Bedok View Secondary School
Department of Mathematics Mark Scheme

Year	2025	**Level & Stream**	4E5N
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P1

Qns	Working
1	$4x^2-6xy-6y=4$ ----(1) $y-x=4$ ------(2) Sub (2) into (1): $4x^2-6x(4+x)-6(4+x)=4$ $4x^2-24x-6x^2-24-6x=4$ $2x^2+30x+28=0$ $x^2+15x+14=0$ $(x+1)(x+14)=0$ $x=-14$ or $x=-1$ $y=-10$ or $y=3$
2	$9\sin x=-2\sec x$ $9\sin x=-\dfrac{2}{\cos x}$ $\sin x\cos x=-\dfrac{2}{9}$ $2\sin x\cos x=-\dfrac{4}{9}$ $\sin 2x=-\dfrac{4}{9}$ Basic angle $=\sin^{-1}\left(\dfrac{4}{9}\right)$ $=0.46055$ $2x=-0.46055,\ -(\pi-0.46055)$ $=-0.46055$ or -2.68104 $x=-0.230$ or -1.34

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Qns	Working
3a	Principal range of $\cos^{-1}x$ is $0 \le \cos^{-1}x \le \pi$, hence principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$.
3b	L.H.S: $\cot A + \tan A = \frac{1}{\tan A} + \tan A$ $= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$ $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \sec A \operatorname{cosec} A$ = RHS (shown) **OR** RHS: $\sec A \operatorname{cosec} A = \frac{1}{\cos A} \times \frac{1}{\sin A}$ $\frac{1}{\cos A \sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$ $= = \frac{\sin^2 A}{\cos A \sin A} + \frac{\cos^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \tan A + \cot A$ = LHS (shown)

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Qns	Working
4	By long division, $\frac{2x^3+4}{x(x+2)^2} = \frac{2x^3+4}{x^3+4x^2+4x}$ $= 2 + \frac{-8x^2-8x+4}{x(x+2)^2}$ $\frac{-8x^2-8x+4}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ $-8x^2-8x+4 = A(x+2)^2 + Bx(x+2) + Cx$ Let $x=0$: $4 = 4A$ $A = 1$ Let $x=-2$: $-12 = -2C$ $C = 6$ Let $x=1$: $-8-8+4 = 9+3B+6$ $B = -9$ $\therefore \frac{2x^3+4}{x(x+2)^2} = 2 + \frac{1}{x} - \frac{9}{x+2} + \frac{6}{(x+2)^2}$
5a	$f'(x) = \frac{(x^2+3)-2x(x+a)}{(x^2+3)^2}$ $= \frac{-x^2-2ax+3}{(x^2+3)^2}$
5b	$\frac{-x^2-2ax+3}{(x^2+3)^2} > 0,$ for $b < x < 1$ Since $(x^2+3)^2 > 0,$ $-x^2-2ax+3 > 0$ $x^2+2ax-3 < 0$ Given that $b < x < 1$ $(x-b)(x-1) < 0$ $x^2-bx-x+b < 0$ By comparing: $b = -3$ $2a = -b-1$ $2a = 3-1$ $a = 1$

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Qns	Working
6	$y = (k-6)x^2 - 8x + k$ Discriminant $= (-8)^2 - 4(k-6)(k)$ $= 64 - 4k^2 + 24k$ For curve that does not intersect the x-axis, Discriminant < 0 $k^2 - 6k - 16 > 0$ $(k+2)(k-8) > 0$ $k < -2$ or $k > 8$ The curve has a minimum point: $k - 6 > 0$ $k > 6$ $\therefore k > 8$
7a	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right] = \mathrm{e}^{-x} - (x-1)\mathrm{e}^{-x}$ $= 2\mathrm{e}^{-x} - x\mathrm{e}^{-x}$
7b	$\int 2\mathrm{e}^{-x} - x\mathrm{e}^{-x}\mathrm{d}x = (x-1)\mathrm{e}^{-x} + c$ $\int 2\mathrm{e}^{-x}\mathrm{d}x - \int x\mathrm{e}^{-x}\mathrm{d}x = (x-1)\mathrm{e}^{-x} + c$ $-\int x\mathrm{e}^{-x}\mathrm{d}x = -\int 2\mathrm{e}^{-x}\mathrm{d}x + (x-1)\mathrm{e}^{-x} + c$ $\int x\mathrm{e}^{-x}\mathrm{d}x = -2\mathrm{e}^{-x} - (x-1)\mathrm{e}^{-x} + c$ $= -2\mathrm{e}^{-x} - x\mathrm{e}^{-x} + \mathrm{e}^{-x} + c$ $= -\mathrm{e}^{-x} - x\mathrm{e}^{-x} + c$

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Qns	Working
8a	$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{2}{(t+1)^2}$ $r = -\int \frac{2}{(t+1)^2}\mathrm{d}t$ $= -\int 2(t+1)^{-2}\mathrm{d}t$ $= 2(t+1)^{-1} + c$ $r = \frac{2}{t+1} + c$ At $r = 4$ and $t = 0$, $4 = 2 + c$ $c = 2$ $r = \frac{2}{t+1} + 2$
8b	$A = \pi r^2$ $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$ At $r = 2.6$, $\frac{\mathrm{d}A}{\mathrm{d}r} = 5.2\pi$, $2.6 = \frac{2}{t+1} + 2$ $0.6 = \frac{2}{t+1}$ $t = \frac{7}{3}$ $\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{2}{\left(\frac{7}{3}+1\right)^2} = -0.18$ $\frac{\mathrm{d}A}{\mathrm{d}t} = 5.2\pi \times -0.18$ $\frac{\mathrm{d}A}{\mathrm{d}t} = -0.936\pi \text{ cm}^2/\text{min}$

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Qns	Working
9a	$y = kx + 6$ ---- (1) $2x^2 - xy = 3$ -----(2) Sub (1) into (2): $2x^2 - x(kx+6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2-k)x^2 - 6x - 3 = 0$ Discriminant $= (-6)^2 - 4(2-k)(-3)$ $= 60 - 12k$ For tangent, $60 - 12k = 0$ $k = 5$
9b	$\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$ $4x - 2x\sqrt{2} + \sqrt{2} = 3x\sqrt{2} - 1$ $\sqrt{2} + 1 = 3x\sqrt{2} + 2x\sqrt{2} - 4x$ $\sqrt{2} + 1 = x\left(5\sqrt{2} - 4\right)$ $x = \frac{\sqrt{2}+1}{5\sqrt{2}-4} \times \frac{5\sqrt{2}+4}{5\sqrt{2}+4}$ $= \frac{5(2) + 4\sqrt{2} + 5\sqrt{2} + 4}{\left(5\sqrt{2}\right)^2 - 4^2}$ $= \frac{9\sqrt{2}+14}{50-16}$ $= \frac{9}{34}\sqrt{2} + \frac{7}{17}$

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Qns	Working
10a	Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2}[3+4-2]$ $= 2.5 \text{ units}^2$
10b	$m_{PR} = \frac{3-0}{2-1}$ $= 3$ $m_{QS} = -\frac{1}{3}$ Equation of QS: $y-2 = -\frac{1}{3}(x-0)$ $y = -\frac{1}{3}x + 2 \quad ---(1)$ $x - y = 5 \quad ---(2)$ Sub (1) into (2): $x + \frac{1}{3}x - 2 = 5$ $x = \frac{21}{4}$ $y = \frac{1}{4}$ coordinates of $S = \left(\frac{21}{4}, \frac{1}{4}\right)$

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Qns	Working
11a	$2^x + 4^{x-1} = 24$ $2^x + \left(4^x\right)\left(4^{-1}\right) = 24$ $2^x + \frac{2^{2x}}{4} = 24$ Let $y = 2^x$, $y + \frac{y^2}{4} = 24$ $4y + y^2 = 96$ $y^2 + 4y - 96 = 0$ $(y-8)(y+12) = 0$ $y = 8$ or $y = -12$ $2^x = 8$ or $2^x = -12$ $\left(\text{Rejected since } 2^x > 0\right)$ $2^x = 2^3$ $x = 3$
11b	$\log_{\sqrt{3}}(x-1) = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3\sqrt{3}} = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = \log_3(x+5)$ $2\log_3(x-1) = \log_3(x+5)$ $\log_3(x-1)^2 = \log_3(x+5)$ $(x-1)^2 = x+5$ $x^2 - 2x + 1 - x - 5 = 0$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $x + 1 = 0$ $x = -1$ [N.A. as $\log_{\sqrt{3}}(x-1)$ will be undefined] or $x - 4 = 0$ $x = 4$

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Qns	Working
12a	$4x-4y=3$ $y=x-\frac{3}{4}$ m of normal $=1$ m of tangent $=-1$ At $x=\frac{1}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$, $6\left(\frac{1}{2}\right)-\frac{1}{k\left(\frac{1}{2}\right)^3}=-1$ $3-\frac{8}{k}=-1$ $k=2$ (shown)
12b	At stat points, $6x-\frac{1}{2x^3}=0$ $6x=\frac{1}{2x^3}$ $x^4=\frac{1}{12}$ $x=\pm\sqrt[4]{\frac{1}{12}}$ $x=-0.537$ or 0.537
12c	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}$ $6>0$ $\frac{3}{2x^4}>0$ $\therefore \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}>0$ Since $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=6+\frac{3}{2x^4}>0$ the gradient has no turning point.

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Qns	Working
13a	$C = 1.2n^2 - 14.4n + 53.7$ At $n = 0,\ C = \$53.7$ thousands
13b	$C = 1.2n^2 - 14.4n + 53.7$ $= 1.2\left[n^2 - 12n + 44.75\right]$ $= 1.2\left[(n-6)^2 - 6^2 + 44.75\right]$ $= 1.2(n-6)^2 + 10.5$
13c	For 600 pairs of running shoes produced, the minimum cost of 10.5 thousand dollars will be incurred.
13d	$1.2(n-6)^2 + 10.5 = 50$ $1.2(n-6)^2 = 39.5$ $(n-6)^2 = \frac{39.5}{1.2}$ $n - 6 = \pm 5.7373$ $n = 11.7373$ or 0.2627 Maximum number of pairs of running shoes for which the cost is at most 50 thousand dollars = 11.7373 = 11.7 hundreds

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Qns	Working
14a	$m_{AB} = 1$ Equation of AB: $y + 2 = x + 4$ $y = x + 2 ----(1)$ $y = -x + 4 ----(2)$ $(1) = (2) : x + 2 = -x + 4$ $2x = 2$ $x = 1$ $y = 3$ Coordinates of $A = (1, 3)$
14b	$\text{Radius} = \sqrt{(1+4)^2 + (3+2)^2}$ $= \sqrt{50}$ units Equation of circle: $(x+4)^2 + (y+2)^2 = 50$
14c	Let the coordinates of the other end of the diameter passes through $A = C(x, y)$. Since AC is the diameter, B is the midpoint of AC. $\left(\frac{x+1}{2}, \frac{y+3}{2}\right) = (-4, -2)$ $\frac{x+1}{2} = -4, \quad \frac{y+3}{2} = -2$ $x = -9, \ y = -7$ Equation of another tangent which is parallel to $y = -x + 4$: $y + 7 = -1(x + 9)$ $y = -x - 16$

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