**NCHS 2024 EM Paper 2 Solutions**

1 (a) $\frac{9a^2-4b^2}{3ap+6aq-4bq-2bp}$

$=\frac{(3a+2b)(3a-2b)}{(3ap+6aq)-(2bp+4bq)}$

$=\frac{(3a+2b)(3a-2b)}{3a(p+2q)-2b(p+2q)}$

$=\frac{(3a+2b)(3a-2b)}{(p+2q)(3a-2b)}$

$=\frac{3a+2b}{p+2q}$

(b) $\frac{x}{6-7x-5x^2}+\frac{2}{5x-3}$

$=\frac{x}{-(5x-3)(x+2)}+\frac{2}{5x-3}$

$=\frac{-x+2(2+x)}{(5x-3)(x+2)}$

$=\frac{-x+4+2x}{(5x-3)(x+2)}$

$=\frac{x+4}{(5x-3)(x+2)}$

1(c) $-\frac{1}{2}<\frac{2y}{5}-\frac{1+y}{3}\leq\frac{5}{6}$

$-\frac{1}{2}<\frac{3(2y)-5(1+y)}{15}$ *and* $\frac{3(2y)-5(1+y)}{15}\leq\frac{5}{6}$

$-\frac{1}{2}<\frac{6y-5-5y}{15}$ $\frac{6y-5-5y}{15}\leq\frac{5}{6}$

$-\frac{1}{2}<\frac{y-5}{15}$ $\frac{y-5}{15}\leq\frac{5}{6}$

$-15<2(y-5)$ $y-5\leq\frac{25}{2}$

$-15<2y-10$ $y\leq 17\frac{1}{2}$

$-5<2y$

$-2\frac{1}{2}<y\leq 17\frac{1}{2}$

2(a) $\frac{r}{19}=\frac{25}{40}$

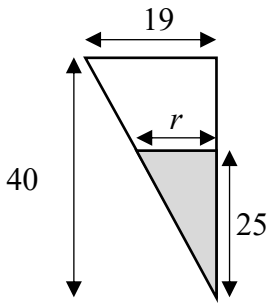
$r=\frac{25}{40}\times 19$

$=11.875$

$V=\frac{1}{3}\pi(11.875)^2(25)$

$=3691.78\ cm^3$

$=3.69\ litres$




(b) **Method 1**

$Volume\ of\ container=\frac{1}{3}\pi(19)^2(40)$

$\left(\frac{h_1}{h_2}\right)^3=\frac{V_1}{V_2}$

$\left(\frac{h_1}{40}\right)^3=\frac{\frac{1}{3}\pi(19)^2(40)-\frac{1}{3}\pi(11.875)^2(25)}{\frac{1}{3}\pi(19)^2(40)}$

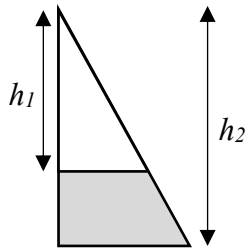
$\frac{h_1}{40}=\sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$

$h_1=40\times\sqrt[3]{\frac{(19)^2(40)-(11.875)^2(25)}{(19)^2(40)}}$

$=36.437$

$Depth\ of\ water=40-36.437$

$=3.56\ cm\ (3sf)$



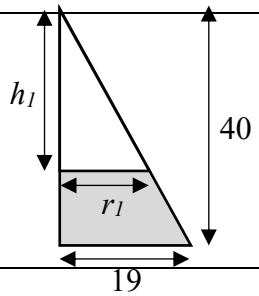

**Method 2**

$\frac{r_1}{19}=\frac{h_1}{40}$

$r_1=\frac{19}{40}h_1$

$\frac{1}{3}\pi(19)^2(40)-\frac{1}{3}\pi(r_1)^2(h_1)=3691.78$

$\frac{1}{3}\pi(r_1)^2(h_1)=11429.75$




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| | | |
|---|---|---|
| | $\left(\frac{19}{40}h_1\right)^2(h_1) = 10914.61$<br>${h_1}^3 = 48375.00277$<br>$h_1 = 36.437$<br>$Depth\ of\ water = 40 - 36.437$<br>$= 3.56\ cm\ (3sf)$ | |
| 3(a) | $\frac{108888-100176.96}{108888} \times 100\%$<br>$= 8\%$ | |
| (b) | Balance owed $= \frac{85}{100} \times 108888$<br>$= \$92554.80$<br>Interest $= \frac{92554.80 \times 2.98 \times 5}{100}$<br>$= \$13790.6652$<br>Monthly instalment $= \frac{92554.80+13790.6652}{5 \times 12}$<br>$= \$1772.42\ (2dp)$ | |
| (c) | $P = 108888 - 18888$<br>$= \$90000$<br>$A = 960 \times 10 \times 12$<br>$= \$115200$<br>$A = P\left(1 + \frac{R}{100}\right)^n$<br>$115200 = 90000\left(1 + \frac{R}{100}\right)^{10}$<br>$\left(1 + \frac{R}{100}\right)^{10} = \frac{115200}{90000}$<br>$1 + \frac{R}{100} = \sqrt[10]{\frac{115200}{90000}}$<br>$R = 100\left(\sqrt[10]{\frac{115200}{90000}} - 1\right)$<br>$= 2.499$<br>$= 2.50$ (3sf) | |
| 4(a) | $Q = \begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$ | |
| (b) | $R = \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$<br>$= \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$ | |
| (c) | The elements in R represent the total cost of baking caramel, strawberry and mint cupcakes in outlet A, B and C respectively | |
| (d) | **Method 1**<br>Selling Price<br>$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 4 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$<br>$= \begin{pmatrix} 3.60 \\ 3.75 \\ 3.80 \end{pmatrix}$ | **Method 2**<br>Profit for each type of cupcake<br>$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} 1.20 \\ 1.50 \\ 0.95 \end{pmatrix}$<br>$= \begin{pmatrix} 2.40 \\ 2.25 \\ 2.85 \end{pmatrix}$ |

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Profit earned in a week

$$= \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix} \begin{pmatrix} 3.60 \\ 3.75 \\ 3.80 \end{pmatrix} - \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$$

$$= \begin{pmatrix} 680 \\ 656.20 \\ 757.30 \end{pmatrix} - \begin{pmatrix} 226.25 \\ 210.25 \\ 244.60 \end{pmatrix}$$

$$= \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix}$$

Total Profit earned in a month

$$= 4\left[ \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix} \right]$$

$$= 4(1412.40)$$

$$= (5649.60)$$

Total Profit = \$5649.60


Profit earned in a week

$$= \begin{pmatrix} 60 & 68 & 55 \\ 49 & 56 & 71 \\ 53 & 70 & 80 \end{pmatrix} \begin{pmatrix} 2.40 \\ 2.25 \\ 2.85 \end{pmatrix}$$

$$= \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix}$$

Total Profit earned in a month

$$= 4\left[ \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 453.75 \\ 445.95 \\ 512.70 \end{pmatrix} \right]$$

$$= 4(1412.40)$$

$$= (5649.60)$$

Total Profit = \$5649.60


5(a)

$$\frac{WY}{\sin 102.6^\circ} = \frac{12.5}{\sin 34^\circ}$$

$$WY = \frac{12.5}{\sin 34^\circ} \times \sin 102.6^\circ$$

$$= 21.815$$

$$XY^2 = 10.3^2 + 21.815^2 - 2(10.3)(21.815)(\cos 45.3^\circ)$$

$$XY = 16.306 \quad or \quad -16.306(NA)$$

$$= 16.3\ (3sf)$$

(b)

**Method 1**

$$\angle N_1WZ = 102.6^\circ + 34^\circ\ (ext \angle\ of\ \Delta)$$

$$= 136.6^\circ$$

$$\angle WZN_2 = 180^\circ - 136.6^\circ\ (int \angle, N_1W // N_2Z)$$

$$= 43.4^\circ$$

$$Bearing\ of\ W\ from\ Z = 360^\circ - 43.4^\circ\ (\angle\ at\ a\ pt)$$

$$= 316.6^\circ$$

**Method 2**

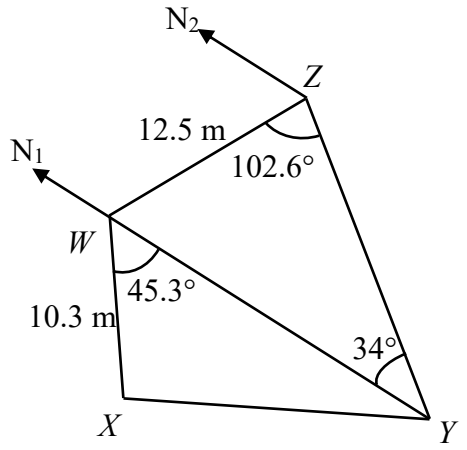
$$\angle YWZ = 180^\circ - (34^\circ + 102.6^\circ)\ (\angle\ sum\ of\ \Delta)$$

$$= 43.4^\circ$$

$$\angle WZN_2 = 43.4^\circ\ (alt \angle, N_1W // N_2Z)$$

$$Bearing\ of\ W\ from\ Z = 360^\circ - 43.4^\circ\ (\angle\ at\ a\ pt)$$

$$= 316.6^\circ$$




(c)

$$\angle YWZ = 43.4^\circ\ (alt \angle, N_1Y // N_2Z)$$

$$l = 12.5 \sin 43.4^\circ \quad \text{OR} \quad \frac{1}{2} \times 21.815 \times l = \frac{1}{2} \times 12.5 \times 21.815 \times \sin 43.4^\circ$$

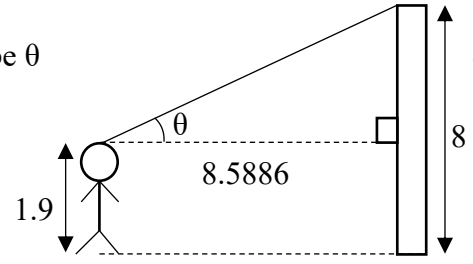
$$= 8.5886 \qquad l = 8.5886$$

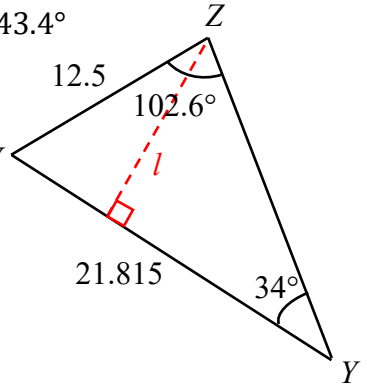
Let the greatest angle of elevation be θ

$$\tan\theta = \frac{8 - 1.9}{12.5 \sin 43.4^\circ}$$

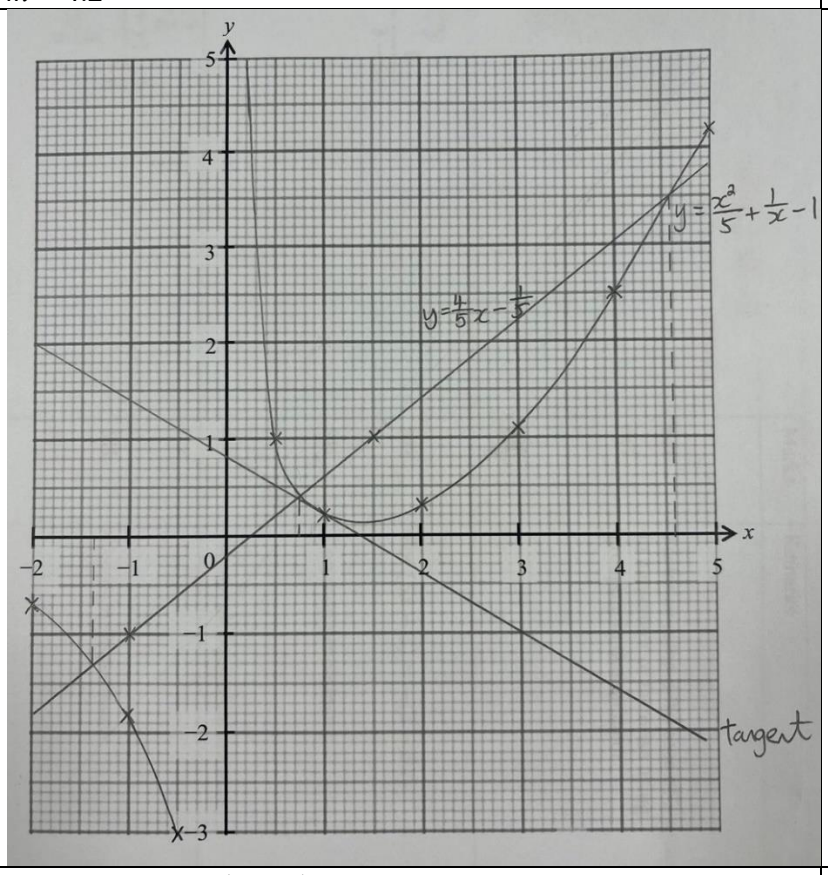
$$\theta = 35.38^\circ$$

$$= 35.4^\circ$$




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| | | |
|---|---|---|
| 6(a) | $m = 4.2$ | |
| (b) |  | |
| (c) | Draw tangent at (1, 0.2)<br>Gradient = −0.5 to −0.7 | |
| (d) | $x^2+\frac{5}{x}-4x-4=0$<br>$\frac{x^2}{5}+\frac{1}{x}-\frac{4}{5}x-\frac{4}{5}=0$<br>$\frac{x^2}{5}+\frac{1}{x}-1=\frac{4}{5}x-\frac{1}{5}$<br>$y=\frac{4}{5}x-\frac{1}{5}$<br>Plot graph of $y=\frac{4}{5}x-\frac{1}{5}$<br>$x=-1.35\pm0.05$ *or* $0.75\pm0.05$ *or* $4.6\pm0.05$ | |
| 7(a) | **Method 1**<br>$\angle BRC=\angle PRQ$ $(common\ \angle)$<br>$\frac{CR}{QR}=\frac{1}{2}$ $(C\ is\ midpt\ of\ QR)$<br>$\frac{BR}{PR}=\frac{1}{2}$ $(diagonals\ of\ parallelogram\ bisect\ each\ other)$<br>$\Delta RBC\ and\ \Delta RPQ\ are\ similar\ (SAS\ Similarity)$<br><br>**Method 2**<br>Since diagonals of a parallelogram bisect each other, *B* is the midpoint of *PR*.<br>In addition *C* is given as the midpoint of *QR*, using midpoint theorem, *BC* is parallel to *PQ*.<br>$\angle BRC=\angle PRQ$ $(common\ \angle)$<br>$\angle BCR=\angle PQR$ $(corresponding\ \angle, BC//PQ)$<br>$\Delta RBC\ and\ \Delta RPQ\ are\ similar\ (AA\ Similarity)$ | |

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(b)

**Method 1**

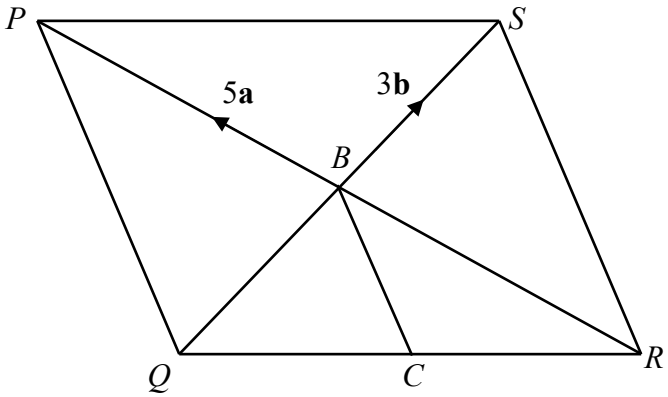
$\overrightarrow{PC} = \overrightarrow{PQ} + \overrightarrow{QC}$

$= \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) + \frac{1}{2}\overrightarrow{QR}$

$= -5\boldsymbol{a} - 3\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ *(Shown)*




**Method 2**

$\overrightarrow{PC} = \overrightarrow{PB} + \overrightarrow{BC}$

$= \overrightarrow{PB} + \frac{1}{2}\overrightarrow{PQ}$

$= -5\boldsymbol{a} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{15}{2}\boldsymbol{a} - \frac{3}{2}\boldsymbol{b}$

$= -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ *(Shown)*

(c)

**Method 1**

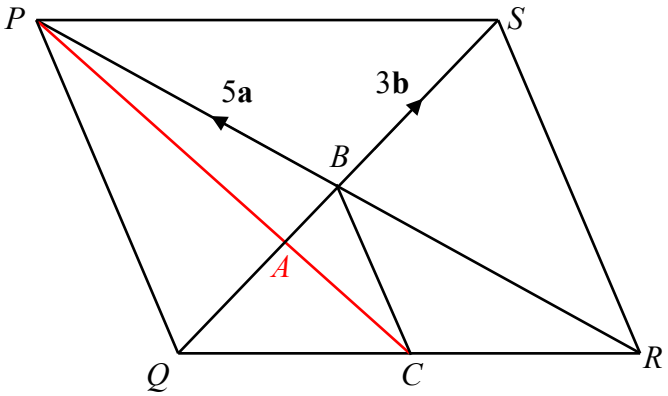
$\overrightarrow{AQ} = \frac{2}{3}(-3\boldsymbol{b})$

$= -2\boldsymbol{b}$

$\overrightarrow{AC} = \overrightarrow{AQ} + \overrightarrow{QC}$

$= -2\boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ **or** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$




**Method 2**

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \boldsymbol{b} + \frac{1}{2}(-5\boldsymbol{a} - 3\boldsymbol{b})$

$= -\frac{5}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$ **or** $-\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

(d)

$\overrightarrow{PC} = -\frac{3}{2}(5\boldsymbol{a} + \boldsymbol{b})$ & $\overrightarrow{AC} = -\frac{1}{2}(5\boldsymbol{a} + \boldsymbol{b})$

$\overrightarrow{PC} = 3\overrightarrow{AC}$

$\therefore$ $C$ is a common point, $P$, $A$ and $C$ are collinear.

(e)

**Method 1**

*BC//PQ (ΔRBC and ΔRPQ are similar)*

*Area ΔBPQ* $= \frac{1}{2} \times PQ \times h$ or $\boldsymbol{Area\ \Delta PBC = \frac{1}{2} \times BC \times h}$

$= Area\ \Delta CPQ$ $= \boldsymbol{Area\ \Delta QBC}$

*Since ΔAPQ is common area* or $\boldsymbol{\Delta ABC\ is\ common\ area}$

*∴ Area ΔPAB and Area ΔQAC are the same*

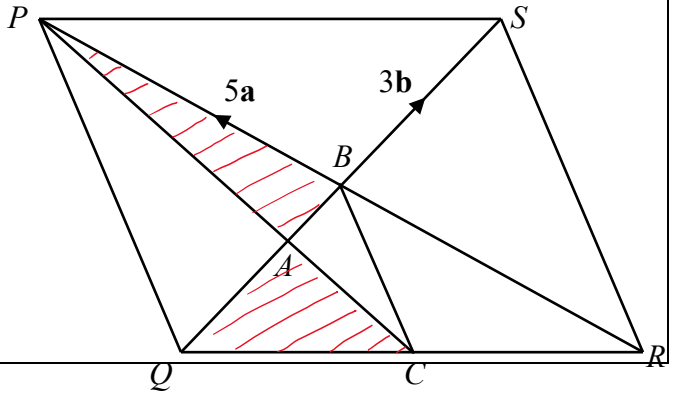
**Method 2**

$\angle PAB = \angle QAC$ *(vert opp ∠)*

$\frac{AB}{QA} = \frac{1}{2}$ *(from (c))* & $\frac{PA}{AC} = \frac{2}{1}$ *(from (d))*

$\frac{Area\ \Delta PAB}{Area\ \Delta QAC} = \frac{\frac{1}{2} \times PA \times AB \times \sin \angle PAB}{\frac{1}{2} \times QA \times AC \times \sin \angle QAC}$

$= \frac{PA}{AC} \times \frac{AB}{QA}$




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| | | |
|---|---|---|
| | $= \frac{2}{1} \times \frac{1}{2}$<br>$= \frac{1}{1}$<br>$\therefore Area\ \Delta PAB\ and\ Area\ \Delta QAC\ are\ the\ same$<br><br>**Method 3**<br>$\frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta QAC} = \frac{Area\ of\ \Delta PAB}{Area\ of\ \Delta ABC} \times \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QAC}$<br>$= \frac{\frac{1}{2} \times PA \times h_1}{\frac{1}{2} \times AC \times h_1} \times \frac{\frac{1}{2} \times AB \times h_2}{\frac{1}{2} \times QA \times h_2}$<br>$= \frac{PA}{AC} \times \frac{AB}{QA}$<br>$= \frac{2}{1} \times \frac{1}{2}$<br>$= \frac{1}{1}$<br>$\therefore Area\ \Delta PAB\ and\ Area\ \Delta QAC\ are\ the\ same$ | |
| (f)<br>(i) | $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} = \frac{1}{3}$ | |
| (ii) | $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta RPQ} = \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta QBC} \times \frac{Area\ of\ \Delta QBC}{Area\ of\ \Delta RBC} \times \frac{Area\ of\ \Delta RBC}{Area\ of\ \Delta RPQ}$<br>$= \frac{1}{3} \times \frac{1}{1} \times \left(\frac{1}{2}\right)^2$<br>$= \frac{1}{12}$ | |
| | | |
| 8(a)<br>(i) | Middle position = $400^{th}$<br>Median Length = 10.6 cm | |
| (ii) | Q1 position = $200^{th}$ Q3 position = $600^{th}$<br>Q1 = 9.95 Q3 = 11.25 or 11.2<br>Interquartile Range = 11.25 – 9.95 or 11.2 – 9.95<br>= 1.3 cm or 1.25 cm | |
| (b) | $Percentage = \frac{15 + 25}{800} \times 100\%$ *or* $\frac{20 + 25}{800} \times 100\%$<br>$= 5\%$ *or* $5.625\%$ | |
| (c) | Lobsters caught in June are **shorter** in length as the median length of **9.3 cm** is **lower** as compared to **10.6 cm** in May.<br>The lengths of the lobsters caught in June has a **wider spread** as its ITR of **3.7 cm** is **higher** as compared to **1.3 cm** in May. | |
| (d) | I disagree with Jill as **25%** of lobsters needed to be released in June because they are shorter than 8.3 cm which is **higher** as compared to the total percentage released of **5.625%** in May.<br>Or<br>I disagree with Jill as **75%** of the lobsters caught in June are within the legal length which is **lower** as compared to **94.375%** of the lobster caught in May are within the legal length. | |
| | | |

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9(a)

(i) $\angle MLO = 47°$ $(base \angle in\ isos\ \Delta)$

$\angle MNH = 180° - 47°$ $(\angle in\ opp\ segment)$

$= 133°$

(ii) $\angle MOH = 47° \times 2$ $(\angle at\ center = 2 \angle at\ circumference)$

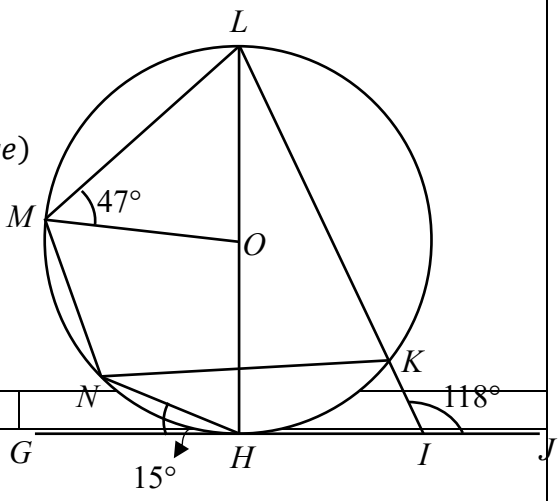
$= 94°$

(iii) $\angle LHJ = 90°$ $(tangent\ \perp radius)$

$\angle HLI = 118° - 90°$ $(ext \angle of\ \Delta)$

$= 28°$

$\angle KNH = 28°$ $(\angle in\ same\ segment)$




(b)

$\angle LHG = 90°$ $(tangent\ \perp radius)$

$\angle OHN = 90° - 15°$

$= 75°$

$\angle OMN = 360° - (133° + 75° + 94°)$ $(\angle sum\ of\ quadrilateral)$

$= 58°$

$\angle NML + \angle MLI = (58° + 47°) + (47° + 28°)$

$= 180°$

Using property of interior angles, $\angle NML$ and $\angle MLI$ add up to 180°, *MN* and *LI* are parallel lines.

10 (a)

**Method 1**

$\sin 60° = \frac{h_1}{10}$ $\quad$ $\tan\left(\frac{\frac{3\times180}{5}}{2}\right)° = \frac{h_2}{5}$

$h_1 = 10 \sin 60°$ $\quad$ $h_2 = 5 \tan 54°$

$= 8.6603$ $\quad$ $= 6.8819$

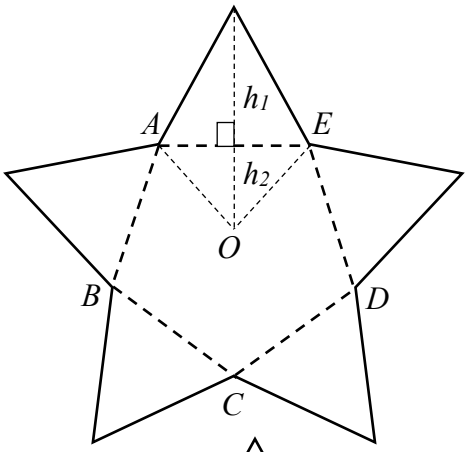
$h_1 + h_2 = 15.5422$

$Area\ PAO = \frac{1}{2} \times 15.5422 \times 5$

$Area\ of\ 5\ pointed\ star = 10 \times \frac{1}{2} \times 15.5422 \times 5$

$= 388.555$

$= 389\ cm^2\ (3sf)$




**Method 2**

Area of 1 equilateral Δ $\quad$ Area of 1 isosceles Δ

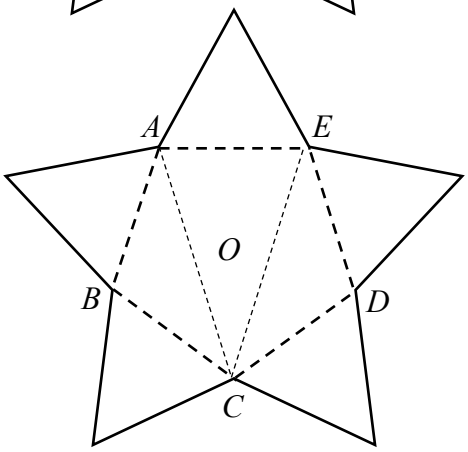
$= \frac{1}{2} \times 10 \times 10 \times \sin 60°$ $\quad$ $= \frac{1}{2} \times 10 \times 5 \tan\left(\frac{\frac{3\times180}{5}}{2}\right)°$

$= 43.301$ $\quad$ $= 34.410$

$Area\ of\ 5\ pointed\ star = 5 \times (43.301 + 34.410)$

$= 388.555$

$= 389\ cm^2\ (3sf)$




**Method 3**

$AC = 16.180\ cm$

$Area\ of\ 5\ pointed\ star$

$= 5 \times Area\ \Delta APE + 2 \times Area\ \Delta ABC + \times Area\ \Delta ACE$

$= 5 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 60°\right) + 2 \times \left(\frac{1}{2} \times 10 \times 10 \times \sin 108°\right) + \frac{1}{2} \times 16.180 \times 10 \times \sin 72°$

$= 216.506 + 95.106 + 76.940$

$= 388.552$

$= 389\ cm^2\ (3sf)$

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(b)

$Density = 900kg/m^3$
$= 0.9g/cm^3$

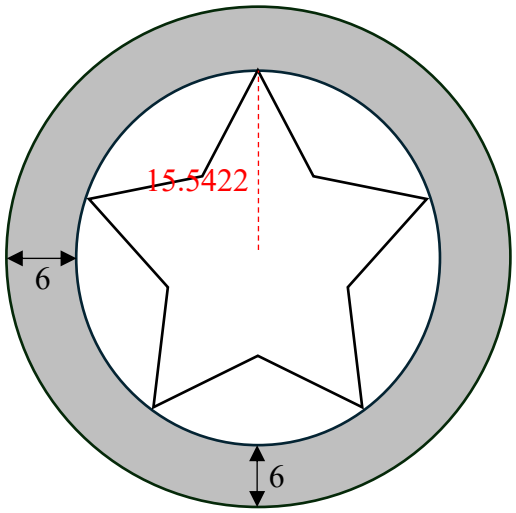
**Design I**

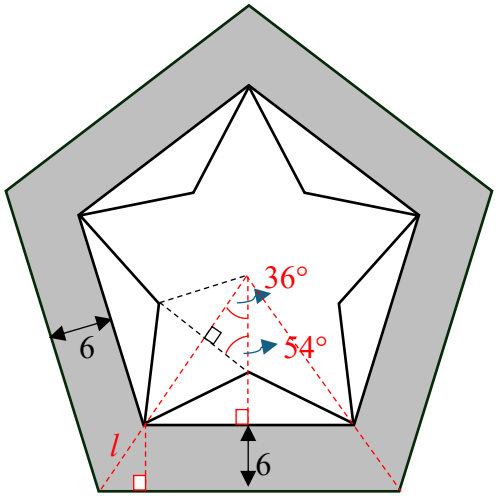
$Outer\ Radius = 15.5422 + 6 = 21.5422$
$Cross\ SA = \pi(21.5422)^2 - \pi(15.5422)^2$
$= 699.024\ cm^2$
$Volume = 699.024 \times 12$
$= 8388.288\ cm^3$
$Mass\ (wood) = 8388.288 \times 0.9$
$= 7549.4592\ grams$
$Total\ Mass = 7549.4592 + 600$
$= 8149.4592\ grams$

**Design II**

$l = \dfrac{6}{\cos 36°}$
$= 7.4164$
$Cross\ SA$
$= 5\left[\frac{1}{2}(15.5422 + 7.4164)^2 \sin 72° - \frac{1}{2}(15.5422)^2 \sin 72°\right]$
$= 678.905\ cm^2$
$Volume = 678.905 \times 12$
$= 8146.86\ cm^3$
$Mass(wood) = 8146.86 \times 0.9$
$= 7332.174\ grams$
$Total\ Mass = 7332.174\ + 600$
$= 7932.174\ grams$

Since $7932.174 < 8000 < 8149.4592\ grams$, Jack should choose **Design II**.




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