}{\sqrt{3}-1} \\
&= \frac{\left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right)}{3-1} \\
&= \frac{4+2\sqrt{3}}{2} \\
&= 2+\sqrt{3}
\end{aligned}$$

(ii) Hence, find the exact value of $\cot^2 75°$. [3]

$$\begin{aligned}
\cot^2 75° &= \frac{1}{\tan^2 75°} \\
&= \frac{1}{\left(2+\sqrt{3}\right)^2} \\
&= \frac{1}{4+4\sqrt{3}+3} \\
&= \frac{1}{7+4\sqrt{3}} \\
&= \frac{7-4\sqrt{3}}{49-48} \\
&= 7-4\sqrt{3}
\end{aligned}$$

Downloaded from SingaKids.com

10 The maximum height that a roller coaster can reach is 90 meters above sea level, and the lowest possible height is 10 meters above sea level. The roller coaster takes 10 seconds to move from its highest position to its lowest position.
The height, h m, of the roller coaster above sea level can be modelled by the function $h = a\cos\left(\frac{\pi}{b}t\right)+c$, where t is the time in seconds.

(a) Show that $h = 40\cos\left(\frac{\pi}{10}t\right)+50$. [3]

Period: $\frac{2\pi}{\frac{\pi}{b}} = 20$

$b = 10$

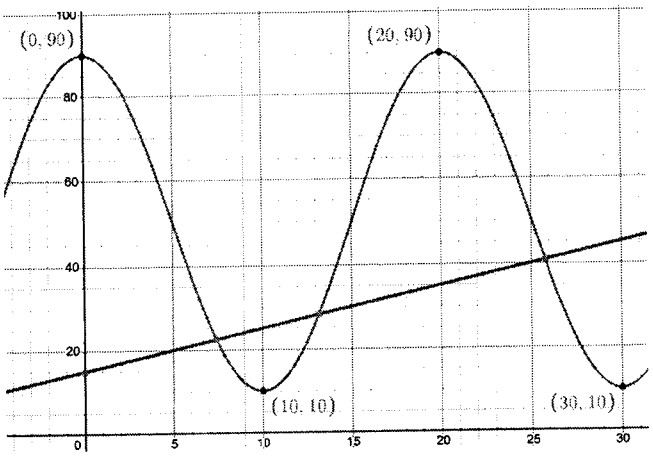
Max $h = 90$
Min $h = 10$

$a + c = 90$
$-a + c = 10$
$2c = 100$
$c = 50$

$a = 90 - 50 = 40$ or $a = \frac{90-10}{2} = 40$

$\therefore h = 40\cos\left(\frac{\pi}{10}t\right)+50$

(b) Sketch the graph of $h = 40\cos\left(\frac{\pi}{10}t\right)+50$ for $0 \le t \le 30$. [2]



Downloaded from SingaKids.com

(c) Find the time when the roller coaster first reaches a height of 62 m. [2]

$$62 = 40\cos\left(\frac{\pi}{10}t\right)+50$$
$$40\cos\left(\frac{\pi}{10}t\right)=12$$
$$\cos\left(\frac{\pi}{10}t\right)=\frac{12}{40}$$
$$\alpha = 1.26610$$
$$\frac{\pi}{10}t = 1.26610$$
$$t = 4.03\text{s} \quad (3\text{sf})$$

(d) **(i)** On your graph in **part (b)**, draw the graph of $h = t+15$. [1]

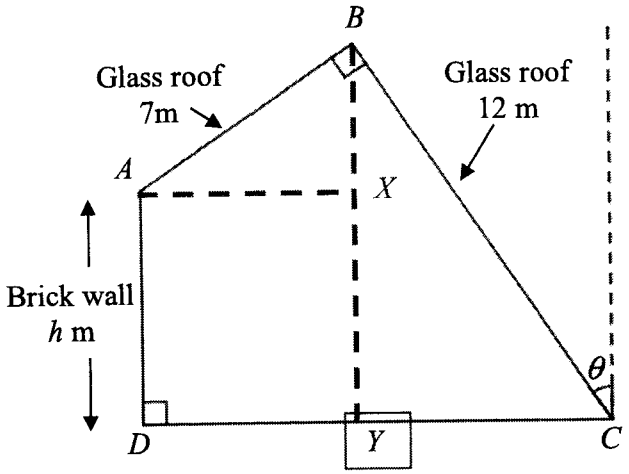
(ii) State for the interval $0 \le t \le 30$, the number of solutions of $t = 40\cos\left(\frac{\pi}{10}t\right)+35$. [1]

$$t = 40\cos\left(\frac{\pi}{10}t\right)+35$$
$$t+15 = 40\cos\left(\frac{\pi}{10}t\right)+50$$

3 solutions

Downloaded from SingaKids.com

11 An architect is designing a building, *ABCD*, using the model shown below.
To allow more natural light into the building, a 12 m glass roof, *BC*, is installed at an angle of θ to the vertical. Another glass roof, *AB*, of 7 m, is installed such that it is perpendicular to *BC*. The vertical height of the brick wall, *AD*, is given as *h* m.



(a) Show that $h = 12\cos\theta - 7\sin\theta$. [2]

$$\sin\theta = \frac{BX}{7}$$
$$BX = 7\sin\theta$$

$$\cos\theta = \frac{BY}{12}$$
$$BY = 12\cos\theta$$

$$h = BY = BX$$
$$= 12\cos\theta - 7\sin\theta$$

(b) Express *h* in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

$$R = \sqrt{12^2 + 7^2} = \sqrt{193}$$
$$\alpha = \tan^{-1}\left(\frac{7}{12}\right) = 0.52807$$
$$h = \sqrt{193}\cos(\theta + 0.528)$$

Downloaded from SingaKids.com

(c) Find the value of θ for which $h = 8\,\text{m}$. [2]

$\sqrt{193}\cos(\theta + 0.52807) = 8$

$\cos(\theta + 0.52807) = \dfrac{8}{\sqrt{193}}$

Basic angle $= 0.95714$

$\theta + 0.52807 = 0.95714$

$\theta = 0.429\,(3\text{sf})$

(d) The building owner makes a request to have the brick wall to have a height of at least 14 m. Is the architect able to meet the building owner's request? Explain your answer. [2]

$\sqrt{193}\cos(\theta + 0.52807) \geq 14$

$\cos(\theta + 0.52807) \geq \dfrac{14}{\sqrt{193}}$

$\cos(\theta + 0.52807) \geq 1.0077$

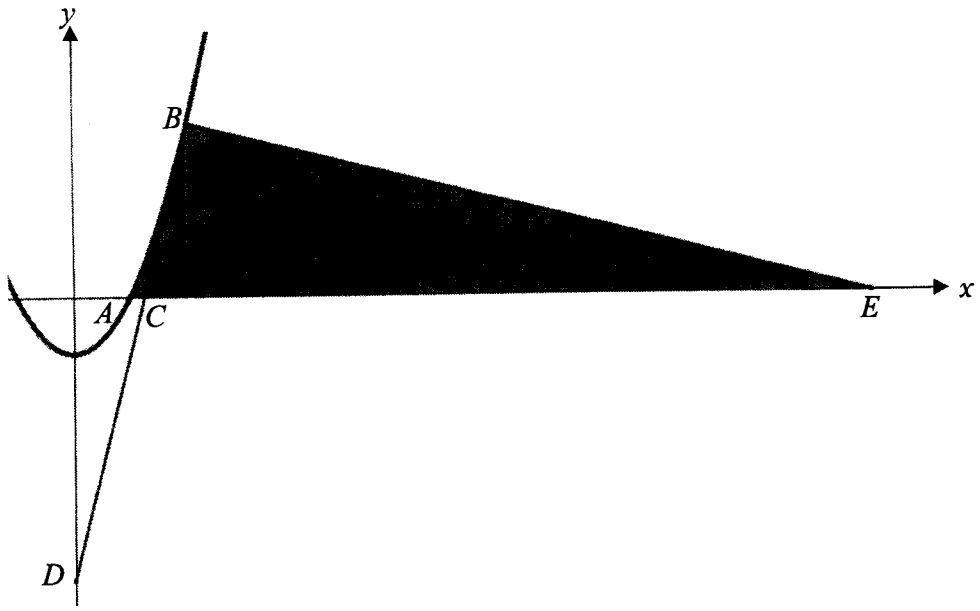
Since $-1 \leq \cos(\theta + 0.52807) \leq 1$, Not possible.

Alternate Solution:

Since max $h = \sqrt{193} = 13.9 < 14$, it is not possible.

Downloaded from SingaKids.com

12 The diagram shows part of the curve $y = x^2 - 1$, cutting the x-axis at $A(1, 0)$. The tangent at point B on the curve cuts the x-axis at $C\left(\frac{5}{4}, 0\right)$ and y-axis at $D(0, -5)$ respectively. The normal at B meets the x-axis at E.



(a) Find the coordinates of B. [3]

Gradient of $BD = \dfrac{0-(-5)}{\frac{5}{4}-0} = 4$

$y = x^2 - 1$

$\dfrac{dy}{dx} = 2x$

At B, gradient = 4

$2x = 4$

$x = 2$

$y = (2)^2 - 1 = 3$

Coordinates of B = (2, 3)

Downloaded from SingaKids.com

(b) Find the coordinates of E. [2]

Gradient of $BE = -\frac{1}{4}$

Equation of normal:

$y-3=-\frac{1}{4}(x-2)$

$y=-\frac{1}{4}x+\frac{7}{2}$

At x-axis, $y=0$

$0=-\frac{1}{4}x+\frac{7}{2}$

$x=14$

Coordinates of E = (14, 0)

(c) Find the area of the shaded region. [3]

$$\text{Area} = \int_1^2 \left(x^2-1\right)dx+\frac{1}{2}(12)(3)$$

$$=\left[\frac{x^3}{3}-x\right]_1^2+18$$

$$=19\frac{1}{3}\text{ units}^2$$

END OF PAPER.

Downloaded from SingaKids.com

BLANK PAGE

Downloaded from SingaKids.com

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

Wednesday 27 August 2025

ADDITIONAL MATHEMATICS
PAPER 2

4049/02

2 hours 15 mins

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

90

This question paper consists of 22 printed pages and 2 blank pages.

Downloaded from SingaKids.com

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

Downloaded from SingaKids.com

1 A piece of raw meat is accidentally left out at room temperature, and bacteria begin to grow on it. Initially, there are 500 bacteria presents, and the number of bacteria doubles every 3 hours. The number of bacteria, N, after t hours can be modelled by the formula $N = 500(2)^{kt}$.

(i) Show that $k = \frac{1}{3}$. [2]

At $t = 3$,

$$1000 = 500(2)^{3k}$$
$$2 = (2)^{3k}$$
$$3k = 1$$
$$k = \frac{1}{3}$$

(ii) It is considered unsafe for consumption when the number of bacteria on the meat exceeds 3 000. Determine the maximum number of hours that it will still be safe to consume the meat.

[3]

$$N \leq 3000$$
$$500(2)^{\frac{t}{3}} \leq 3000$$
$$(2)^{\frac{t}{3}} \leq 6$$
$$\frac{t}{3} \leq \frac{\lg 6}{\lg 2}$$
$$t \leq 7.75$$

Maximum number of hours that it will still be safe to consume = 7 hours

Downloaded from SingaKids.com

2 **(a)** Given that $y = x^3 - x$, find the range of values of x such that $x\frac{d^2y}{dx^2} + \frac{dy}{dx} > 0$. [4]

$$y = x^3 - x$$

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\frac{d^2y}{dx^2} = 6x$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x(6x) + 3x^2 - 1$$

$$= 9x^2 - 1$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} > 0$$

$$9x^2 - 1 > 0$$

$$(3x - 1)(3x + 1) > 0$$

$$x < -\frac{1}{3} \text{ or } x > \frac{1}{3}$$

Downloaded from SingaKids.com

2 **(b)** A curve is such that $\frac{d^2y}{dx^2} = \frac{1}{3\sqrt{2x-1}}$. Given that the curve has a gradient of 3 at the point $(1,6)$, find the equation of the curve. [4]

$$\frac{d^2y}{dx^2} = \frac{1}{3\sqrt{2x-1}}$$
$$= \frac{1}{3}(2x-1)^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \int \frac{1}{3}(2x-1)^{-\frac{1}{2}}\,dx$$
$$= \frac{1}{3}\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2}(2)} + c \quad , c \text{ is an arbitrary constant}$$
$$= \frac{1}{3}\sqrt{(2x-1)} + c$$

When $x = 1$, $\frac{dy}{dx} = 3$,

$$3 = \frac{1}{3} + c$$
$$c = \frac{8}{3}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{3}\sqrt{(2x-1)} + \frac{8}{3}$$

$$y = \int \left[\frac{1}{3}\sqrt{(2x-1)} + \frac{8}{3}\right] dx$$
$$= \frac{1}{3}\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}(2)} + \frac{8}{3}x + c_1 \quad c_1 \text{ is an arbitrary constant}$$

$$y = \frac{(2x-1)^{\frac{3}{2}}}{9} + \frac{8}{3}x + c_1$$

At $(1,6)$,

$$6 = \frac{1}{9} + \frac{8}{3} + c_1$$
$$c_1 = \frac{29}{9}$$

$$\therefore \quad y = \frac{1}{9}(2x-1)^{\frac{3}{2}} + \frac{8}{3}x + \frac{29}{9}$$

Downloaded from SingaKids.com

3 A circle C_1 passes through $A(-1,-3)$. Its centre, B, lies on the line $4y+3x=10$. AB is perpendicular to $4y+3x=10$.

(a) Find the equation of the circle. [5]

$$y+3=\frac{4}{3}(x+1)$$
$$y=\frac{4}{3}x-\frac{5}{3}$$

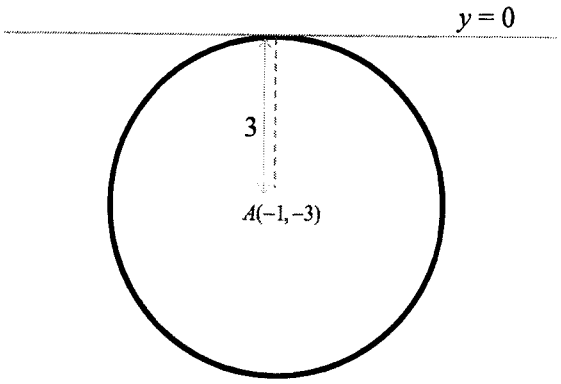
$$3y-4x=-5\cdots(\text{i})$$
$$4y+3x=10\cdots(\text{ii})$$
$$9y-12x=-15\cdots(\text{i})$$
$$16y+12x=40\cdots(\text{ii})$$
$$25y=25$$
$$y=1$$
$$x=2$$

Centre (2, 1) and radius $\sqrt{(2+1)^2+(1+3)^2}=5$

Equation of circle is $(x-2)^2+(y-1)^2=25$

Downloaded from SingaKids.com

3 **(b)** A second circle C_2 has centre at $A(-1,-3)$. The line $y=0$ is a tangent to circle C_2. Find the equation of the circle in the form of $x^2+y^2+px+qy+r=0$, where p, q and r are integers. [2]

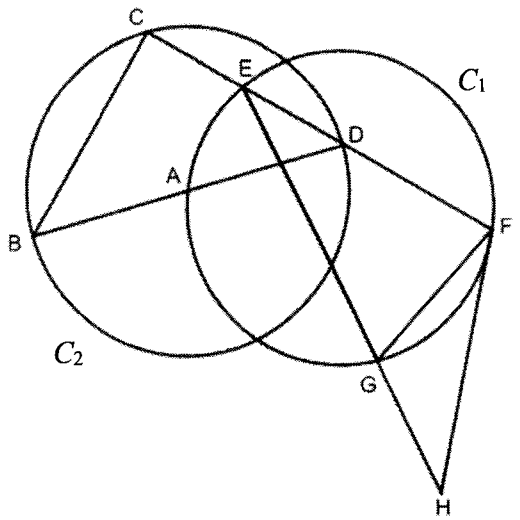


radius of $C_2 = 3$

$$(x+1)^2+(y+3)^2=3^2$$
$$x^2+2x+1+y^2+6y+9=9$$
$$x^2+y^2+2x+6y+1=0$$

Downloaded from SingaKids.com

4



The diagram shows two intersecting circles C_1 and C_2. The line CF intersects C_1 at E, and C_2 at D, such that $CE = ED$. BAD is a straight line, where A is the centre of C_2. The tangent to C_1 at F meets EG produced at H.

Stating your reasons clearly, prove that

(i) ΔGFH and ΔFEH are similar, [2]

$\angle GHF = \angle FHE$ (Common)

$\angle HFG = \angle HEF$ (alternate segment or tangent-chord theorem)

Hence ΔGFH and ΔFEH are similar

Downloaded from SingaKids.com

4 **(ii)** $HF^2 = HE \times GH$, [2]

Since ΔGFH and ΔFEH are similar,

$$\frac{HF}{HE} = \frac{GH}{HF}$$
$$(HF)^2 = HE \times GH$$

(iii) $AF^2 = AE^2 + EF^2$ [3]

In ΔAED and ΔBCD,
A and E are midpoints of BD and CD respectively.

By mid-point theorem,

$$AE // BC \text{ and } AE = \frac{1}{2}BC$$
$$\angle BCD = 90° (\text{right } \angle \text{ in semicircle})$$
$$\angle AEF = 90° (\text{corresponding } \angle \text{s})$$

Therefore, by Pythagoras Theorem,

$$AF^2 = AE^2 + EF^2$$

Alternative method:

$$\frac{CE}{CD} = \frac{1}{2} (\text{given})$$
$$\frac{BA}{BD} = \frac{1}{2} (A \text{ is the centre})$$
$$\angle EDA = \angle CDB (\text{common})$$
$$\therefore \Delta EDA \text{ and } \Delta CDB \text{ are similar}$$
$$\angle BCD = 90° (\text{rt } \angle \text{in semicircle})$$
$$\angle AED = 90°$$

Therefore, by Pythagoras Theorem,

$$AF^2 = AE^2 + EF^2$$

Downloaded from SingaKids.com

5 **(a)** Show that $6\log_4 x - \log_2 y = 3$ can be expressed as $y = ax^n$, where a and n are constants to be determined. [4]

$$6\log_4 x - \log_2 y = 3$$
$$6\frac{\log_2 x}{\log_2 4} - \log_2 y = 3$$
$$\frac{6}{2}\log_2 x - \log_2 y = 3$$
$$\log_2 x^3 - \log_2 y = 3$$
$$\log_2 \frac{x^3}{y} = 3$$
$$\frac{x^3}{y} = 8$$
$$y = \frac{x^3}{8}$$
$$a = \frac{1}{8}, n = 3$$

(b) Solve the equation $3e^y - 5 = 2e^{-y}$, giving values of y in logarithmic form. [4]

$$3e^y - 5 = 2e^{-y}$$
$$\text{let } p = e^y$$
$$3p - 5 = \frac{2}{p}$$
$$3p^2 - 5p - 2 = 0$$
$$(p-2)(3p+1) = 0$$

$e^y = 2$ or $e^y = -\frac{1}{3}$ (NA)

$y = \ln 2$

Downloaded from SingaKids.com

6 **(a)** Find the range of values of m such that $(m-6)x^2-8x+m>0$ for all values of x. [4]

$m-6>0$

$m>6$

$b^2-4ac<0$

$(-8)^2-4(m-6)(m)<0$

$64-4m^2+24m<0$

$m^2-6m-16>0$

$(m-8)(m+2)>0$

$m>8$ or $m<-2$

Since $m>6$, $m>8$

(b) Given that the quadratic equation $px^2-2(p-q)x-(r-p)=0$ has equal roots, prove that $p=\dfrac{q^2}{2q-r}$. [4]

$b^2-4ac=0$

$[-2(p-q)]^2-4p[-(r-p)]=0$

$4(p-q)^2+4p(r-p)=0$

$4(p^2-2pq+q^2)+4pr-4p^2=0$

$4p^2-8pq+4q^2+4pr-4p^2=0$

$pr-2pq+q^2=0$

$q^2=2pq-pr$

$=p(2q-r)$

$p=\dfrac{q^2}{2q-r}$

Downloaded from SingaKids.com

7 A particle moves in a straight line so that, t seconds after passing through a fixed point O, its velocity, v m/s, is given by $v = k\cos 2t$, where k is a constant. When $t = \frac{\pi}{12}$, the acceleration is -6 m/s^2.

(a) Find the value of k. [2]

$$v = k\cos 2t$$
$$a = \frac{dv}{dt}$$
$$= -2k\sin 2t$$
$$t = \frac{\pi}{12},\ a = -6$$
$$-6 = -2k\sin 2\left(\frac{\pi}{12}\right)$$
When $$-3 = -k\sin\left(\frac{\pi}{6}\right)$$
$$3 = k\left(\frac{1}{2}\right)$$
$$k = 6$$

(b) The particle is first at rest at point P. Find the value of t when the particle is at point P. [2]

At rest $\Rightarrow v = 0$

$$v = 6\cos 2t$$
$$6\cos 2t = 0$$
$$\cos 2t = 0$$
$$2t = \frac{\pi}{2}$$
$$t = \frac{\pi}{4}$$

Downloaded from SingaKids.com

7 **(c)** Find the distance travelled by the particle from O to P. [3]

Distance from O to P =

$$\int_0^{\frac{\pi}{4}} 6\cos 2t \ \mathrm{d}t$$

$$= \left[\frac{6\sin 2t}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 3\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= 3\text{m}$$

Alternative:

$$s = \int 6\cos 2t \ \mathrm{d}t$$

$$= \frac{6\sin 2t}{2} + c$$

At $t = 0$, $s = 0$

$$\frac{6\sin 2(0)}{2} + c = 0$$

$$c = 0$$

At $t = \frac{\pi}{4}$,

$$s = \frac{6\sin 2\left(\frac{\pi}{4}\right)}{2} = 3$$

Downloaded from SingaKids.com

8 The equation of the curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x+4)\sqrt{2x-3}$, $x > \frac{3}{2}$.

(a) Show that $\mathrm{f}'(x) = \frac{px+q}{\sqrt{2x-3}}$, where p and q are integers. [3]

$$\mathrm{f}'(x) = (x+4)\frac{1}{2}(2x-3)^{-\frac{1}{2}}(2) + \sqrt{2x-3}$$
$$= \frac{x+4}{\sqrt{2x-3}} + \frac{2x-3}{\sqrt{2x-3}}$$
$$= \frac{3x+1}{\sqrt{2x-3}}$$

Downloaded from SingaKids.com

8 **(b)** Find the x-coordinates of the points where the gradient of the normal is $-\frac{1}{7}$. [5]

gradient of the normal is $-\frac{1}{7}$

gradient of the tangent is 7

$$\frac{3x+1}{\sqrt{2x-3}} = 7$$

$$3x+1 = 7\sqrt{2x-3}$$

$$(3x+1)^2 = 49(2x-3)$$

$$9x^2+6x+1 = 98x-147$$

$$9x^2-92x+148 = 0$$

$$x = \frac{92 \pm \sqrt{(92)^2-4(9)(148)}}{2(9)}$$

$$x = \frac{74}{9} \quad \text{or} \quad x = 2$$

(c) Determine whether f is an increasing or a decreasing function. Explain your answer. [2]

$$f'(x) = \frac{3x+1}{\sqrt{2x-3}}$$

$$x > \frac{3}{2}$$

$$3x+1 > 0$$

$$f'(x) > 0$$

Hence, f is an increasing function.

Downloaded from SingaKids.com

9 **(a)** Prove that $\cot x + \dfrac{\sin x}{1+\cos x} = \operatorname{cosec} x$. [3]

$$\begin{aligned}
\cot x + \frac{\sin x}{1+\cos x} &= \frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x} \\
&= \frac{\cos x(1+\cos x)+\sin^2 x}{(\sin x)(1+\cos x)} \\
&= \frac{\cos x+\cos^2 x+\sin^2 x}{(\sin x)(1+\cos x)} \\
&= \frac{\cos x+1}{(\sin x)(1+\cos x)} \\
&= \frac{1}{(\sin x)} \\
&= \cos ecx
\end{aligned}$$

Downloaded from SingaKids.com

9 **(b)** Hence solve $\cot x + \dfrac{\sin x}{1+\cos x} = 4\cos x$, for $0 \le x \le 2\pi$. [4]

$$\cos ecx = 4\cos x$$
$$\frac{1}{\sin x} = 4\cos x$$
$$1 = 4\sin x\cos x \qquad 0 \le 2x \le 4\pi$$
$$2\sin x\cos x = \frac{1}{2}$$
$$\sin 2x = \frac{1}{2}$$

$$\alpha = \frac{\pi}{6}$$
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

(c) Using the result in part **(a)**, find the set of values of the k, such that the equation $\cot x + \dfrac{\sin x}{1+\cos x} = \dfrac{1}{k}$, $k \neq 0$has solutions. [2]

$$\cos ecx = \frac{1}{k}, \; k \neq 0$$
$$\sin x = k$$
$$-1 \le k \le 1$$

Downloaded from SingaKids.com

10 **(i)** The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is exactly divisible by $x+2$ and leaves a remainder of 4 when divided by $x+1$.

Find the value of a and of b. [4]

$$f(x) = ax^3 + 4x^2 + bx - 2$$

$$f(-2) = 0$$

$$-8a + 4(4) - 2b - 2 = 0$$

$$8a + 2b = 14$$

$$4a + b = 7 \cdots\cdots\cdots (1)$$

$$f(-1) = 4$$

$$-a + 4 - b - 2 = 4$$

$$-a - b = 2 \cdots\cdots\cdots (2)$$

$(1)+(2)$

$$3a = 9$$

$$a = 3$$

$$4(3) + b = 7$$

$$b = -5$$

Downloaded from SingaKids.com

10 **(ii)** Factorise $f(x)$ completely. [3]

$$f(x) = 3x^3 + 4x^2 - 5x - 2$$
$$= (x+2)(3x^2 + bx - 1)$$

By comparing,

$$4 = 6 + b$$
$$b = -2$$

$$f(x) = 3x^3 + 4x^2 - 5x - 2$$
$$= (x+2)(3x^2 - 2x - 1)$$
$$= (x+2)(3x+1)(x-1)$$

(iii) By substituting a suitable value of x, find the remainder when 3 003 994 998 is divided by 1002. [2]

$$x = 1000$$
$$f(x) = 3(1000)^3 + 4(1000)^2 - 5(1000) - 2$$
$$= 3\ 003\ 994\ 998$$

Hence,

$$f(1000) = (1000+2)(3000+1)(1000-1)$$

Remainder = 0

Downloaded from SingaKids.com

11 It is believed that the number of views, V, of a viral video online after t hours are related by the equation $V = ab^{2t}$, where a and b are constants.

The table below shows a set of survey results for the variables V and t.

Time **(hours)**	1	2	3	4	5
Views **(in thousands)**	2.16	3.11	4.48	6.45	9.29

(a)(i) Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

$V = ab^{2t}$

$\lg V = \lg a + 2t \lg b$

Plot $\lg V$ against t

$\lg a$ = vertical intercept

$2\lg b$ = gradient

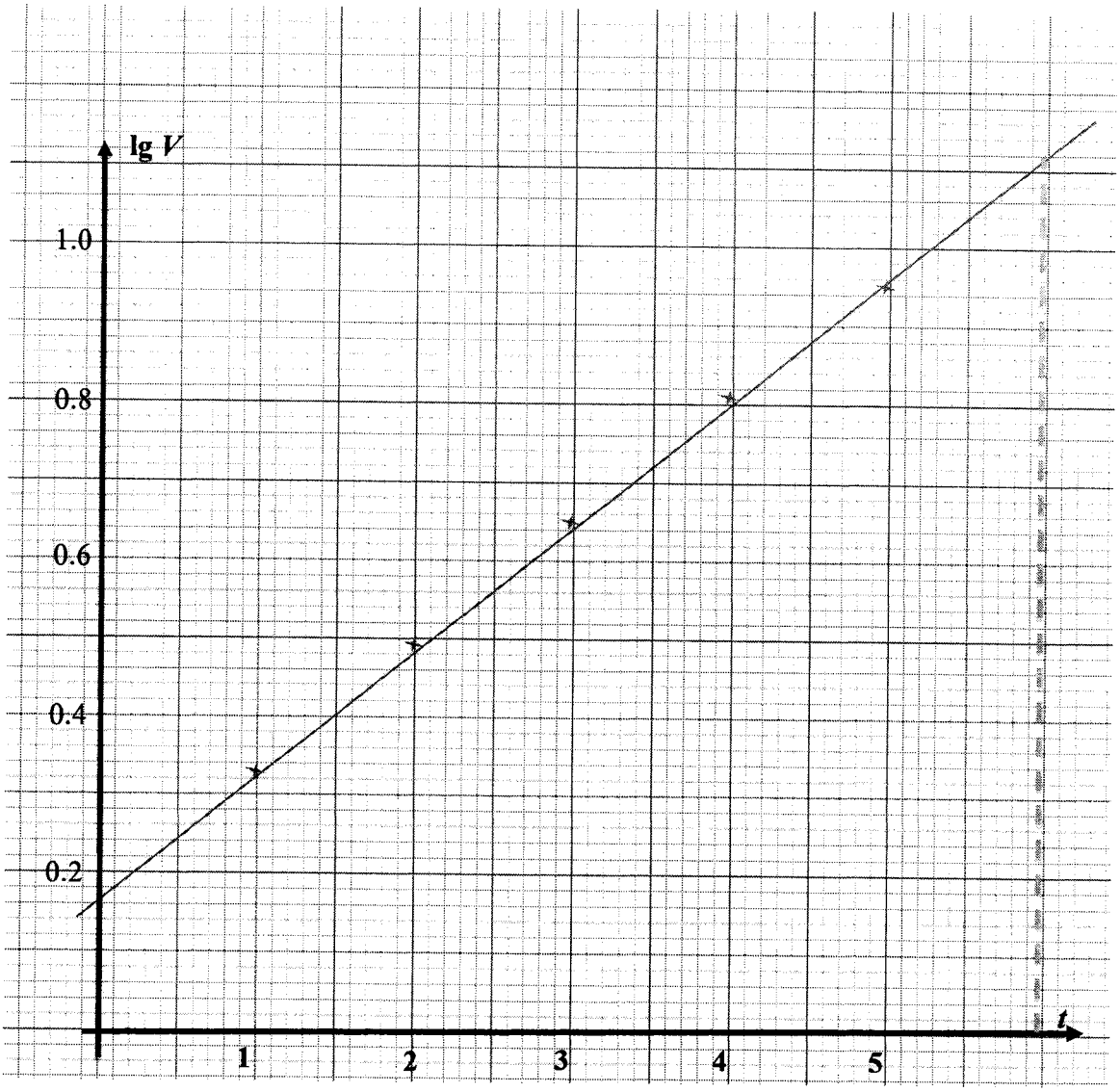
Time **(hours)**	1	2	3	4	5
$\lg V$ **(in thousands)**	0.33	0.49	0.65	0.81	0.97

Alternative:

Time **(hours)**	1	2	3	4	5
$\ln V$ **(in thousands)**	0.77	1.13	1.5	1.86	2.23

Downloaded from SingaKids.com

(a)(ii) On the grid below, draw the graph and estimate the value of a and of b. [3]



$\lg a = 0.17$ [0.16 – 0.18]

$a = 10^{0.17}$

$= 1.48$ [1.445 to 1.513]

$2\lg b = \frac{1-0.24}{5.3-0.5} = \frac{19}{120} = 0.158$ [0.14 to 0.18]

$\lg b = \frac{19}{240}$

$b = 1.20$ [1.17 to 1.23]

(iii) Using your graph to estimate the number of views when time is 6 hours. [2]

From the graph, at $t = 6$

$\lg V = 1.12$

$V = 13.2$ [13.2 to 13.5]

When $t = 6$ hours, there are **13.2 thousands** of views

Downloaded from SingaKids.com

11 **(b)** The variables x and y are related in such a way that when a graph of xy is plotted against x^2, a straight line graph which passes through the points (1, 9) and (5, 1) is obtained. Express y in terms of x. [3]

$$Y = xy$$
$$X = x^2$$
$$m = \frac{9-1}{1-5} = -2$$

$$Y - 9 = -2(X - 1)$$
$$Y = -2X + 11$$

$$xy = -2x^2 + 11$$
$$y = \frac{-2x^2 + 11}{x}$$
$$= -2x + \frac{11}{x}$$

Downloaded from SingaKids.com