DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS
Paper 1

4052/01
20 August 2025
2 hours 15 minutes

Solutions

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Question	Answer
1	$x-4=\dfrac{2}{x-3}$
	$(x-4)(x-3)=2$
	$x^2-7x+12=2$
	$x^2-7x+10=0$
	$(x-2)(x-5)=0$
	$x=2$ or 5
	When $x=2$, $y=-2$
	When $x=5$, $y=1$
	Length $=\sqrt{(2-5)^2+(-2-1)^2}$ $=4.24$ unit
2a	$-90° < \tan^{-1} p < 90°$ or $-\dfrac{\pi}{2} < \tan^{-1} p < \dfrac{\pi}{2}$
2b	$\pi - x$
3	$3x^2+15x+20 = 3(x^2+5x)+20$
	$= 3\left[(x+2.5)^2-(2.5)^2\right]+20$
	$= 3(x+2.5)^2+1.25$
	Since $3(x+2.5)^2+1.25 \geq 1.25$, $3x^2+15x+20$ cannot be smaller than 1.

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Question	Answer
4a	$\frac{x^2+3x+2}{x^2(2x+1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x+1}$
	$x^2+3x+2 = Ax(2x+1)+B(2x+1)+Cx^2$
	When $x=-\frac{1}{2}$, $\frac{1}{4}-\frac{3}{2}+2=C\left(\frac{1}{4}\right)$ $C=3$
	When $x=0$, $B=2$
	Comparing coefficients of x^2, $A=-1$
	$\frac{x^2+3x+2}{x^2(2x+1)} = -\frac{1}{x}+\frac{2}{x^2}+\frac{3}{2x+1}$
4b	$\int \frac{x^2+3x+2}{x^2(2x+1)}\,dx = \int -\frac{1}{x}+\frac{2}{x^2}+\frac{3}{2x+1}\,dx$ $= -\ln x-\frac{2}{x}+\frac{3}{2}\ln(2x+1)+c$
5	$3\log_5 y-\log_y 5=2$
	$3\log_5 y-\frac{\log_5 5}{\log_5 y}=2$
	$3\log_5 y-\frac{1}{\log_5 y}=2$
	$3(\log_5 y)^2-2\log_5 y-1=0$
	$(3\log_5 y+1)(\log_5 y-1)=0$
	$\log_5 y=1$ or $-\frac{1}{3}$
	$y=5$ or $y=5^{-\frac{1}{3}}=0.585$ (3s.f.)
Question	**Answer**
6a	$\sin(105°)=\sin(60°+45°)$
	$=\sin 60°\cos 45°+\cos 60°\sin 45°$
	$=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}+\frac{1}{2}\cdot\frac{1}{\sqrt{2}}$
	$=\frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$
	$=\frac{\sqrt{6}+\sqrt{2}}{4}$
6b	$\frac{1}{2}(4)(BC)\sin 105°=10+2\sqrt{3}$

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	$\frac{1}{2}(4)(BC)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=10+2\sqrt{3}$
	$BC=\frac{2\left(10+2\sqrt{3}\right)}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	$=\frac{4\left(5+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)}{6-2}$
	$=5\sqrt{6}+\sqrt{18}-5\sqrt{2}-\sqrt{6}$
	$=4\sqrt{6}+3\sqrt{2}-5\sqrt{2}$
	$=4\sqrt{6}-2\sqrt{2}$
	$=\sqrt{2}\left(4\sqrt{3}-2\right)$

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Question	Answer
7a	$9^x+6^x=6\left(4^x\right)$
	$\frac{9^x}{4^x}+\frac{6^x}{4^x}=6$
	$\frac{3^{2x}}{2^{2x}}+\left(\frac{6}{4}\right)^x-6=0$
	$\left(\frac{3}{2}\right)^{2x}+\left(\frac{3}{2}\right)^x-6=0$
	When $u=\left(\frac{3}{2}\right)^x$, $u^2+u-6=0$
7b	$(u+3)(u-2)=0$
	$u=2$ or -3 (n.a.)
	$\left(\frac{3}{2}\right)^x=2$
	$x=\frac{\ln 2}{\ln\frac{3}{2}}$ or $\frac{\lg 2}{\lg\frac{3}{2}}=1.71$
8	When $x=-2$, $2(-2)^3+a(-2)^2+b(-2)+10=0$
	$4a-2b=6$
	$2a-b=3$ - eqn 1
	When $x=3$, $2(3)^3+a(3)^2+b(3)+10=10$
	$9a+3b=-54$
	$3a+b=-18$ - eqn 2
	eqn 1 + eqn 2: $5a=-15$
	$a=-3$
	$\therefore b=-9$
Question	**Answer**
9a	$12t-\frac{3}{2}t^2=0$
	$t\left(12-\frac{3}{2}t\right)=0$
	$t=0$ or 8
	$\therefore 8$ s
9b	distance $=\int_0^8 12t-\frac{3}{2}t^2\ \mathrm{d}x$
	$=\left[6t^2-\frac{1}{2}t^3\right]_0^8$

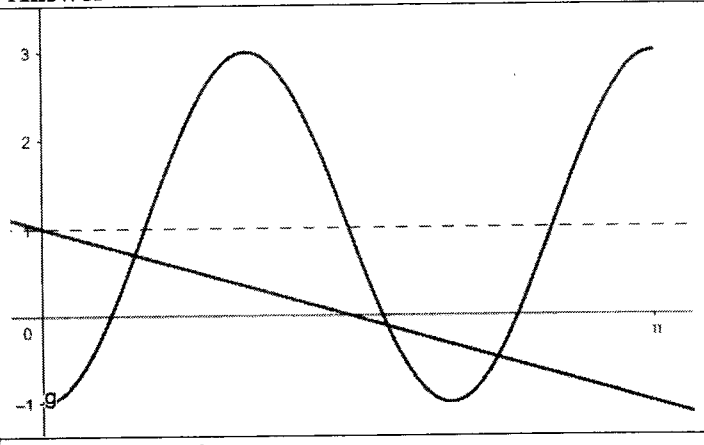
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	$=[128]-[0]$
	$=128$ m
9c	$a=12-3t$
	When $t=5$, $a=-3$ m/s^2

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Question	Answer
10	$y = xe^{-3x}$
	$\frac{dy}{dx} = e^{-3x}(1) + x(-3e^{-3x})$
	$= e^{-3x}(1-3x)$
	$e^{-3x}(1-3x) = 0$
	$x = \frac{1}{3}$
	$y = \frac{1}{3e}$ or 0.123
	$\frac{d^2y}{dx^2} = (1-3x)(-3e^{-3x}) + e^{-3x}(-3)$
	When $x = \frac{1}{3}$, $\frac{d^2y}{dx^2} < 0$
	$\left(\frac{1}{3}, -\frac{1}{3e}\right)$ is a maximum point.

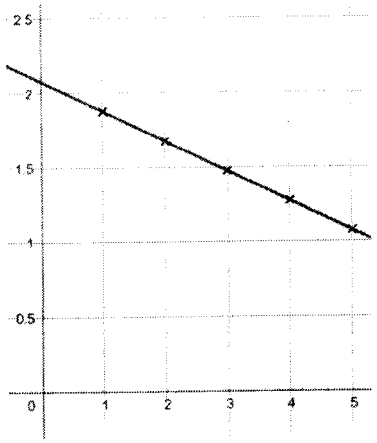
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Question	Answer
11a	
11b	The $2\cos 3x = \frac{2}{\pi}x$ is equivalent to $-2\cos 3x + 1 = -\frac{2}{\pi}x + 1$ and so its solutions can be found from the interception points of the two graphs.

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Question	Answer
12a	Midpoint of $PQ = \left(\frac{1+9}{2}, \frac{-2+10}{2}\right) = (5,4)$
	Gradient of $PQ = \frac{-2-10}{1-9} = \frac{3}{2}$
	Gradient of perpendicular bisector $= -\frac{2}{3}$
	$y-4=-\frac{2}{3}(x-5)$
	$y=-\frac{2}{3}x+\frac{22}{3}$
12b	$-\frac{2}{3}x+\frac{22}{3}=8x-10$
	$-\frac{26}{3}x=-\frac{52}{3}$
	$x=2$
	$R(2,6)$
12c	Area $= \frac{1}{2}\begin{vmatrix} 9 & 2 & 1 & 9 \\ 10 & 6 & -2 & 10 \end{vmatrix}$
	$= \frac{1}{2}\left[(54-4+10)-(20+6-18)\right]$
	$= 26$ unit^2

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<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>13a</td><td>
<table>
<tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>m</td><td>6.55</td><td>5.36</td><td>4.40</td><td>3.60</td><td>2.94</td></tr>
<tr><td>$\ln m$</td><td>1.88</td><td>1.68</td><td>1.48</td><td>1.28</td><td>1.08</td></tr>
</table>


</td></tr>
<tr><td>13b</td><td>$\ln M = 2.08$</td></tr>
<tr><td></td><td>$M = 8.00$</td></tr>
<tr><td>13c</td><td>Gradient = -0.2</td></tr>
<tr><td></td><td>$k = 0.2$</td></tr>
</table>

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Question	Answer
14a	$\frac{d^2y}{dx^2} = -4\sin(2x+\pi)$
	$\frac{dy}{dx} = 2\cos(2x+\pi)+c_1$
	$3 = 2\cos\left[2\left(\frac{\pi}{2}\right)+\pi\right]+c_1$
	$3 = 2(1)+c_1$
	$c_1 = 1$
	$\frac{dy}{dx} = 2\cos(2x+\pi)+1$
	$y = \sin(2x+\pi)+x+c_2$
	$\pi = \sin\left[2\left(\frac{\pi}{2}\right)+\pi\right]+\frac{\pi}{2}+c_2$
	$c_2 = \frac{\pi}{2}$
	$y = \sin(2x+\pi)+x+\frac{\pi}{2}$
14b	When $x = 3$, $\frac{dy}{dx} = 2\cos[2(3)+\pi]+1 = -0.920$
	y is decreasing when $x = 3$.

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Question	Answer
15	$y=\frac{16}{(x+1)^2}-1=16(x+1)^{-2}-1$
	$\frac{dy}{dx}=-32(x+1)^{-3}$
	$-4=-32(x+1)^{-3}$
	$(x+1)^3=8$
	$x=1$
	At L, $y=\frac{16}{(1+1)^2}-1=3$
	$y-3=-4(x-1)$
	$y=-4x+7$
	At N, $0=-4x+7$
	$x=1.75$
	At M, $0=\frac{16}{(x+1)^2}-1$
	$x=3$ or -5 (n.a.)
	Area $=\left[\int_1^3 16(x+1)^{-2}-1\,dx\right]-\int_1^{1.75}-4x+7\,dx$
	$=\left[-16(x+1)^{-1}-x\right]_1^3-\left[-2x^2+7x\right]_1^{1.75}$
	$=2-1.125$
	$=0.875$ unit^2

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PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS
Paper 2

4052/02
25 August 2025
2 hours 15 minutes

Marking Scheme

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Question	Answer
1a	$h = 7 + 20t - 5t^2$
	$= -5\left(t^2 - 4t\right) + 7$
	$= -5\left[(t-2)^2 - 4\right] + 7$
	$= -5(t-2)^2 + 27$
	maximum height = 27
	OR
	$\frac{dh}{dt} = -10t + 20$
	$-10t + 20 = 0$
	$t = 2$
	When $t = 2$, $h = 7 + 20(2) - 5(2)^2 = 27$ m
1b	$x^2 + 6x + k = k(x-1)$
	$x^2 + (6-k)x + 2k = 0$
	$(6-k)^2 - 4(1)(2k) < 0$
	$36 - 12k + k^2 - 8k < 0$
	$k^2 - 20k + 32 < 0$
	$(k-2)(k-18) < 0$
	$\therefore 2 < x < 18$
2a	$\tan A = \frac{4}{3}$
	$\tan 2A = \frac{2\left(\frac{4}{3}\right)}{1-\left(\frac{4}{3}\right)^2}$
	$= -\frac{24}{7}$
Question	**Answer**
2b	$\sin B = \frac{12}{13}$, $\cos B = \frac{5}{13}$
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin B \cos\frac{3\pi}{2} + \sin\frac{3\pi}{2}\cos B$
	$= \frac{12}{13} \times 0 + (-1) \times \frac{5}{13}$
	$= -\frac{5}{13}$
	OR
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin\left(2\pi - \frac{\pi}{2} + B\right)$

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	$= \sin\left(-\frac{\pi}{2}+B\right) = -\sin\left(\frac{\pi}{2}-B\right)$
	$= -\cos B = -\frac{5}{13}$
2c	$\cos B = 2\cos^2\frac{B}{2}-1$
	$\frac{5}{13} = 2\cos^2\frac{B}{2}-1$
	$2\cos^2\frac{B}{2} = \frac{18}{13}$
	$\cos^2\frac{B}{2} = \frac{9}{13}$
	$\cos\frac{B}{2} = \frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$(rej)

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Question	Answer
3a	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PQC = \angle PBC$ (angles in same segment)
	Hence $\angle PBC = \angle PSQ$
	By alternate angles of parallel lines, BC is parallel to QS
3b	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PMQ = \angle QMS$ (Common angle)
	ΔPQM is similar to ΔQSM (AA Similarity Test)
3c	Since ΔPQM is similar to ΔQSM, $\frac{QM}{SM} = \frac{PM}{QM}$
	$(QM)^2 = PM \times SM$

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4a	$\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$
	$= \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$
	$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$
	$= -\frac{1}{\sin^2 x}$
	$= -\text{cosec}^2\ x$ (Shown)
	OR
	$\frac{d}{dx}(\cot x) = \frac{d}{dx}(\tan x)^{-1}$
	$= -(\tan x)^{-2}(\sec^2 x)$
	$= -\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$
	$= -\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
	$= -\frac{1}{\sin^2 x}$
	$= -\text{cosec}^2\ x$ (Shown)
4b	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x\ dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1 - \text{cosec}^2 x\ dx$
	$= \left[x + \cot x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
	$= \left[\frac{\pi}{3} + \cot\frac{\pi}{3}\right] - \left[\frac{\pi}{4} + \cot\frac{\pi}{4}\right]$
	$= \frac{\pi}{3} + \frac{1}{\sqrt{3}} - \frac{\pi}{4} - 1$
	$= \frac{\pi}{12} + \frac{1}{\sqrt{3}} - 1$ or -0.161

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Question	Answer
5a	General term $= \binom{8}{r} x^{8-r} \left(-\frac{k}{x^3}\right)^r$
	$= \binom{8}{r} x^{8-r} (-1)^r k^r \left(x^{-3}\right)^r$
	$= \binom{8}{r} (-1)^r k^r x^{8-4r}$
	When $8-4r=0$, $r=2$
	$\binom{8}{2}(-1)^2 k^2 x^{8-4(2)} = 7$
	$28k^2 = 7$
	$k^2 = \frac{1}{4}$
	$k = \frac{1}{2}$
5b	When $8-4r=-4$, $r=3$
	$T_4 = \binom{8}{3}(-1)^3 \left(\frac{1}{2}\right)^3 x^{-4}$
	$=-7x^{-4}$
	$\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8 = \left(1+x^4\right)\left(\ldots+7-7x^{-4}+\ldots\right)$
	$=\ldots+7-7+\ldots=\ldots+0+\ldots$
	There is no constant term in the expansion. (Shown)

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Question	Answer
6a	$AM = 2\cos\theta$
	$BM = 2\sin\theta$
	$BC = 4\sin\theta$
	Total area $= (4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$
	$= 16\sin^2\theta + 4\sin\theta\cos\theta$
6b	$16\sin^2\theta + 4\sin\theta\cos\theta$
	$= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$
	$= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$
	$= 2\sin 2\theta - 8\cos 2\theta + 8$
6c	$R = \sqrt{2^2 + 8^2} = \sqrt{68}$
	$\tan\alpha = \frac{8}{2}$
	$\alpha = 76.0°$
	$2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$
7a	$LHS = \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$
	$= \frac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$
	$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$
	$= \frac{\cos x - \sin x}{\cos x + \sin x}$
	$= \frac{1 - \tan x}{1 + \tan x} = RHS$

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Question	Answer
7b	$\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$
	$3(1-\tan x)=2\tan x+\tan^2 x$
	$2\tan^2 x+5\tan x-3=0$
	$(2\tan x-1)(\tan x+3)=0$
	$\tan x=\dfrac{1}{2}$ or -3
	$x=0.464$ or 3.61
	$x=1.89$ or 5.03
8a	$x^2-\frac{3x}{2}-2$ $2x-4 \overline{)\,2x^3-7x^2+2x+3}$ $-\quad 2x^3-4x^2$ $-3x^2+2x+3$ $-\quad -3x^2+6x$ $3-4x$ $-\quad 8-4x$ -5

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Question	Answer
8b	$2x^3-7x^2+2x+3=0$
	Let $f(x)=2x^3-7x^2+2x+3$
	When $x=3$, $f(3)=2(3)^3-7(3)^2+2(3)+3=0$
	By factor theorem, $(x-3)$ is a factor of f(x).
	$(x-3)(2x^2-x-1)=0$
	$(x-3)(2x+1)(x-1)=0$
	$x=-\frac{1}{2}$, 1 or 3
8c	$3y^6+2y^4-7y^2+2=0$
	$3+\frac{2}{y^2}-\frac{7}{y^4}+\frac{2}{y^6}=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^4}\right)+2\left(\frac{1}{y^6}\right)=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^2}\right)^2+2\left(\frac{1}{y^2}\right)^3=0$
	$\therefore x=\frac{1}{y^2}$, Kun Ye is correct

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Question	Answer
9a	Height $= 3x^2$
	Base $= 2x$
	Area $= \frac{1}{2}(2x)(3x^2) = 3x^3$
9b	$\frac{dy}{dx} = 6x$
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$
	$3 = 6x \times \frac{dx}{dt}$
	$3 = 6x \times \frac{dx}{dt}$
	$\frac{dx}{dt} = \frac{1}{2x}$
	When $x = 4$, $\frac{dx}{dt} = \frac{1}{8}$
	Rate of change of $PQ = \frac{1}{4}$ unit/s
9c	$\frac{dA}{dx} = 9x^2$
	$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$
	$\frac{dA}{dt} = 9x^2 \times \frac{1}{8}$
	When $x = 4$, $\frac{dA}{dt} = 18$ unit^2/s

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Question	**Answer**
10a	y-coordinate of centre $= \frac{-1+9}{2} = 4$
10b	Let the x-coordinate of the centre be a,
	$(x-a)^2+(y-4)^2=25$
	From the line, $y=\frac{4x+9}{3}$
	$(x-a)^2+\left(\frac{4x+9}{3}-4\right)^2=25$
	$(x-a)^2+\left(\frac{4}{3}x-1\right)^2=25$
	$x^2-2ax+a^2+\frac{16}{9}x^2-\frac{8}{3}x+1=25$
	$\frac{25}{9}x^2+\left(-2a-\frac{8}{3}\right)x+a^2-24=0$
	Since the line is a tangent, $b^2-4ac=0$
	$\left(-2a-\frac{8}{3}\right)^2-4\left(\frac{25}{9}\right)\left(a^2-24\right)=0$
	$4a^2+\frac{32}{3}a+\frac{64}{9}-\frac{100}{9}a^2+\frac{2400}{9}=0$
	$36a^2+96a+64-100a^2+2400=0$
	$-64a^2+96a+2464=0$
	$2a^2-3a-77=0$
	$(2a+11)(a-7)=0$
	$a=7$ or -5.5 (rej)

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Question	Answer
10c	Distance between point and centre $=\sqrt{(7-3)^2+(4-0)^2}$
	$=5.66$ unit
	Since the distance is longer than the radius, the point lies outside.
11a	$CX=\sqrt{(30-x)^2+20^2}$
	$=\sqrt{x^2-60x+1300}$
	$T=\frac{x}{4}+\frac{1}{2}\sqrt{x^2-60x+1300}$
11b	$\frac{dT}{dx}=\frac{1}{4}+\frac{1}{2}\left[\frac{1}{2}\left(x^2-60x+1300\right)^{-\frac{1}{2}}(2x-60)\right]$
	$=\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}$
11c	$\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}=0$
	$\sqrt{x^2-60x+1300}+2x-60=0$
	$\sqrt{x^2-60x+1300}=60-2x$
	$x^2-60x+1300=3600-240x+4x^2$
	$3x^2-180x+2300=0$
	$x=\frac{180\pm\sqrt{180^2-4(3)(2300)}}{2(3)}$
	$x=41.5$ (rej) or 18.5

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