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JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
Secondary 4 Express

STUDENT NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

19 AUGUST 2025

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **ALL** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

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90

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1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 The curve has the equation $y = x^2 - 4x + 7$.

A line with gradient 1 is a tangent to the curve.

Find the equation of the tangent line. [3]

2 Find the set of values of the constant p for which the curve $y = px^2 + (p-2)x + 5$ lies entirely above the x-axis. [5]

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3 (a) Prove that $\dfrac{\cot^2\theta - \tan^2\theta}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)} = \cos 2\theta$. [5]

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3 **(b)** Hence, solve the equation $\dfrac{\left(\cot^2\theta - \tan^2\theta\right)\left(2\sin 2\theta\right)}{\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right)} = 0.75$ for $-90° \le \theta \le 90°$. [5]

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4 The line $y - x = 5$ intersects the curve $x^2 + xy + y^2 = 91$ at two points.
Find the coordinates of these two points. [5]

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5 The function f is defined for $x > 0$ by $f(x) = \dfrac{x^2+4}{x+1}$.

(a) Find $f'(x)$ in the form $\dfrac{ax^2+bx+c}{(x+1)^2}$. [2]

(b) Hence determine the values of x for which f is increasing. [4]

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6 A balloon is being inflated such that its volume increases and the radius, r cm of the balloon, changes over time.

The rate of change of the radius at time t seconds is given by $\frac{dr}{dt} = \frac{4}{t+2}$, $t > 0$.

Initially, at $t = 0$, the radius is 3 cm.

(a) Find an expression for r in terms of t. [2]

(b) The volume V of the balloon is given by $V = \frac{4}{3}\pi r^3$.

Find the rate of increase of the volume of the balloon at $t = 2$. [4]

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7 **(a)** Express $\frac{5x^2+3x+7}{(x+1)(x-2)^2}$ in partial fractions. [6]

(b) Hence $\int \frac{5x^2+3x+7}{(x+1)(x-2)^2}\,dx$. [3]

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8 A curve has equation $y = x\sqrt{x+k} + 3$, where k is a positive constant.

(a) Show that $\frac{dy}{dx} = \frac{3x+2k}{2\sqrt{x+k}}$. [3]

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(b) Determine the nature of the stationary point on the curve. [6]

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9 A curve $y = \dfrac{1+3\ln x}{x^2}$ has a minimum point at P.

(a) Find $\dfrac{dy}{dx}$. [2]

(b) Show that the x-coordinate of P is $e^{\frac{1}{6}}$. [2]

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(c) Find the equation of normal to the curve at the point $x = e$.
Leave your answer in terms of e. [5]

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10 The table shows experimental values of two variables x and y.

x	1	2	3	4	5	6	7
y	56.2	31.5	25.1	9.96	5.60	3.14	1.76

The variables x and y are related by the equation $y = \frac{h}{k^x}$, where h and k are constants.

It is believed that an error was made in one of the experimental values of y.

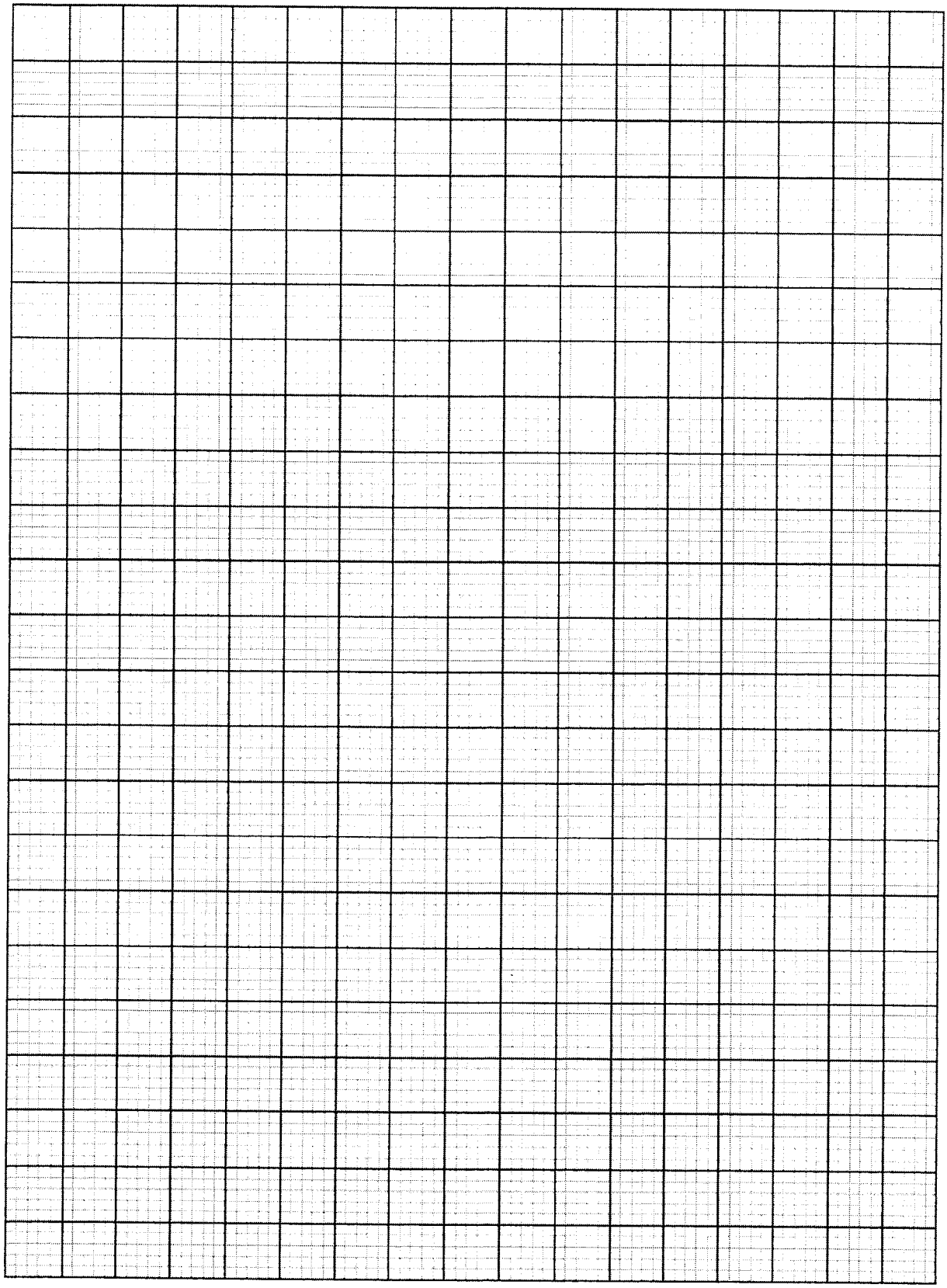
(a) On the grid, on the next page, plot lg y against x and draw a straight line graph to illustrate the information using a scale of 2 cm to represent 1 unit on the horizontal axis and 8 cm to represent 1 unit on the vertical axis. [3]

(b) Use your graph to

(i) identify the abnormal reading of y and estimate the correct value of y, [2]

(ii) find the value of h and k. [4]

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11 A circle, C, has equation $x^2 + y^2 - 10x + 6y + 9 = 0$.

(a) Find the coordinates of the centre and radius of C. [3]

(b) Explain why the y-axis is a tangent to C. [1]

(c) Given that the circle C crosses the x-axis at the point $P(1, 0)$, find the equation of the tangent to the circle at P. [3]

(d) Determine whether the point $A(3, -7)$ lies inside, outside or on the circle C? [2]

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12 **(a)** Find the equation of the perpendicular bisector of the points $P(a, 4)$ and $Q(6, -2)$ in terms of a. [4]

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(b) The perpendicular bisector of line segment PQ passes through the point $R(2,\ 1)$.
Find the possible values of a. [2]

(c) S is the reflection of point Q across the perpendicular bisector of PQ.
By using the positive value of a found in part (b) calculate the area of the quadrilateral $PQRS$. [4]

End of Paper

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JURONGVILLE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025
Secondary 4 Express

STUDENT NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **20 AUGUST 2025**

2 hours 15 minutes

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Mathematical Formulae

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$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
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$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
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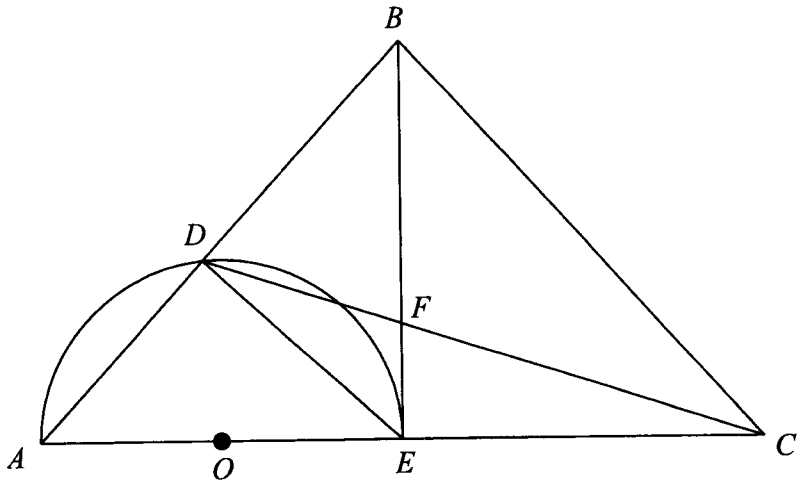
1 **(a)** Find the first 3 terms in the expansion of $(1+3x)^8$ in ascending powers of x. [2]

(b) Hence determine the first 3 terms in the expansion of $(2+6x+3x^2)^8$. [3]

(c) Explain why there is no constant term in the expansion of $\left(2-\frac{1}{3x}\right)^2(1+3x)^8$. [3]

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2



ADE is a semicircle with centre O. It is given that D and E are midpoints of AB and AC respectively and $AB = BC$.

(a) Prove that $\Delta ABE \equiv \Delta CBE$. [3]

(b) Prove that $\angle BEA = 90°$. [1]

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(c) Determine, with explanation, whether ΔADE is similar to ΔABC. [3]

(d) Show that $\frac{DF}{FC} = \frac{1}{2}$. [3]

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3 A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

(a) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

(b) Find the mass of radioactive substance left at the end of 1 week. [1]

(c) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

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(d) Find the rate at which the substance decays at $t = 50$ hours. [2]

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4 **(a)** Prove that $\cot(45° - x) = \dfrac{\cot x + 1}{\cot x - 1}$. [3]

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(b) Hence express $\cot 15°$ in the form $p+q\sqrt{3}$, where p and q are integers. [3]

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5 It is given that $\cos\theta = \frac{5}{13}$, where $0° < \theta < 360°$.

(a) Show that the value of $2 - \sin^2\theta$ is $\frac{194}{169}$. [2]

(b) By finding the value of $\sin^2\theta$, find the exact value of $\tan^2(90° - \theta)$. [4]

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6 The function $f(x) = x^3 + px^2 + qx - 12$, where p and q are constants, has a factor $x+1$ and leaves a remainder of 36 when divided by $x-3$.

(a) Show that $p = 5$ and $q = -8$. [4]

(b) Using these values of p and q, solve the equation $f(x) = 0$. [2]

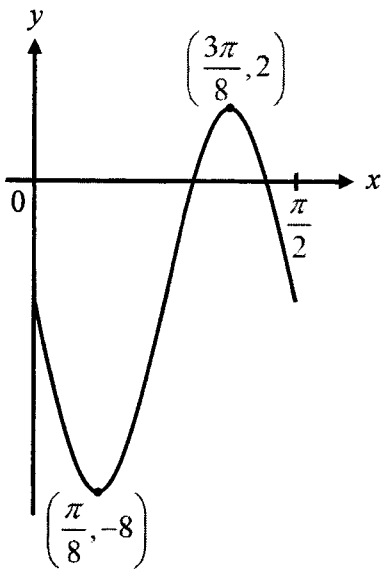
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7 **(a)** Given that $\log_7 x = m$ and $\log_7 y = 2n$, express xy in terms of m and n. [2]

(b) Solve the equation $\log_3 3x^2 + 4 = \log_x 27$. [5]

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8



The diagram shows the curve $y = a\sin bx + c$ for $0 \le x \le \frac{\pi}{2}$ radians.

The curve has a minimum point at $\left(\frac{\pi}{8}, -8\right)$ and a maximum point at $\left(\frac{3\pi}{8}, 2\right)$.

(a) Explain why $c = -3$. [2]

(b) Explain why $b = 4$. [2]

(c) Hence find the equation of the curve. [2]

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9 A boat travelling in a straight line passes a buoy, B, with a speed of v m/s. The velocity of the boat after passing B is given by $v = t^2 - 5t + 4$. The boat comes to rest momentarily, first at point C and then at point D.

(a) Find the distance CD. [5]

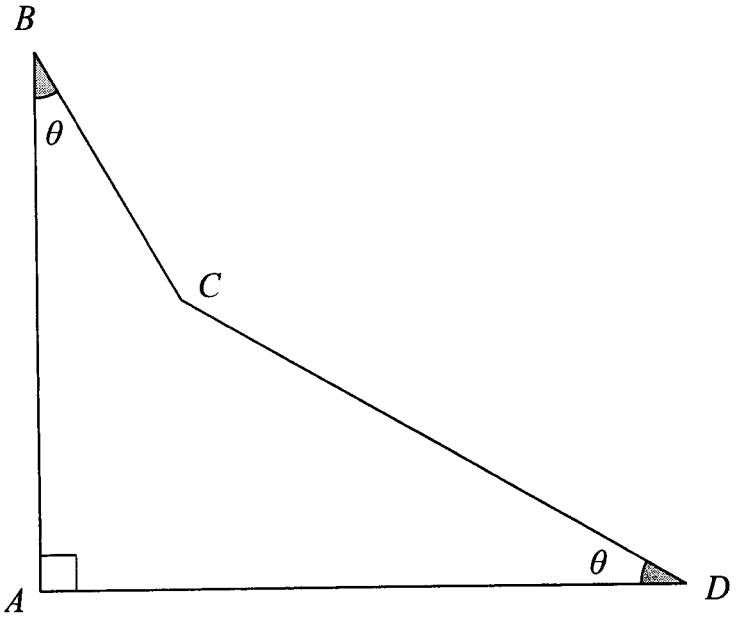
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(b) Find the total distance travelled by the boat in the first 5 seconds after passing B. [3]

(c) Given that A is the point at which the boat has zero acceleration, determine with full working, whether A is nearer to B or to D. [3]

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10



The diagram shows a drawing of a mini garden $ABCD$ in the school. It is given that $BC = 4$ m, $CD = 8$ m and $\angle ABC = \angle ADC = \theta$, where θ is an acute angle measured in degrees.

(a) Show that the area of the mini garden is $A = 20\sin 2\theta - 16\cos 2\theta + 16$. [4]

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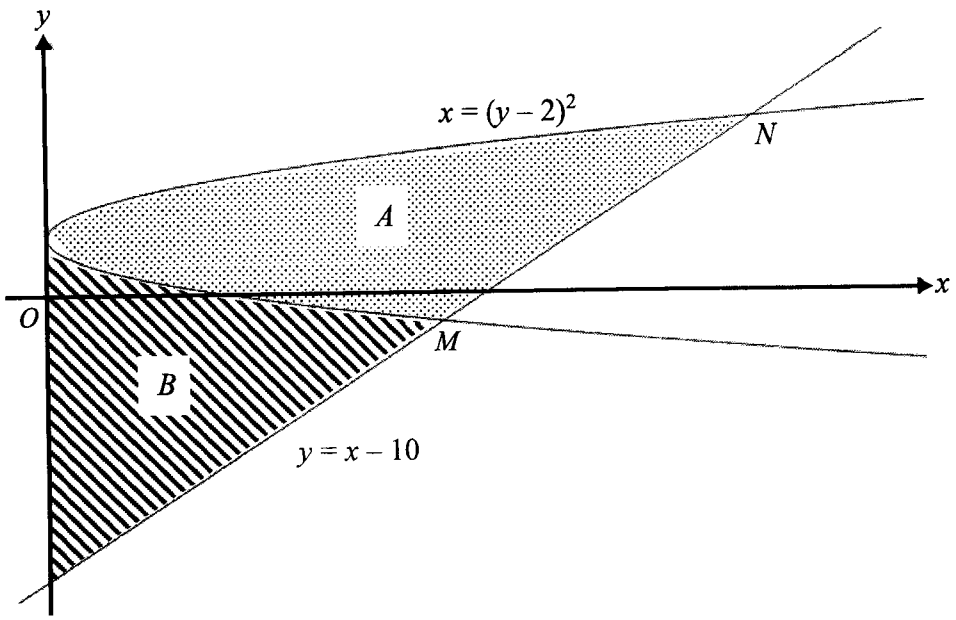
(b) Express A in the form $R\sin(2\theta-\alpha)+16$, where $R>0$ and α is an acute angle. [3]

(c) Find the value of θ when the area of the garden is 25 m^2. [3]

(d) Determine the maximum value of A and the corresponding value of θ. [3]

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11



The curve $x = (y-2)^2$ and the line $y = x - 10$ intersects at the points M and N.

(a) Find the coordinates of M and N. [3]

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(b) Find the area of shaded region A and the area of the shaded region B. [6]

END OF PAPER

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