*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** Solve the simultaneous equations

[4]



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$$y - x = 4,$$
$$4x^2 - 6xy - 6y = 4.$$

**2** Solve the equation $9 \sin x = -2 \sec x$ for $-\pi \leq x \leq 0$. [4]

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**3** **(a)** Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

**[Turn over**

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**(b)** Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$. [4]

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**4** Express $\dfrac{2x^3 + 4}{x(x+2)^2}$ in partial fractions. [5]



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**5** The function f is given by $\mathrm{f}(x) = \dfrac{x + a}{x^2 + 3}$.

**(a)** Find $\mathrm{f}'(x)$. [2]

It is given that f increases for $b < x < 1$.

**(b)** Find the values of $a$ and $b$. [4]

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**6** Find the set of values of the constant $k$ for which the curve $y = (k-6)x^2 - 8x + k$ does not intersect the $x$-axis and has a minimum point. [6]

**[Turn over**

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**7** **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x-1\right)\mathrm{e}^{-x}\right]$. [2]

**(b)** Hence find $\int x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

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**8** The radius of a circle, $r$ cm, decreases at a rate of $\frac{2}{(t+1)^2}$ cm per minute.

**(a)** Given that the initial radius is 4 cm, find an expression for $r$ in terms of $t$. [3]

**(b)** Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]



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**9** **(a)** Find the value of the constant $k$ such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [3]

**(b)** Solve $\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1,$ giving your answer in the form $a\sqrt{2} + b,$ where $a$ and $b$ are rational numbers. [4]

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**10** An isosceles triangle $PQR$ has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
$PQ = QR$.

**(a)** Find the area of the triangle $PQR$. [2]

**(b)** If $S$ is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of $S$. [5]

**11** **(a)** Solve the equation $2^x + 4^{x-1} = 24$. [4]

**[Turn over**

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**(b)** Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$. [4]

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**12** The gradient at any point on a curve $y$ is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

**(a)** Show that $k = 2$. [2]

**(b)** Hence find the $x$-coordinates of the stationary points. [2]

**[Turn over**

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**(c)** Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and explain whether the gradient $6x - \frac{1}{kx^3}$ has a turning point. [4]

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**13** The cost in thousands of dollars, $C$, of producing $n$ hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

**(a)** Find the initial cost of production. [1]

**(b)** Express $C$ in the form $a(x + h)^2 + k$ where $a$, $h$ and $k$ are constants to be determined. [3]

**(c)** Explain the meaning of the values of $h$ and $k$ in the formula as expressed in **part (b)**. [1]

**(d)** Find the maximum number of pairs of running shoes for which the cost will be [3]

**[Turn over**

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50 thousand dollars.

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**14** The line $y = -x + 4$ is a tangent to a circle at the point $A$ and the centre of the circle is at $B(-4, -2)$.

**(a)** Find coordinates of $A$. [3]

**(b)** Find the radius and the equation of the circle. [3]



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**(c)** Find the equation of another tangent to the circle which is parallel to $y = -x + 4$. [4]

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