North Vista Secondary
Secondary 4 Express Mathematics (4052)
2024
Paper 2 Marking Scheme

Qn	Solutions		Mark	AO	Total
1(a)	$-5 < 3x - 2 < 13$ $-3 < 3x < 15$ $-1 < x < 5$	$-3 < 3x$ and $3x < 15$ $-1 < x$ and $x < 5$ $-1 < x < 5$	M1 A1	**AO1**	**2**
1(b)	$\frac{1}{2}x + y = 5$ --- (1) $2x - 3y = 13$ --- (2) $(1) \times 4$ $2x + 4y = 20$ --- (3) $(2) - (3)$ $-7y = -7$ $y = 1$ When $y = 1$, $2x - 3(1) = 13$ $x = 8$ $\therefore x = 8,\ y = 1$		M1 A1 A1	**AO1**	**3**
1(c)(i)	When $r = 7$ and $q = -15$,			**AO1**	**1**

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Qn	Solutions	Mark	AO	Total
	$p = \sqrt[3]{\frac{r+q}{4r-1}}$ $= \sqrt[3]{\frac{7-15}{4(7)-1}}$ $= -\frac{2}{3}$	B1		
1(c)(ii)	$p = \sqrt[3]{\frac{r+q}{4r-1}}$ $p^3 = \frac{r+q}{4r-1}$ $p^3(4r-1) = r+q$ $4p^3r - p^3 = r+q$ $4p^3r - r = p^3 + q$ $r(4p^3 - 1) = p^3 + q$ $r = \frac{p^3+q}{4p^3-1}$	M1 M1 A1	**AO1**	**3**
1(d)	$\frac{15}{2x-1} = x+3$ $(2x-1)(x+3) = 15$ $2x^2 + 5x - 18 = 0$ $(2x+9)(x-2) = 0$ $x = -4\frac{1}{2}$ *or* $x = 2$	M1 M1 A1	**AO1**	**3**
2(a)(i)	Total Food Waste Output		**AO1**	**1**

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Qn	Solutions	Mark	AO	Total
	$= 565\ 000 + 605\ 000 + 640\ 000$ $= 1.81 \times 10^6$ tonnes	B1		
2(a)(ii)	Percentage increase (2009-2010) $= \frac{99000 - 75000}{75000} \times 100$ $= 32\%$ Food Waste Recycled in 2011 $= \frac{132}{100} \times 99000$ $= 130680$ tonnes	M1 A1	**AO1**	**2**
2(a)(iii)	Per capital food waste $= \frac{565000 \times 1000}{4.84 \times 1000000 \times 365}$ $= 0.319823$ $= 0.320$ kg/day Per capital food waste $= \frac{565000 \times 1000}{4.84 \times 1000000 \times 366}$ $= 0.318949$ $= 0.319$ kg/day	M1 A1	**AO1**	**2**
2(a)(iv)	Food Waste Output $= \frac{509000}{100 - 8.6} \times 100$ $= 556892.779$ $= 557000$ tonnes	M1 A1	**AO1**	**2**
2(b)(i)	Perimeter of the lake on map A $= \frac{1700}{612} \times 9$ $= 25$ cm	M1 A1	**AO1**	**2**
2(b)(ii)	Actual area of lake $= 36 \times (68)^2$ $= 166464\ \text{m}^2$	M1	**AO2**	**2**

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Qn	Solutions	Mark	AO	Total
	Area of lake on Map B $= \frac{166464}{51^2}$ $= 64\ \text{cm}^2$	A1		
3(a)	$2\pi r = 120 - 4x$ $r = \frac{120 - 4x}{2\pi}$ $= \frac{2(60 - 2x)}{2\pi}$ $= \frac{60 - 2x}{\pi}$ (shown)	M1 A1	**AO2**	**2**

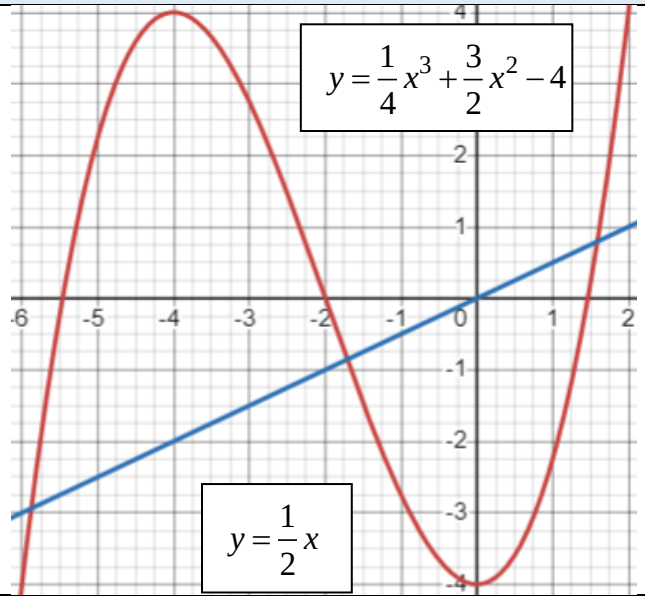
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Qn	Solutions	Mark	AO	Total
3(b)	$x^2 = \pi\left(\frac{60-2x}{\pi}\right)^2$ $x^2 = \frac{(60-2x)^2}{\pi}$ $\pi x^2 = 3600 - 240x + 4x^2$ $(4-\pi)x^2 - 240x + 3600 = 0$ (shown)	M1 A1	**AO2**	**2**
3(c)	$x = \frac{-(-240) \pm \sqrt{(-240)^2 - 4(4-\pi)(3600)}}{2(4-\pi)}$ $= \frac{240 \pm \sqrt{45238.93421}}{2(4-\pi)}$ $= 263.68$ or 15.90	M1 M1 A1, A1	**AO1**	**4**
3(d)	Reject $x = 263.68$ because - the perimeter of the square ($4x$) must be less than the length of the wire - the radius of the circle cannot be negative	B1	**AO3**	**1**
4(a)(i)	Mean = 166 cm	B1	**AO1**	**1**
4(a)(ii)	Standard deviation $= \sqrt{\frac{331532}{12} - (166)^2}$ $= 8.47$ cm	B1	**AO1**	**1**

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Qn	Solutions	Mark	AO	Total
4(a)(iii)	1. Since the **mean height** of Group B **(168 cm)** is **greater mean height** of Group A (**166 cm**), the **students in Group B are generally taller on average**. 2. Since standard deviation of heights of Group B (**9.5 cm**) is **greater** standard deviation of heights of Group A (**8.47 cm**), the **heights of students in Group B have a wider spread / less consistent.**	B1 B1	**AO3**	**2**
4(b)(i)	$\frac{12}{21}=\frac{4}{7}$	B1	**AO1**	**1**
4(b)(ii)	$\frac{9}{17}\times\frac{8}{16}$ $=\frac{9}{34}$	M1 A1	**AO1**	**2**
4(b)(iii)	$\left(\frac{23}{40}\times\frac{17}{39}\times\frac{16}{38}\right)\times 3$ $=\frac{391}{1235}$	M1 A1	**AO1**	**2**
5(a)	–4	B1	**AO1**	**1**

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Qn	Solutions	Mark	AO	Total
5(b)		P2 (9 points plotted correctly) P1 (7-8 points plotted correctly) C1 (Smooth curve through at least 7 points) Tolerate 1mm for plotting and drawing curve through points	**AO1**	**3**
5(c)	$k=-5$ or $k=5$	B2	**AO2**	**2**
5(d)	$x^3+6x^2-2x-16=0$ $\frac{x^3}{4}+\frac{3x^2}{2}-\frac{1}{2}x-4=0$ $\frac{x^3}{4}+\frac{3x^2}{2}-4=\frac{1}{2}x$ Draw the line $y=\frac{1}{2}x$. (Must label equation of line on graph) $x=-5.85,-1.7,1.6\ (\pm 0.1)$	M1 M1 A1	**AO2**	**3**
5(e)	$3.75\ (\pm 0.5)$	M1 – tangent line A1	**AO2**	**2**
6(a)(i)	$T_n=2^{2n}$ (or $T_n=4^n$)	B1	**AO2**	**1**

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Qn	Solutions	Mark	AO	Total
6(a)(ii)	$R_n = 4^{3n+1}$ (or $R_n = 2^{2+6n}$)	B1	AO2	1
6(a)(iii)(a)	$Q_n = \dfrac{4^{3n+1}}{2^{2n}}$ $= \dfrac{2^{2(3n+1)}}{2^{2n}}$ $= 2^{6n+2-2n}$ $= 2^{4n+2}$ (shown)	 B1	AO2	1
6(a)(iii)(b)	$2^{4n+2} = 128$ $2^{4n+2} = 2^7$ $4n + 2 = 7$ $n = 1.25$ 128 is not a term of sequence Q_n as n = 1.25 is not a **positive integer**. (or positive whole number)	 M1 A1	AO3	2
6(b)(i)	$\overrightarrow{AB} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 12 \end{pmatrix}$ $= \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{6^2 + (-8)^2}$ $= 10$ units or *OR* $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(4-10)^2 + (12-4)^2}$ $= 10$ units $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(10-4)^2 + (4-12)^2}$ $= 10$ units	 M1 A1	AO1	2

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Qn	Solutions	Mark	AO	Total
6(b)(ii)	$\overrightarrow{AC} = \frac{1}{2}\overrightarrow{BA}$ $\overrightarrow{OC} - \begin{pmatrix} 4 \\ 12 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -6 \\ 8 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 12 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ 16 \end{pmatrix}$ $C\,(1, 16)$	M1 A1	**AO1**	**2**
7(a)	$\frac{\sin \angle ACB}{30} = \frac{\sin 95°}{38}$ $\angle ACB = \sin^{-1}\left(\frac{30 \sin 95°}{38}\right)$ $= 51.856°$ $\angle ABC = 180 - 95 - 51.856$ $= 33.144$ $= 33.1°$	M1 M1 A1	**AO2**	**3**
7(b)	Let the shortest distance from A to BC be h and greatest angle of depression be θ. $\sin 33.144 = \frac{h}{30}$ $h = 30 \times \sin 33.144$ $= 16.402$ m	M1[ECF on $\angle ABC$]	**AO2**	**3**

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Qn	Solutions	Mark	AO	Total
	$\tan\theta = \frac{2.5}{16.402}$ $\theta = 8.6663°$ $= 8.7°$ (1 d.p.) Note: Students who found the length of *AC* in 7(b) will be awarded **M2 in 7c only if *AC* is used to calculate the speed of Ali,** otherwise, **M2 is not awarded** for working to find *AC* in 7b	M1 A1		
7(c)	$\frac{AC}{\sin 33.144} = \frac{38}{\sin 95}$ $AC = \frac{38\sin 33.144}{\sin 59}$ $= 20.855$ m Time taken by Ken $= \frac{38}{4}$ $= 9.5$ s Speed of Ali $= \frac{20.855}{9.5-3}$ $= 3.2085$ m/s $=3.21$ m/s (3 s.f.)	M1 [ECF on $\angle ABC$] M1 M1 A1	**AO2**	**4**
8(a)	Reflex $\angle COA = 360 - 92$($\angle$s at a point) $= 268^o$ $\angle CDA = \frac{268}{2}$(angle at centre = 2 angle at circumferene) $= 134^o$	M1		**2**

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Qn	Solutions	Mark	AO	Total
	Alternative Method $\angle CBA = \frac{92}{2}$ (angle at centre = 2 angle at circumferene) $= 46^o$ $\angle CDA = 180 - 46$ (angless in opp segments) $= 134^o$	A1 (M1) (A1)		
8(b)	$\angle OBT = \angle OAT = 90^o$ (tangent $\perp$ radius) $\angle AOB = 360 - 90 - 90 - 38$($\angle$ sum of quad) $= 142^o$ $\angle BOC = 360 - 92 - 142$($\angle$s at a point) $= 126^o$ $\angle BCO = \frac{180 - 126}{2}$ (base angles of isos triangle) $= 27^o$ Alternative Method $\angle OBT = \angle OAT = 90^o$ (tangent $\perp$ radius) $\angle AOB = 360 - 90 - 90 - 38$($\angle$ sum of quad) $= 142^o$ $\angle OBA = \frac{180 - 142}{2}$ (base angles of isos triangle) $= 19^o$ $\angle CBA = \frac{92}{2}$ (angle at centre = 2 angle at circumferene) $= 46^o$	M1 M1 A1 (M1) (M1)	**AO1**	**3**

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Qn	Solutions	Mark	AO	Total
	$\angle BCO = 46 - 19$ $= 27^o$ Alternative Method to find $\angle AOB$ $\angle OBT = \angle OAT = 90^o$ (tangent $\perp$ radius) $\angle BTO = \angle AT) = \frac{38}{2} = 19^o$ (tangents from ext point) $\angle AOT = 180 - 90 - 19$ ($\angle$ sum of triangle) $= 71^o$ $\angle AOB = 71 \times 2$ (tangents from ext point) $= 142^o$ Note: Maximum of 1 marks awarded in 8(b) for 2 or more incorrect or missing reasons.	(A1)		
8(c)	$\tan 19° = \frac{7}{BT}$ $BT = \frac{7}{\tan 19°}$ $= 20.329$ cm Area of quad $TBOA = 2 \times \frac{1}{2} \times 20.329 \times 7$ $= 142.306$ cm^2 Area of major sector $OBCA = \frac{92 + 126}{360} \times \pi \times 7^2$ $= 93.218$ cm^2	M1 M1 M1	**AO2**	**4**

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Qn	Solutions	Mark	AO	Total
	Requied area $= 142.306 + 93.218$ $= 235.524$ $= 236\ \text{cm}^2$	A1		
9(a)	Volume of 1st sculpture $= \frac{4}{3} \times \pi \times 14^3 + \pi \times 2.5^2 \times 6$ $= 11611.85\ \text{cm}^3$ Volume of 2nd sculpture $= \left(\frac{1}{2}\right)^3 \times 11611.85$ $= 1451.481\ \text{cm}^3$ Mass of 2 sculptures $= (11611.85 + 1451.481) \times 1.5$ $= 19594\ \text{g}$ $= 19.594\ \text{kg}$ (show at least 5 sf) $= 19.59\ \text{kg}$ (shown) Notes: **A1 not awarded** for students **did not show** the value of the total mass to **at least 5 sig fig** before rounding off to 4 sig fig. <u>Alternative Method</u>	M1 M1 M1 A1 (M1)	**AO2**	**4**

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Qn	Solutions	Mark	AO	Total
	Volume of 1st sculpture $= \frac{4}{3} \times \pi \times 14^3 + \pi \times 2.5^2 \times 6$ $= 11611.85\ \text{cm}^3$ Mass of 1st sculpture $= 11611.85 \times 1.5$ $= 17417.775\ \text{g}$ Mass of 2nd sculpture $= \left(\frac{1}{2}\right)^3 \times 17417.775$ $= 2177.221\ g$ Mass of 2 sculptures $= 17417.775 + 2177.221$ $= 19594\ \text{g}$ $= 19.594\ \text{kg}$ (show at least 5 sf) $= 19.59\ \text{kg}$ (shown)	(M1) (M1) (A1)		
9(b)	Packs of clay for 2 sculptures $= \frac{19.5975}{5}$ $= 3.9195$ $= 4$ Total cost of clay for 2 sculptures $= 4 \times 41.50$ $= \$166$ Total cost of postage for 2 sculptures $= 40 + (20 - 5) \times 7$ $= \$145$ <u>Carton size: XXL</u> Volume of empty space in carton $= (55 \times 32 \times 32) - (11611 + \frac{11611}{8})$ $= 43257.625\ \text{cm}^3$	M1[cost of clay] M1[Airmail cost] M1[empty space in carton]	**AO3**	**6**

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Qn	Solutions	Mark	AO	Total
	Total cost of clay + delivery + carton + foam peanuts $=166+145+8.90+(6\times5)$ $=\$349.90$ <u>Carton sizes: S and XL</u> Volume of empty space in carton $=(30\times25\times15)+(45\times30\times30)-(11611+\frac{11611}{8})$ $=38687.625\ \text{cm}^3$ Total cost of clay + delivery + carton + foam peanuts $=166+145+4.90+7.90+(5\times5+2)$ $=\$350.80$ XXL carton should be used. Selling price = a value > total costs, with reasonable justification e.g. make a profit, cover labour cost, cover cost of materials and delivery. Notes: **A1 not awarded** for students **did not provide justification** for their proposed selling price.	M1[empty space in 2 cartons] M1 [Comparison of total cost of 2 options & decision on carton size] A1[selling price with justification]		

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