# DUNMAN SECONDARY SCHOOL

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| **CANDIDATE NAME** | | | |
| **CLASS** | | **INDEX NUMBER** | |

## PRELIMINARY EXAMINATION 2025
## SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

**MATHEMATICS** **4052/02**

Paper 2 25 August 2025

**2 hours 15 minutes**

# Marking Scheme

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

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| Question | Answer |
|---|---|
| 1a | $h = 7 + 20t - 5t^2$ |
| | $= -5\left(t^2 - 4t\right) + 7$ |
| | $= -5\left[\left(t-2\right)^2 - 4\right] + 7$ |
| | $= -5\left(t-2\right)^2 + 27$ |
| | maximum height = 27 |
| | OR |
| | $\frac{\mathrm{d}h}{\mathrm{d}t} = -10t + 20$ |
| | $-10t + 20 = 0$ |
| | $t = 2$ |
| | When $t = 2,\ h = 7 + 20\left(2\right) - 5\left(2\right)^2 = 27$ m |
| 1b | $x^2 + 6x + k = k\left(x - 1\right)$ |
| | $x^2 + \left(6 - k\right)x + 2k = 0$ |
| | $\left(6-k\right)^2 - 4\left(1\right)\left(2k\right) < 0$ |
| | $36 - 12k + k^2 - 8k < 0$ |
| | $k^2 - 20k + 32 < 0$ |
| | $\left(k-2\right)\left(k-18\right) < 0$ |
| | $\therefore 2 < x < 18$ |
| 2a | $\tan A = \frac{4}{3}$ |
| | $\tan 2A = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$ |
| | $= -\frac{24}{7}$ |
| **Question** | **Answer** |
| 2b | $\sin B = \frac{12}{13}\ ,\ \cos B = \frac{5}{13}$ |
| | $\sin\left(B + \frac{3\pi}{2}\right) = \sin B \cos\frac{3\pi}{2} + \sin\frac{3\pi}{2}\cos B$ |
| | $= \frac{12}{13} \times 0 + \left(-1\right) \times \frac{5}{13}$ |
| | $= -\frac{5}{13}$ |
| | OR |
| | $\sin\left(B + \frac{3\pi}{2}\right) = \sin\left(2\pi - \frac{\pi}{2} + B\right)$ |

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| | |
|---|---|
| | $= \sin\left(-\frac{\pi}{2}+B\right) = -\sin\left(\frac{\pi}{2}-B\right)$ |
| | $= -\cos B = -\frac{5}{13}$ |
| 2c | $\cos B = 2\cos^2\frac{B}{2}-1$ |
| | $\frac{5}{13} = 2\cos^2\frac{B}{2}-1$ |
| | $2\cos^2\frac{B}{2} = \frac{18}{13}$ |
| | $\cos^2\frac{B}{2} = \frac{9}{13}$ |
| | $\cos\frac{B}{2} = \frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ (rej) |



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| **Question** | **Answer** |
|---|---|
| 3a | $\angle PQC = \angle PSQ$ (Alternate segment theorem) |
| | $\angle PQC = \angle PBC$ (angles in same segment) |
| | Hence $\angle PBC = \angle PSQ$ |
| | By alternate angles of parallel lines, $BC$ is parallel to $QS$ |
| 3b | $\angle PQC = \angle PSQ$ (Alternate segment theorem) |
| | $\angle PMQ = \angle QMS$ (Common angle) |
| | $\Delta PQM$ is similar to $\Delta QSM$ (AA Similarity Test) |
| 3c | Since $\Delta PQM$ is similar to $\Delta QSM$, $\frac{QM}{SM} = \frac{PM}{QM}$ |
| | $(QM)^2 = PM \times SM$ |



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| | |
|---|---|
| 4a | $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)$ |
| | $=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$ |
| | $=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$ |
| | $=-\frac{1}{\sin^2 x}$ |
| | $=-\text{cosec}^2\ x$ (Shown) |
| | OR |
| | $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}(\tan x)^{-1}$ |
| | $=-(\tan x)^{-2}\left(\sec^2 x\right)$ |
| | $=-\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$ |
| | $=-\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$ |
| | $=-\frac{1}{\sin^2 x}$ |
| | $=-\text{cosec}^2\ x$ (Shown) |
| 4b | $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x\ \mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1-\ \text{cosec}^2 x\ \mathrm{d}x$ |
| | $=\left[x+\cot x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ |
| | $=\left[\frac{\pi}{3}+\cot\frac{\pi}{3}\right]-\left[\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$ |
| | $=\frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{4}-1$ |
| | $=\frac{\pi}{12}+\frac{1}{\sqrt{3}}-1$ or $-0.161$ |



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| Question | Answer |
|---|---|
| 5a | General term $= \binom{8}{r} x^{8-r} \left(-\frac{k}{x^3}\right)^r$ |
| | $= \binom{8}{r} x^{8-r} (-1)^r k^r \left(x^{-3}\right)^r$ |
| | $= \binom{8}{r} (-1)^r k^r x^{8-4r}$ |
| | When $8-4r=0$ , $r=2$ |
| | $\binom{8}{2}(-1)^2 k^2 x^{8-4(2)} = 7$ |
| | $28k^2 = 7$ |
| | $k^2 = \frac{1}{4}$ |
| | $k = \frac{1}{2}$ |
| 5b | When $8-4r=-4$ , $r=3$ |
| | $T_4 = \binom{8}{3}(-1)^3 \left(\frac{1}{2}\right)^3 x^{-4}$ |
| | $= -7x^{-4}$ |
| | $\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8 = \left(1+x^4\right)\left(\ldots+7-7x^{-4}+\ldots\right)$ |
| | $= \ldots+7-7+\ldots = \ldots+0+\ldots$ |
| | There is no constant term in the expansion. (Shown) |



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| Question | Answer |
|---|---|
| 6a | $AM = 2\cos\theta$ |
| | $BM = 2\sin\theta$ |
| | $BC = 4\sin\theta$ |
| | Total area $= (4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$ |
| | $= 16\sin^2\theta + 4\sin\theta\cos\theta$ |
| 6b | $16\sin^2\theta + 4\sin\theta\cos\theta$ |
| | $= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$ |
| | $= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$ |
| | $= 2\sin 2\theta - 8\cos 2\theta + 8$ |
| 6c | $R = \sqrt{2^2 + 8^2} = \sqrt{68}$ |
| | $\tan\alpha = \frac{8}{2}$ |
| | $\alpha = 76.0°$ |
| | $2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$ |
| 7a | $LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$ |
| | $= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$ |
| | $= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$ |
| | $= \dfrac{\cos x - \sin x}{\cos x + \sin x}$ |
| | $= \dfrac{1 - \tan x}{1 + \tan x} = RHS$ |



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| Question | Answer |
|---|---|
| 7b | $\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$ |
| | $3(1-\tan x)=2\tan x+\tan^2 x$ |
| | $2\tan^2 x+5\tan x-3=0$ |
| | $(2\tan x-1)(\tan x+3)=0$ |
| | $\tan x=\dfrac{1}{2}$ or $-3$ |
| | $x=0.464$ or $3.61$ |
| | $x=1.89$ or $5.03$ |
| 8a | $x^2-\frac{3x}{2}-2$ <br> $2x-4$ ) $2x^3-7x^2+2x+3$ <br> $-$ <br> $2x^3-4x^2$ <br> $-3x^2+2x+3$ <br> $-$ <br> $-3x^2+6x$ <br> $3-4x$ <br> $-$ <br> $8-4x$ <br> $-5$ |



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| Question | Answer |
|---|---|
| 8b | $2x^3-7x^2+2x+3=0$ |
| | Let $\mathrm{f}(x)=2x^3-7x^2+2x+3$ |
| | When $x=3$, $\mathrm{f}(3)=2(3)^3-7(3)^2+2(3)+3=0$ |
| | By factor theorem, $(x-3)$ is a factor of f($x$). |
| | $(x-3)(2x^2-x-1)=0$ |
| | $(x-3)(2x+1)(x-1)=0$ |
| | $x=-\frac{1}{2}$ , 1 or 3 |
| 8c | $3y^6+2y^4-7y^2+2=0$ |
| | $3+\frac{2}{y^2}-\frac{7}{y^4}+\frac{2}{y^6}=0$ |
| | $3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^4}\right)+2\left(\frac{1}{y^6}\right)=0$ |
| | $3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^2}\right)^2+2\left(\frac{1}{y^2}\right)^3=0$ |
| | $\therefore x=\frac{1}{y^2}$ , Kun Ye is correct |



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| Question | Answer |
|---|---|
| 9a | Height = $3x^2$ |
| | Base = $2x$ |
| | Area = $\frac{1}{2}(2x)(3x^2) = 3x^3$ |
| 9b | $\frac{dy}{dx} = 6x$ |
| | $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ |
| | $3 = 6x \times \frac{dx}{dt}$ |
| | $3 = 6x \times \frac{dx}{dt}$ |
| | $\frac{dx}{dt} = \frac{1}{2x}$ |
| | When $x = 4$, $\frac{dx}{dt} = \frac{1}{8}$ |
| | Rate of change of $PQ = \frac{1}{4}$ unit/s |
| 9c | $\frac{dA}{dx} = 9x^2$ |
| | $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ |
| | $\frac{dA}{dt} = 9x^2 \times \frac{1}{8}$ |
| | When $x = 4$, $\frac{dA}{dt} = 18$ unit$^2$/s |



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| **Question** | **Answer** |
|---|---|
| 10a | $y$-coordinate of centre = $\frac{-1+9}{2} = 4$ |
| 10b | Let the $x$-coordinate of the centre be $a$, |
| | $(x-a)^2 + (y-4)^2 = 25$ |
| | From the line, $y = \frac{4x+9}{3}$ |
| | $(x-a)^2 + \left(\frac{4x+9}{3} - 4\right)^2 = 25$ |
| | $(x-a)^2 + \left(\frac{4}{3}x - 1\right)^2 = 25$ |
| | $x^2 - 2ax + a^2 + \frac{16}{9}x^2 - \frac{8}{3}x + 1 = 25$ |
| | $\frac{25}{9}x^2 + \left(-2a - \frac{8}{3}\right)x + a^2 - 24 = 0$ |
| | Since the line is a tangent, $b^2 - 4ac = 0$ |
| | $\left(-2a - \frac{8}{3}\right)^2 - 4\left(\frac{25}{9}\right)\left(a^2 - 24\right) = 0$ |
| | $4a^2 + \frac{32}{3}a + \frac{64}{9} - \frac{100}{9}a^2 + \frac{2400}{9} = 0$ |
| | $36a^2 + 96a + 64 - 100a^2 + 2400 = 0$ |
| | $-64a^2 + 96a + 2464 = 0$ |
| | $2a^2 - 3a - 77 = 0$ |
| | $(2a+11)(a-7) = 0$ |
| | $a = 7$ or $-5.5$ (rej) |



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| Question | Answer |
|---|---|
| 10c | Distance between point and centre $= \sqrt{(7-3)^2 + (4-0)^2}$ |
| | $= 5.66$ unit |
| | Since the distance is longer than the radius, the point lies outside. |
| 11a | $CX = \sqrt{(30-x)^2 + 20^2}$ |
| | $= \sqrt{x^2 - 60x + 1300}$ |
| | $T = \frac{x}{4} + \frac{1}{2}\sqrt{x^2 - 60x + 1300}$ |
| 11b | $\frac{dT}{dx} = \frac{1}{4} + \frac{1}{2}\left[\frac{1}{2}\left(x^2 - 60x + 1300\right)^{-\frac{1}{2}}(2x - 60)\right]$ |
| | $= \frac{1}{4} + \frac{2x - 60}{4\sqrt{x^2 - 60x + 1300}}$ |
| 11c | $\frac{1}{4} + \frac{2x - 60}{4\sqrt{x^2 - 60x + 1300}} = 0$ |
| | $\sqrt{x^2 - 60x + 1300} + 2x - 60 = 0$ |
| | $\sqrt{x^2 - 60x + 1300} = 60 - 2x$ |
| | $x^2 - 60x + 1300 = 3600 - 240x + 4x^2$ |
| | $3x^2 - 180x + 2300 = 0$ |
| | $x = \frac{180 \pm \sqrt{180^2 - 4(3)(2300)}}{2(3)}$ |
| | $x = 41.5$ (rej) or 18.5 |



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