BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME | CLASS | REGISTER NUMBER

ADDITIONAL MATHEMATICS
Paper 2

4049 / 02
2 hours 15 minutes

90

Answer **all** the questions.

1 **(a)** Solve the equation $2(4^x)-8=11(2^x)-2^{2x+3}$, giving your answer correct to 3 significant figures. [5]

(b) Show that the solution from **part (a)** may be written in the form $m-\log_n p$ where m, n and p are integers to be determined. [2]

2 The mass, M grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $M=200e^{-kt}$, where k is a constant.
The mass of the substance was 123.7 g after being observed for 2 years.

(a) **(i)** State the initial mass of the substance. [1]

(ii) Show that k is approximately 0.240, correct to 3 significant figures. [1]

(iii) Find the mass of the substance when $t = 5$, [1]

(iv) Find the value of t when the mass of the substance is 15% of its initial mass. Give your answers correct to three significant figures. [2]

(b) Explain, with clear working, why the mass of the substance can never be more than 200 grams. [1]

(c) Sketch the graph of M against t. [2]

3 **(a)** It is given that $y=x^2\sqrt{2x-5}$, where $x \geq h$.

Show that $\frac{dy}{dx}=\frac{kx(x-2)}{\sqrt{2x-5}}$, where k is an integer and determine the value of h and of k. [4]

(b) Hence evaluate $\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\} dx$. [4]

4 **(a)** **(i)** Factorise completely x^3+64. [2]

(ii) **Hence**, express $\frac{3x^2}{x^3+64}$ in partial fractions. [5]

(b) Using the results in **part (a)**, or otherwise, find

$$\int \frac{2x^3+3x^2+128}{x^3+64}\,dx.$$ [3]

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5 A particle moves in a straight line so that, at time t seconds after passing a fixed point O, its velocity is v m/s, where $v = 4 + 8\cos 2t$. Find

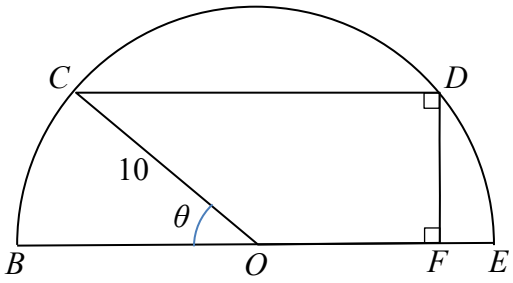
(a) the velocity of the particle at the instant it passes O, [1]

(b) the least value of the particle's acceleration, [1]

(c) the values of t, in terms of π, when the particle is at rest for $0 \le t \le 3$, [4]

(d) the distance travelled in the first 2 seconds. [4]

6



The diagram shows right angled trapezium $OCDF$ inside a semicircle with centre O and radius 10 cm such that angle BOC is θ radians, and angle CDF and angle OFD are right angles.

(a) Show that the perimeter, P cm, of trapezium $OCDF$ is given by

$$P = 10 + 30\cos\theta + 10\sin\theta$$ [2]

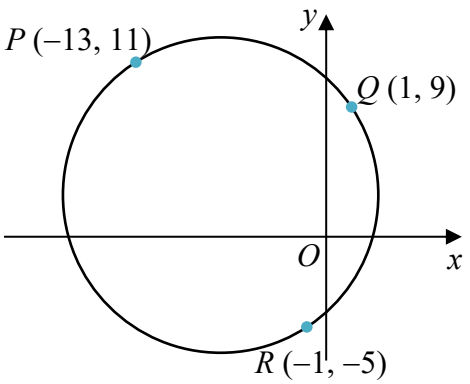
(b) Find the value of R when $10\sin\theta + 30\cos\theta$ is expressed as $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum perimeter of the trapezium. [3]

(c) Show that the area, A cm^2, of trapezium $OCDF$ is given by

$$A = 75\sin 2\theta$$ [2]

(d) The area of the trapezium varies with the value of θ. Find the value of θ for which the area has a stationary value and determine whether this area is a maximum or a minimum. [4]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, P, Q and R are points on the circle.

(a) Explain, with geometrical reason, why PR is the diameter of the circle. [3]

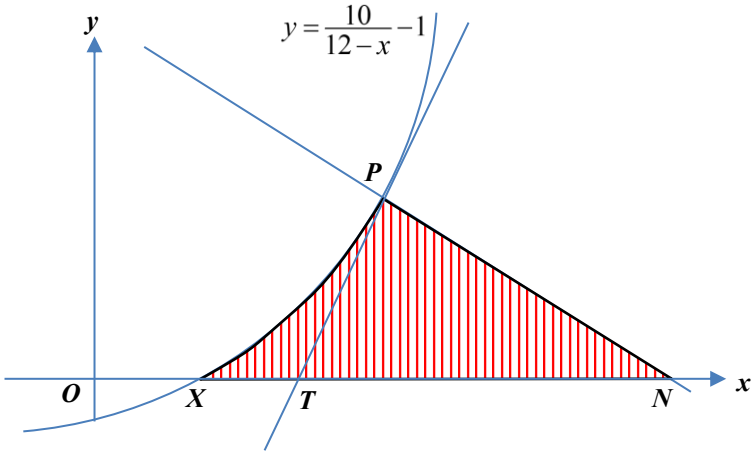
(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers. [3]

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(c) Find the equation of the perpendicular bisector of PQ. [3]

(d) The point S lies on the circle such that it is furthest from the point Q.
Show that the coordinates of S are $(-15, -3)$ and hence calculate the area of the quadrilateral $PQRS$. [3]

8



The diagram shows part of the curve $y = \frac{10}{12-x} - 1$ passing through the point $P(2k, k-1)$, where k is a constant. The curve meets the x-axis at the point X. The tangent and normal at P meet the x-axis at the points T and N respectively.

(a) Find the equation of the normal at P. [6]

(b) Find the **exact** area of the shaded region. [6]

9 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.83	1.87	1.96

The variables x and y are related by the equation $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$, where a and b are constants. One value of y has been recorded incorrectly.

(a) Show how $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ is transformed to plot a graph of xy against $x\sqrt{x}$. [1]

(b) Draw a straight line graph of xy against $x\sqrt{x}$ for the given data. [2]

(c) Using your graph,

(i) find an approximate value of y to replace the incorrect value, [2]

(ii) estimate the value of a and of b, [3]

(iii) find the value of y when $x = 2.52$. [2]

(d) A pair of values of x and y is considered acceptable only if the xy value is within 2% vertical difference from the straight line. A student recorded a pair of values such that $x = 4.50$ and $y = 1.86$.
Verify whether the recorded values by the student are acceptable. [2]

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\, r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

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ANSWERS

(Bedok South) 4E5N AM P2 (Prelim 2025) (w Ans)

1. (a) $x = 0.678$ (to 3 sig. fig.)

(b) $m = 3,\ n = 2,\ p = 5$

2. (a) **(i)** $M_0 = 200$ g

(ii) $k = 0.240$ (to 3 sig. fig.)

(iii) $M = 60.2$ g (to 3 sig. fig.)

(iv) $t = 7.90$ years (to 3 sig. fig.)

(c)



3. (a) $h = \frac{5}{2},\ k = 5$

(b) 300

4. (a) **(i)** $(x + 4)(x^2 - 4x + 16)$

(ii) $\frac{1}{x+4} + \frac{2x-4}{x^2-4x+16}$

(b) $2x + \ln(x^3 + 64) + c$

5. (a) $v = 12$ m/s

(b) Least $a = -16$ m/s^2

(c) $t = \frac{1}{3}\pi,\ \frac{2}{3}\pi$

(d) 10.3 m (to 3 sig. fig.)

6. (a) **Hint:** Draw a line $CG \perp BE$

(b) $R = 10\sqrt{10}$ or 31.6 (to 3 sig. fig.)
Max $P = 41.6$ cm (to 3 sig. fig.)

(c) **Hint:** Apply Double Angle formula

(d) $\theta = \frac{\pi}{4}$

7. (a) **Hint:** Right angle in semicircle

(b) $x^2 + y^2 + 14x - 6y - 42 = 0$

(c) $y = 7x + 52$

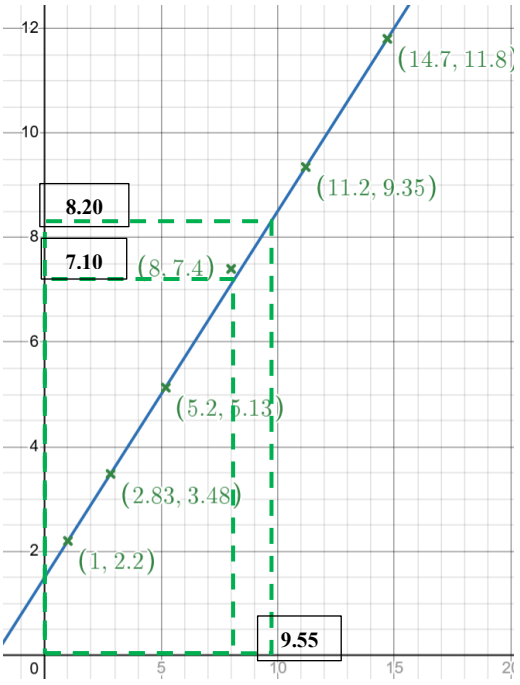
(d) **Hint:** The furthest two points on a circle is its diameter.
$Area\ of\ PQRS = 200$ units^2

8. (a) $y = -\frac{2}{5}x + 8$

(b) $10 \ln 5 + 12$

9. (a) **Hint:** Multiply equation with ax to get $xy = ab(x\sqrt{x}) + a$

(b)



(c) **(i)** $y = 1.78$ (to 3 sig. fig.)

(ii) $a = 1.50$ (to 3 sig. fig.)

$b = 0.467$ (to 3 sig. fig.)

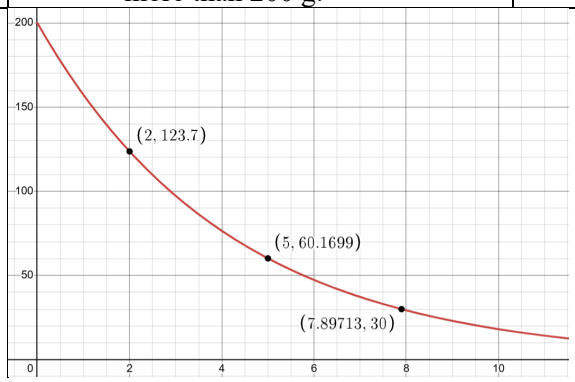
(iii) $y = 1.71$ (to 3 sig. fig.)

(d) Unacceptable,
Vertical difference $= 2.073\%$

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MARKING SCHEME

(Bedok South) 4E5N AM P2 (Prelim 2023) (w Ans)

Qn	Solution	Testing
1.	**Exponential Equations**	
(a)	$2(4^x) - 8 = 11(2^x) - 2^{2x+3}$ $2(2^{2x}) - 8 = 11(2^x) - (2^{2x})2^3$ $2(2^x)^2 - 8 = 11(2^x) - 8(2^x)^2$	Convert to base 2
	Let $u = 2^x$ $2u^2 - 8 = 11u - 8u^2$ $10u^2 - 11u - 8 = 0$ $(2u+1)(5u-8) = 0$ $u = -\frac{1}{2}$ or $u = \frac{8}{5}$	Apply substitution correctly Solving for u
	$2^x = -\frac{1}{2}$ (reject) $\quad 2^x = \frac{8}{5}$ $\lg 2^x = \lg \frac{8}{5}$ $x = \frac{\lg \frac{8}{5}}{\lg 2}$ $= 0.678072$ $= \mathbf{0.678}$ (to 3 sig. fig.)	Apply logarithm appropriately
(b)	From (a), $2^x = \frac{8}{5}$ $\log_2 2^x = \log_2 \frac{8}{5}$ $x \log_2 2 = \log_2 8 - \log_2 5$ $x = \log_2 2^3 - \log_2 5$ $= 3\log_2 2 - \log_2 5$ $= 3 - \log_2 5$ $= m - \log_n p$ $m = \mathbf{3},\ n = \mathbf{2},\ p = \mathbf{5}$	Apply $\log_2$ onto equation Power law Quotient law Identity property
2.	**Exponential Functions (Word Problems)**	
(a) (i)	When $t = 0$, $M = 200e^{-kt}$ $M_0 = 200e^{-k(0)}$ $= \mathbf{200\ g}$	
(a) (ii)	When $t = 2$, $M = 123.7$ $M = 200e^{-kt}$ $123.7 = 200e^{-k(2)}$ $\frac{123.7}{200} = e^{-2k}$ $\ln\left(\frac{123.7}{200}\right) = -2k$	Take natural logarithm on both sides of equation
	$k = -\frac{1}{2}\ln\left(\frac{123.7}{200}\right)$ $= 0.240229$ $= \mathbf{0.240}$ (to 3 sig. fig.)	
(a) (iii)	When $t = 5$, For $k = 0.24$: $M = 200e^{-kt}$, $= 200e^{-0.24(5)}$, $= 60.23884$ For $k = 0.240229$: $M = 200e^{-kt}$, $= 200e^{-0.240229(5)}$, $= 60.16991$ $= \mathbf{60.2\ g}$ (to 3 sig. fig.)	
(a) (iv)	$M = M_0 e^{-kt}$ $\frac{M}{M_0} = e^{-kt}$ $15\% = e^{-kt}$ $0.15 = e^{-kt}$ For $k = 0.24$: $0.15 = e^{-0.24t}$, $\ln 0.15 = -0.24t$, $t = -\frac{\ln 0.15}{0.24}$, $= 7.90467$ For $k = 0.240229$: $0.15 = e^{-0.240229t}$, $\ln 0.15 = -0.240229t$, $t = -\frac{\ln 0.15}{0.240229}$, $= 7.89713$ $t = \mathbf{7.90}$ **years** (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(b)	Since for $t \geq 0$, $e^{kt} \geq 1$ $\frac{1}{e^{kt}} \leq 1$ $e^{-kt} \leq 1$ $200e^{-kt} \leq 200$ $M \leq 200$ $\therefore$ mass of substance can never be more than 200 g.	
(c)	 G1 – Initial state (0, 200) G1 – Smooth curve decreasing towards asymptote t-axis	

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Qn	Solution	Testing
3.	**Differentiation (Product Rule)**	
(a)	For square root to be defined, $2x-5 \geq 0 \quad \Rightarrow \quad x \geq \frac{5}{2}$ $\therefore\ \boldsymbol{h = \frac{5}{2}}$	Square root criterion
	$y = x^2\sqrt{2x-5}$ $y = x^2\,(2x-5)^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x(2x-5)^{\frac{1}{2}} + x^2(2)\frac{1}{2}(2x-5)^{-\frac{1}{2}}$	Apply Product rule
	$\frac{dy}{dx} = 2x\sqrt{2x-5} + \frac{x^2}{\sqrt{2x-5}}$ $= \frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$ $= \frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$ $= \frac{5x^2-10x}{\sqrt{2x-5}}$ $\frac{dy}{dx} = \frac{5x(x-2)}{\sqrt{2x-5}}$ $\therefore\ k = \mathbf{5}$	Combine to single fraction
	Integration (Reverse of Differentiation)	
(b)	$\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}} + 6\right\} dx$ $= 2\int_3^7 \frac{5x(x-2)}{\sqrt{2x-5}}\,dx + \int_3^7 6\,dx$	Split into 2 integrals
	$= 2\left[x^2\sqrt{2x-5}\right]_3^7 + 6[x]_3^7$ $= 2\left\{7^2\sqrt{2(7)-5} - 3^2\sqrt{2(3)-5}\right\}$ $+ 6(7-3)$ $= 2\{49\sqrt{9} - 9\sqrt{1}\} + 24$ $= 276 + 24$ $= \mathbf{300}$	Integration as reversed differentiation Substitution of boundaries
	OR $\frac{dy}{dx} = \frac{5x(x-2)}{\sqrt{2x-5}}$ $\int_3^7 \frac{5x(x-2)}{\sqrt{2x-5}}\,dx = \left[x^2\sqrt{2x-5}\right]_3^7$	Integration as reversed differentiation

Qn	Solution	Testing
	$\int_3^7 \frac{10x(x-2)}{\sqrt{2x-5}}\,dx = 2\left[x^2\sqrt{2x-5}\right]_3^7$ $\int_3^7 \frac{10x(x-2)}{\sqrt{2x-5}}\,dx + \int_3^7 6\,dx$ $= 2\left\{7^2\sqrt{2(7)-5} - 3^2\sqrt{2(3)-5}\right\}$ $+ \int_3^7 6\,dx$ $\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}} + 6\right\} dx$ $= 2\{49\sqrt{9} - 9\sqrt{1}\} + 6[x]_3^7$ $= 276 + 6(7-3)$ $= \mathbf{300}$	Transform to required integral Substitution of boundaries
4.	**Cubic Identities**	
(a) (i)	$x^3 + 64$ $= (x)^3 + (4)^3$ $= (x+4)((x)^2 - (x)(4) + (4)^2)$ $= \boldsymbol{(x+4)(x^2-4x+16)}$	Apply sum of cubes identity
	Partial Fractions	
(a) (ii)	$\frac{3x^2}{x^3+64}$ $= \frac{3x^2}{(x+4)(x^2-4x+16)}$ $= \frac{A}{x+4} + \frac{Bx+C}{x^2-4x+16}$ $A(x^2-4x+16)$ $+ (Bx+C)(x+4) = 3x^2$	Express proper fraction in partial fractions with unknown coefficients
	Let $x = -4$, $A((-4)^2 - 4(-4) + 16) = 3(-4)^2$ $A(16+16+16) = 3(16)$ $A = 1$ Let $x = 0$, $A(16) + C(4) = 0$ $1(16) + 4C = 0$ $16 + 4C = 0$ $C = -4$ Let $x = 1$, $A(1-4+16)$ $+ (B+C)(1+4) = 3$ $1(13) + (B-4)(5) = 3$ $13 + 5B - 20 = 3$	Use Substitution and/or Equating coefficients methods to find unknowns Award 1 mark for at least 2 correct values

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Qn	Solution	Testing
	$B = 2$	
	OR Equating coefficient of x^2 : $A + B = 3$, $B = 3 - A \cdots (1)$ x : $-4A + 4B + C = 0 \cdots (2)$ x^0 : $16A + 4C = 0$, $C = -4A \cdots (3)$ Subst (1) & (3) into (2), $-4A + 4(3 - A) + (-4A) = 0$ $-4A + 12 - 4A - 4A = 0$ $12A = 12$ $A = 1$ $B = 3 - 1 = 2$ $C = -4(1) = -4$	
	$\therefore \dfrac{3x^2}{x^3 + 64} = \dfrac{\mathbf{1}}{\boldsymbol{x + 4}} + \dfrac{\mathbf{2x - 4}}{\boldsymbol{x^2 - 4x + 16}}$	
	Indefinite Integral	
(b)	$\int \dfrac{2x^3 + 3x^2 + 128}{x^3 + 64}\,dx$ $= \int \dfrac{2(x^3 + 64)}{x^3 + 64}\,dx + \int \dfrac{3x^2}{x^3 + 64}\,dx$ $= \int 2\,dx + \int \dfrac{3x^2}{x^3 + 64}\,dx$ $\boldsymbol{= 2x + \ln(x^3 + 64) + c}$	Split into 2 integrals Correct integration
	5. Kinematics (Trigonometric Functions)	
(a)	When particle passes O, $t = 0$, Hence, $v = 4 + 8\cos 2(0)$ $\boldsymbol{= 12\ m/s}$	
(b)	$v = 4 + 8\cos 2t$ $\dfrac{dv}{dt} = -(2)8\sin 2t$ $a = -16\sin 2t$ Least acceleration $\boldsymbol{= -16\ m/s^2}$	
(c)	$0 \le t \le 3$ $0 \le 2t \le 6$ When particle is at rest, $v = 0$ m/s,	

Qn	Solution	Testing
	$4 + 8\cos 2t = 0$ $\cos 2t = -\dfrac{1}{2}$ Basic angle $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\pi$ For $\cos 2t < 0$, $2t$ lies in Q2, Q3 $2t = \pi - \dfrac{1}{3}\pi,\ \pi + \dfrac{1}{3}\pi$ $= \dfrac{2}{3}\pi,\ \dfrac{4}{3}\pi$ $t = \dfrac{1}{3}\pi,\ \dfrac{2}{3}\pi$	Correct Substitution Basic Angle Angles in correct quadrants
(d)	$s = \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t)\,dt$ $s = 4t + 4\sin 2t + c$ t: 0, $\frac{1}{3}\pi$, 2 s: 0, 7.65289, 4.97279 Distance travelled in the first 2 sec $= (7.65289 - 0)$ $+(7.65289 - 4.97279)$ $= 10.33299$ $\boldsymbol{= 10.3\ m}$ (to 3 sig. fig.)	Correct Integration
(d)	Distance travelled in the first 2 sec $= \left\lvert \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t)\,dt \right\rvert$ $+ \left\lvert \int_{\frac{\pi}{3}}^{2} (4 + 8\cos 2t)\,dt \right\rvert$ $= \left\lvert [4t + 4\sin 2t]_0^{\frac{\pi}{3}} \right\rvert$ $+ \left\lvert [4t + 4\sin 2t]_{\frac{\pi}{3}}^{2} \right\rvert$ $= \left\lvert 4\left(\frac{\pi}{3}\right) + 4\left(\sin\frac{2\pi}{3}\right) \right\rvert$ $+ \left\lvert 4\left(2 - \frac{\pi}{3}\right) + 4\left(\sin 4 - \sin\frac{2\pi}{3}\right) \right\rvert$ $= 7.65289 + 2.6801$ $= 10.33299$ $\boldsymbol{= 10.3\ m}$ (to 3 sig. fig.)	Correct Expressions Correct Integration Correct Substitution

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Qn	Solution	Testing
6.	**R-Formulae**	
(a)	Draw a line $CG \perp BE$. In ΔCGO, $\angle CGO = \frac{\pi}{2}$, $OC = 10$ $CG = 10\sin\theta$ $GO = 10\cos\theta$ $DF = CG = 10\sin\theta$ $CD = GF = 2GO$ $= 20\cos\theta$ $FO = GO = 10\cos\theta$ $P = OC + CD + DF + FO$ $= 10 + 20\cos\theta + 10\sin\theta$ $+ 10\cos\theta$ $P = 10 + 30\cos\theta + 10\sin\theta$	
(b)	$30\cos\theta + 10\sin\theta$ $= R\cos(\theta - \alpha)$ $= \sqrt{30^2 + 10^2}\cos(\theta - \alpha)$ $= 10\sqrt{10}\cos(\theta - \alpha)$ where $R = \mathbf{10\sqrt{10}}$ $= 31.6227$ $\mathbf{= 31.6}$ (to 3 s.f.) $Max\ P = 10 + R$ $\mathbf{= 41.6\ cm}$ (to 3 s.f.)	
(c)	$A = \frac{1}{2}DF(CD + FO)$ $= \frac{1}{2}(10\sin\theta)(20\cos\theta + 10\cos\theta)$ $A = 5\sin\theta\,(30\cos\theta)$ $A = 150\sin\theta\cos\theta$ $A = 75\sin 2\theta$	Area $= \frac{1}{2} \times base \times ht$ Double angle formula
	Stationary Values	
(d)	$\frac{dA}{d\theta} = 150\cos 2\theta$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ For stationary value, $\frac{dA}{d\theta} = 0$ $150\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$	
	$\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ $= -300\sin\frac{\pi}{2}$ $= -300$ $\frac{d^2A}{d\theta^2} < 0$	
	$A = 75\sin 2\theta$ $= 75\sin\frac{\pi}{2}$ $= 75$	Value of A is not required
	A has a maximum area (of 75 cm^2) when $\theta = \frac{\pi}{4}$.	
7.	**Circles & Coordinate Geometry**	
(a)	$P(-13, 11)$ $Q(\ 1,\ 9)$ $R(-1, -5)$	
	Gradient of PQ: $m_{PQ} = \frac{11-9}{-13-1} = -\frac{1}{7}$ Gradient of QR: $m_{QR} = \frac{9-(-5)}{1-(-1)} = 7$	
	Since $m_{PQ} \times m_{QR} = -\frac{1}{7} \times 7 = -1$ $\Rightarrow PQ$ is perpendicular to QR $\Rightarrow \angle PQR = 90°$ **(right angle in semicircle)** $\Rightarrow PR$ is the diameter of circle.	Applying $m_1 \times m_2 = -1$
	OR $PQ^2 = (-13-1)^2 + (11-9)^2$ $= 200$ $QR^2 = (1-(-1))^2 + (9-(-5))^2$ $= 200$ $PR^2 = (-13+1)^2 + (11+5)^2$ $= 400$ Since $PR^2 = PQ^2 + QR^2 = 400$, by converse of Pythagoras' Theorem, $\angle PQR = 90°$. **(right angle in semicircle)** $\Rightarrow PR$ is the diameter of circle.	

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Qn	Solution	Testing
	OR Midpoint of PR, $M = \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7,\ 3)$ ... [1] $PM = \sqrt{(-13+1)^2 + (11-3)^2}$ $= 10$ $QM = \sqrt{(1+7)^2 + (9-3)^2}$ $= 10$ $RM = (-1+7)^2 + (-5-3)^2 = 10$ $\therefore PM = QM = RM = 10$ ... [2] From [1] and [2], **midpoint of chord *PR* is equidistant to *P*, *Q* & *R*** $\Rightarrow PR$ is the diameter of circle.	
(b)	As centre is midpoint of diameter, centre $= \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7, 3)$ radius $= \frac{1}{2}PR$ $= \frac{1}{2}\sqrt{(-13-(-1))^2 + (11-(-5))^2}$ $= \frac{1}{2}\sqrt{400}$ $= 10$ Equation of circle $(x-(-7))^2 + (y-3)^2 = 10^2$ $x^2 + 14x + 49 + y^2 - 6y + 9 = 100$ $\boldsymbol{x^2 + y^2 + 14x - 6y - 42 = 0}$	Award 1 mark for either centre or radius correct Correct substitution
(c)	Let $T(x, y)$ be a general point on the perpendicular bisector of PQ. $PT = QT$ $\sqrt{(x-(-13))^2 + (y-11))^2}$ $= \sqrt{(x-1)^2 + (y-9)^2}$ $(x+13)^2 + (y-11)^2$ $= (x-1)^2 + (y-9)^2$ $x^2 + 26x + 169 + y^2 - 22y + 121$ $= x^2 - 2x + 1 + y^2 - 18y + 81$ $4y = 28x + 208$ $\boldsymbol{y = 7x + 52}$	
	OR Perpendicular bisector passes through the centre. Using centre $(-7,\ 3)$ and $m_{\perp PQ} = 7$	

Qn	Solution	Testing
	Equation of perpendicular bisector of QR is $y - 3 = 7(x-(-7))$ $\boldsymbol{y = 7x + 52}$	
	OR Perpendicular bisector passes through the midpoint of chord. Midpoint of PQ $= \left(\frac{-13+1}{2}, \frac{11+9}{2}\right)$ $= (-6,\ 10)$ $m_{\perp PQ} = 7$ Equation of perpendicular bisector of QR is $y - 10 = 7(x-(-6))$ $\boldsymbol{y = 7x + 52}$	
(d)	The furthest two points on a circle is the diameter of the circle. $\therefore$ centre $(-7, 3)$ is midpoint of diameter QS, $\frac{x_Q + x_S}{2} = -7$ | $\frac{y_Q + y_S}{2} = 3$ $1 + x_S = -14$ | $9 + y_s = 6$ $x_S = -15$ | $y_S = -3$ $\therefore \boldsymbol{S(-15, -3)}$	Diameter is the longest chord Mid-point
	Area of quadrilateral $PQRS$ $= \frac{1}{2}\begin{vmatrix} P & Q & R & S & P \\ -13 & 1 & -1 & -15 & -13 \\ 11 & 9 & -5 & -3 & 11 \end{vmatrix}$ $= \frac{1}{2}\begin{vmatrix} (-117 - 5 + 3 - 165) \\ -(11 - 9 + 75 + 39) \end{vmatrix}$ $\boldsymbol{= 200\ \text{units}^2}$	Shoelace method
8.	**Tangent & Normal**	
(a)	$y = \frac{10}{12-x} - 1$ ... ... [1] Subst $P(2k, 4)$ into [1] $4 = \frac{10}{12-2k} - 1$ $5 = \frac{5}{6-k}$ $6 - k = 1$ $k = 5$ $P(10, 4)$ $\frac{dy}{dx} = (-10)(12-x)^{-2}(-1)$ $\frac{dy}{dx} = \frac{10}{(12-x)^2}$ ... ... [2]	Chain rule

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Qn	Solution	Testing
	Subst $P(10,4)$ into [2] $\frac{dy}{dx} = \frac{10}{(12-10)^2} = \frac{5}{2}$ $m_{PT} = \frac{5}{2}$ and $m_{PN} = -\frac{2}{5}$ Using $P(10,4)$ and $m_{PN} = -\frac{2}{5}$, **Equation of normal at *P*** $y - 4 = -\frac{2}{5}(x-10)$ $y - 4 = -\frac{2}{5}x + 4$ $\boldsymbol{y = -\frac{2}{5}x + 8}$ …[3]	
	Area under Curve	
(b)	Subst $X(x_X, 0)$ into [1] $0 = \frac{10}{12 - x_X} - 1$ $\frac{10}{12 - x_X} = 1$ $10 = 12 - x_X$ $x_X = 2$ Subst $N(x_N, 0)$ into [3] $0 = -\frac{2}{5}x_N + 8$ $\frac{2}{5}x_N = 8$ $x_N = 20$ Area of shaded region $\int_2^{10} \left(\frac{10}{12-x} - 1\right) dx + \frac{1}{2}(20-10)(4)$ $= [-10\ln(12-x) - x]_2^{10} + 20$ $= -[10\ln(12-x) + x]_2^{10} + 20$ $= [10\ln(12-x) + x]_{10}^{2} + 20$ $= 10\ln\left(\frac{12-2}{12-10}\right) + \{2-10\} + 20$ $= 10\ln 5 + 12$ **OR** Area of shaded region $\int_2^{10} \left(\frac{10}{12-x} - 1\right) dx + \int_{10}^{20} \left(-\frac{2}{5}x + 8\right) dx$ $= [-10\ln(12-x) - x]_2^{10} +$ $\left[-\frac{1}{5}x^2 + 8x\right]_{10}^{20}$	Integration Substitution of boundaries Integration

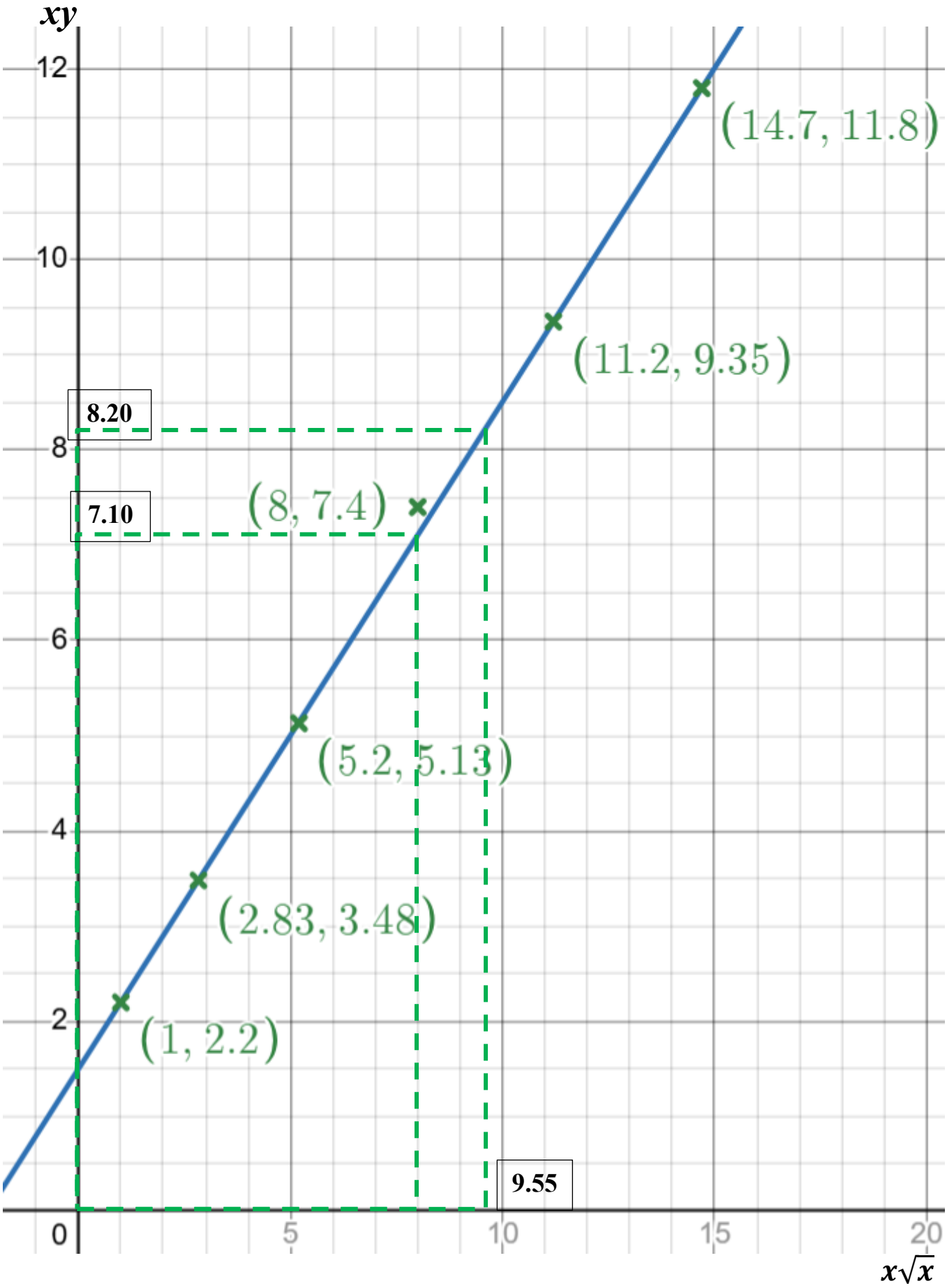
Qn	Solution	Testing
	$= -10\ln\left(\frac{12-10}{12-2}\right) - \{10-2\} -$ $\frac{1}{5}\{20^2 - 10^2\} + 8\{20-10\}$ $= 10\ln 5 + 12$	Substitution of boundaries
	9. Linear Law	
(a)	$\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ Multiply equation with ax, $xy = a + abx\sqrt{x}$ $xy = ab(x\sqrt{x}) + a$	Linearise equation $Y = mX + C$
(b)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>2.20</td><td>1.74</td><td>1.71</td><td>1.85</td><td>1.87</td><td>1.96</td></tr><tr><td>$x\sqrt{x}$</td><td>1.00</td><td>2.83</td><td>5.20</td><td>8.00</td><td>11.2</td><td>14.7</td></tr><tr><td>xy</td><td>2.20</td><td>3.48</td><td>5.13</td><td>7.40</td><td>9.35</td><td>11.8</td></tr></table>	Tabulate values for Y vs. X
		G1 – Plotted points G1 – Best-fit line
(c) **(i)**	From the graph, the incorrect point is (8, 7.40). Corresponding to $x = 4$ (in the table), For $x\sqrt{x} = 8$ Correct $xy = 7.05$ *to* 7.15 $4y = 7.05$ *to* 7.15 $y = 1.7625$ *to* 1.7875 $\boldsymbol{y = 1.76}$ *to* $\boldsymbol{1.79}$ (to 3 sig. fig.)	Read from graph
(c) **(ii)**	From the graph, xy intercept $= 1.5$ $\boldsymbol{a = 1.50}$ (to 3 sig. fig.) Gradient $= \frac{11.8 - 2.20}{14.7 - 1.00}$ $ab = 0.7007$ $1.50\,b = 0.7007$ $b = 0.467153$ $\boldsymbol{b = 0.467}$ (to 3 sig. fig.)	Read from graph
(c) **(iii)**	When $x = 2.52$, for $x\sqrt{x} = 2.52\sqrt{2.52}$ $= 4.00037$ From the graph, for $x\sqrt{x} = 4.00$ $xy = 4.3$ $2.52\,y = 4.3$ $y = 1.70634$ $\boldsymbol{y = 1.71}$ (to 3 sig. fig.)	Read from graph

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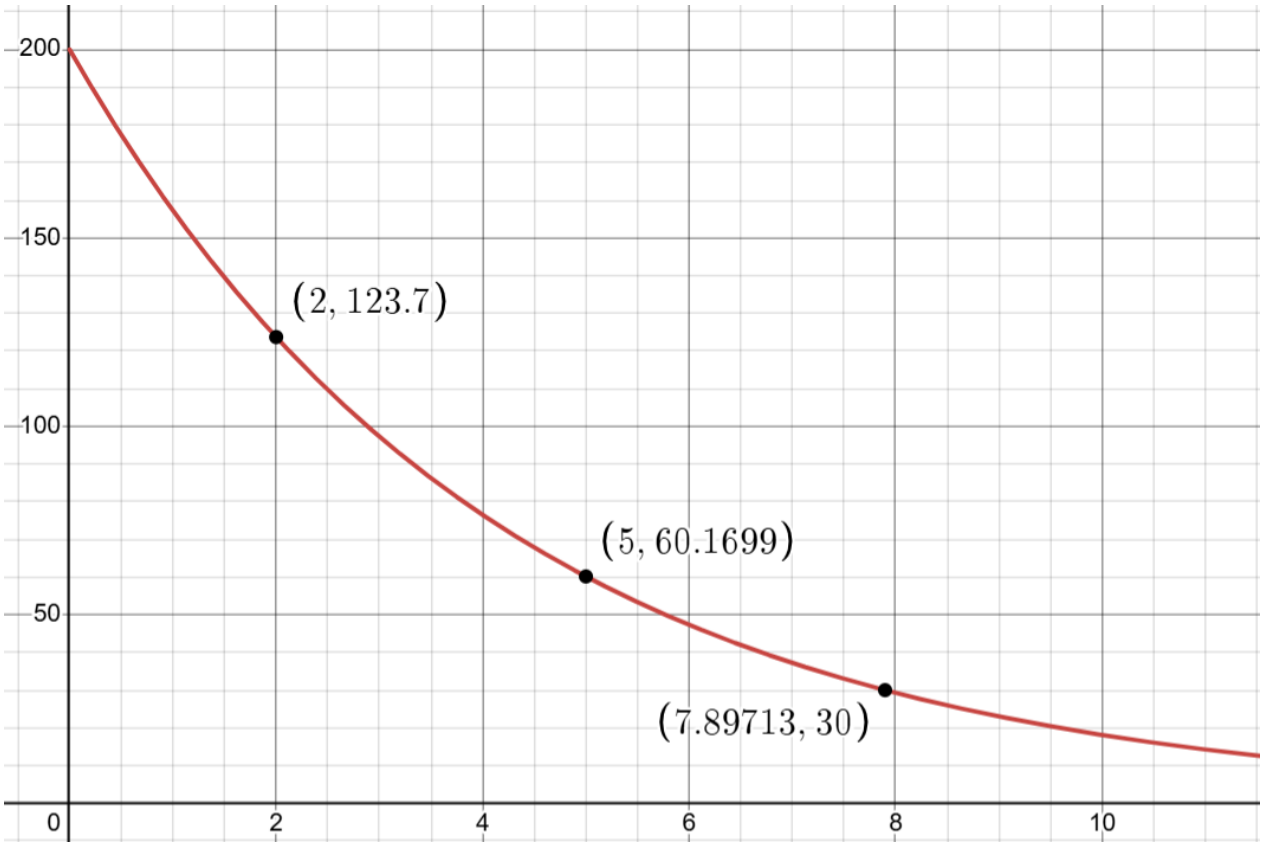
Qn	Solution	Testing
(d)	From readings of experiment, $x = 4.50$ and $y = 1.86$. $x\sqrt{x} = 9.5459$ $xy = (4.50)(1.86)$ $= 8.37$ From the graph, for $x\sqrt{x} = 9.5459$ $xy = 8.20$ Vertical difference $= \frac{8.37 - 8.20}{8.20} \times 100\%$ $= 2.073\% > 2\%$ Since vertical difference is **more than 2%**, the experimental readings are **unacceptable**.	Read from graph

Qn	Solution	Testing
	OR Acceptable range $= 8.20 \times (100 \pm 2)\%$ $= 8.036 \ to \ 8.364$ Since $xy = 8.37$ is **out of** the acceptable **range**, the experimental readings are **unacceptable**.	

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