Geylang Methodist School (Secondary)
Preliminary Examination 2024

Candidate Name	**WORKED SOLUTIONS & SUGGESTED ANSWER SCHEME**
Class	**Index Number**

MATHEMATICS

4052/02

Paper 2

4 Express
5 Normal (Academic)

Candidates answer on the Question Paper.

2 hours 15 minutes

Setter: Ms Nainee Ismail and Mr Kenneth Tan

Thursday, 15 August 2024

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use
90

This document consists of **20** printed pages including the cover page.

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ANSWER SCHEME

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Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

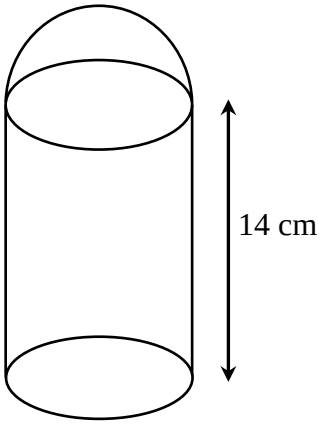
$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

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1 A model of a thermos flask consists of a solid hemisphere attached to a solid cylinder as shown. The height of the cylinder is 14 cm.





The curved surface area of the cylinder is twice the curved surface area of the hemisphere.

(a) Calculate the radius of the base of the cylinder.

Curved Surface area of cylinder $= 2 \times$ Curved Surface area of hemisphere

$$2\pi rh = 2\left(2\pi r^2\right)$$
$$2\pi r(14) = 4\pi r^2$$
$$28(\pi r) = 4r(\pi r)$$
$$\frac{28}{4} = r$$
$$r = 7 \text{ cm}$$

Answer ..... cm [2]

(b) Calculate the height of the model of the thermos flask.

Height of the model $= 14 + 7 = 21$ cm

Answer ..... cm [1]

(c) Find the ratio volume of hemisphere : volume of cylinder.

volume of hemisphere : volume of cylinder

$$\frac{2}{3}\pi r^3 : \pi r^2 h$$
$$\frac{2}{3}\pi(7)^3 : \pi(7)^2(14)$$
$$\frac{2}{3}\pi(7)^3 : \pi(7)^3(2)$$
$$\frac{2}{3} : (2)$$
$$2 : 6$$
$$1 : 3$$

Answer ..... : ..... [2]

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2 **(a)** **(i)** Solve the inequalities $-9 < 5-4x \leq 6$.

$$\begin{aligned} -9 < 5-4x &\leq 6 \\ -9-5 < -4x &\leq 6-5 \\ -14 < -4x &\leq 1 \\ \frac{-14}{-4} > x &\geq \frac{1}{-4} \\ -\frac{1}{4} \leq x &< 3\frac{1}{2} \end{aligned}$$

Answer ..... [2]

(ii) Write down all the integers that satisfy $-9 < 5-4x \leq 6$.

$-\frac{1}{4} \leq x < 3\frac{1}{2}$

Integers $x = 0, 1, 2, 3$

Answer ..... [1]

(b) $y = \dfrac{5-x^2}{x^2+w}$

(i) Evaluate the value of y when $x = 2.41$ and $w = 1.908$.
Write your answer correct to three decimal places.

$$\begin{aligned} y &= \frac{5-x^2}{x^2+w} \\ &= \frac{5-(2.41)^2}{(2.41)^2+1.908} = \frac{-0.8081}{7.7161} \\ y &= -0.104729073 = -0.105 \end{aligned}$$

Answer $y =$ ..... [2]

(ii) Rearrange the formula to make x the subject.

$$\begin{aligned} y &= \frac{5-x^2}{x^2+w} \\ y(x^2+w) &= 5-x^2 \\ yx^2+yw &= 5-x^2 \\ yx^2+x^2 &= 5-yw \\ x^2(y+1) &= 5-yw \\ x^2 &= \frac{5-yw}{y+1} \\ x &= \pm\sqrt{\frac{5-yw}{y+1}} \end{aligned}$$

Answer $x =$ ..... [4]

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2 **(c)** Solve $\frac{6}{x-1} - \frac{4-x}{3-x} = 1$.

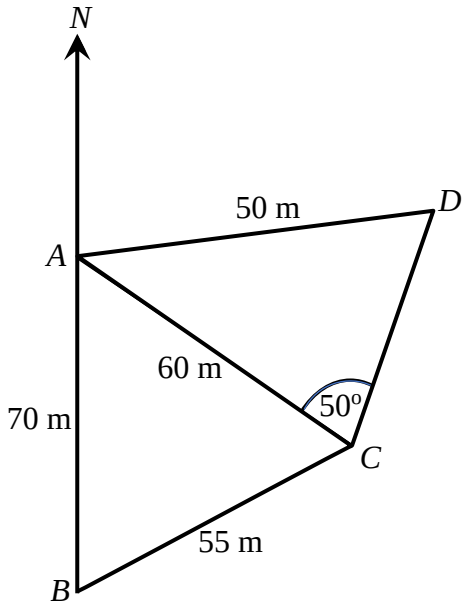
$$\frac{6}{x-1} - \frac{4-x}{3-x} = 1$$
$$\frac{6(3-x)}{(x-1)(3-x)} - \frac{(4-x)(x-1)}{(3-x)(x-1)} = 1$$
$$\frac{18-6x-(4x-4-x^2+x)}{(x-1)(3-x)} = 1$$
$$18-6x-4x+4+x^2-x = 1(x-1)(3-x)$$
$$x^2-11x+22 = 3x-x^2-3+x$$
$$x^2+x^2-11x-3x-x+22+3 = 0$$
$$2x^2-15x+25 = 0$$
$$(2x-5)(x-5)=0$$
$$x = 2\frac{1}{2} \text{ or } x = 5$$

[Accept working by general formula – correct substitution]

Answer $x =$ ..... or ..... [4]

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3



A, *B*, *C* and *D* are four points on horizontal ground.
AB = 70 m, *BC* = 55 m, *CA* = 60 m, *AD* = 50 m, angle *ACD* = $50°$ and angle *ADC* is acute.

(a) Calculate angle *BAC*.

$$55^2 = 60^2 + 70^2 - 2(60)(70)\cos\angle BAC$$
$$\cos\angle BAC = \frac{60^2 + 70^2 - 55^2}{2(60)(70)}$$
$$\text{Angle } BAC = 49.324° = 49.3°$$

Answer Angle *BAC* = ... 49.3° ..... [3]

(b) Calculate angle *DAC*.

$$\frac{\sin\angle ADC}{60} = \frac{\sin 50°}{50}$$
$$\sin\angle ADC = \frac{\sin 50°}{50} \times 60$$
$$= 0.9192533317$$
$$\text{Angle } ADC = 66.817°$$
$$\text{Angle } DAC = 180° - 66.817° - 50°$$
$$= 63.183°$$
$$\text{Angle } DAC = 63.2°$$

Answer Angle *DAC* = ... 63.2° ..... [3]

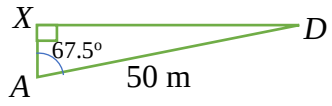
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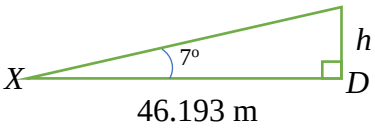
3 **(c)** Calculate the bearing of A from D.

N_1 : North is drawn at D
$\angle ADN_1 = 63.2° + 49.3°$
$= 112.5°$
(Alternate Angles, $AN // DN_1$)
Bearing of A from D = $360^o - 112.5^o$
$= 247.5^o$
(Sum of angles at a point)

Answer ..... 247.5° ..... [2]

(d) A man walks northwards from point A. A vertical pole is placed at point D.
The greatest angle of elevation of the pole when viewed along this path is $7°$.
Calculate the height of the pole.





Angle $XAD = 180° - 63.2° - 49.30°$ (angles on a straight line)
$= 67.5°$

$$\sin 67.5° = \frac{XD}{50}$$
$$XD = 50 \sin 67.5° = 46.193 \text{ m}$$
$$\tan 7° = \frac{h}{46.193}$$
$$h = 46.193 \times \tan 7° = 5.67 \text{ m}$$

Answer ..... 5.67 m ..... m [2]

[Turn Over

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4 Mr Tan planned to travel to Kuala Lumpur for a family trip.
Before the trip, he exchanged Singapore Dollars (S$) for Malaysian Ringgit (RM).
The exchange rate is S$ 1 = RM x.

(a) Write down an expression, in terms of x, for the Singapore Dollars he exchanged if he received RM 2000.

$$\text{RM } x = \text{S\$ } 1$$
$$\text{RM } 1 = \text{S\$ } \left(\frac{1}{x}\right) \quad \Rightarrow \quad \text{RM } 2000 = \text{S\$ } \left(\frac{2000}{x}\right)$$

Answer S$ $\frac{2000}{x}$ [1]

(b) After the trip, Mr Tan found out that he still had RM 300.
He exchanged the Malaysian Ringgit back to Singapore Dollars.
The exchange rate is now S$ 1 = RM $(x + 0.05)$.
Write down an expression, in terms of x, for the Singapore Dollars he exchanged for RM 300.

$$\text{RM } (x+0.05)x = \text{S\$ } 1$$
$$\text{RM } 1 = \text{S\$ } \left(\frac{1}{x+0.05}\right) \Rightarrow \text{RM } 300 = \text{S\$ } \left(\frac{300}{x+0.05}\right)$$

Answer S$ $\frac{300}{x+0.05}$ [1]

(c) Mr Tan used up S$ 485 for the trip.
Form an equation in x and show that it reduces to $1940x^2 - 6703x - 400 = 0$.

Answer

$$\text{RM } 2000 - \text{RM } 300 = \text{S\$ } 485$$
$$\frac{2000}{x} - \frac{300}{x+0.05} = 485$$
$$\frac{2000(x+0.05) - 300x}{x(x+0.05)} = 485$$
$$2000x + 100 - 300x = 485x(x+0.05)$$
$$0 = 485x^2 + 24.25x - 2000x - 100 + 300x$$
$$0 = 485x^2 - \frac{6703}{4}x - 100$$
$$1940x^2 - 6703x - 400 = 0 \quad \text{(Shown)}$$

[3]

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4 **(d)** Solve the equation $1940x^2 - 6703x - 400 = 0$,
giving your solutions correct to two decimal places.

$$x = \frac{-(-6703) \pm \sqrt{(-6703)^2 - 4(1940)(-400)}}{2(1940)}$$
$$= \frac{6703 \pm \sqrt{48034209}}{3880}$$
$$= 3.51383289 \quad \textbf{or} \quad -0.05867825064$$
$$x = 3.51 \quad \textbf{or} \quad -0.06 \quad (2 \text{ dp})$$

Answer $x =$ ..... 3.51 ..... or ..... – 0.06 ..... [3]

(e) From your solution to **part (d)**, state the exchange rate in the form is S$ 1 = RM *x*, to two decimal places, and hence, find the total amount of Singapore Dollars (S$) he initially converted to Malaysian Ringgit (RM).

$$\text{S\$ 1 = RM 3.51}$$
$$\text{RM } 2000 = \text{S\$} \left(\frac{2000}{x} \right)$$
$$= \text{S\$} \left(\frac{2000}{3.51} \right)$$
$$= \text{S\$ } 569.8005698$$
$$\text{Amount of S\$} = \frac{2000}{3.51} = \text{S\$ } 569.80 \quad (2\text{dp})$$

Answer S$ 1 = RM ..... 3.51 .....

S$ ..... 569.80 ..... [2]

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5 A shop sells two flavours of ice-cream, Raisins (R) and Chunkies (C) in three different sizes, small (S), medium (M) and large (L).
The sales on two successive days are given in the table below.

	Saturday			Sunday		
Size	small	medium	large	small	medium	large
Number of cups sold (Raisins)	12	17	8	14	12	10
Number of cups sold (Chunkies)	18	15	11	13	21	16

The information for Saturday's sales can be represented by the matrix

$$\mathbf{M} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix} \end{matrix}.$$

(a) Represent the information for Sunday's sales in a 2×3 matrix $\mathbf{N}$.

$$\mathbf{N} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix} \end{matrix}$$

Answer $\mathbf{N} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} & & \\ & & \end{pmatrix} \end{matrix}$ [1]

(b) The cost of each cup of ice-cream is \$2.50 for small (S), \$3.20 for medium (M) and \$4.50 for large (L).

Represent the cost of ice cream in a 3×1 matrix $\mathbf{C}$.

$$\mathbf{C} = \begin{matrix} \\ \text{S} \\ \text{M} \\ \text{L} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix} \end{matrix}$$

Answer $\mathbf{C} = \begin{matrix} \\ \text{S} \\ \text{M} \\ \text{L} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} \\ \\ \end{pmatrix} \end{matrix}$ [1]

(c) Evaluate the matrix $\mathbf{P} = \mathbf{M} + \mathbf{N}$.

$$\mathbf{P} = \mathbf{M} + \mathbf{N} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 12 & 17 & 8 \\ 18 & 15 & 11 \end{pmatrix} \end{matrix} + \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 14 & 12 & 10 \\ 13 & 21 & 16 \end{pmatrix} \end{matrix}$$

$$\mathbf{P} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix} \end{matrix}$$

Answer $\mathbf{P} =$ [1]

(d) State what the elements of $\mathbf{P}$ represent.

The elements represent the **total number** of **different sizes** of cups (S, M, L) of **Raisins and Chunkies** flavoured ice-cream sold on **Saturday and Sunday**. [1]

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5 **(e)** Evaluate the matrix $\mathbf{Q} = \frac{1}{2}\mathbf{PC}$.

$$\mathbf{Q} = \frac{1}{2}\mathbf{PC} = \frac{1}{2}\begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \text{S} & \text{M} & \text{L} \end{matrix} \\ \begin{pmatrix} 26 & 29 & 18 \\ 31 & 36 & 27 \end{pmatrix} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 2.5 \\ 3.2 \\ 4.5 \end{pmatrix} \end{matrix}\begin{matrix} \\ \text{S} \\ \text{M} \\ \text{L} \end{matrix}$$

$$\mathbf{Q} = \frac{1}{2}\mathbf{PC} = \frac{1}{2}\begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \end{matrix}$$

$$\mathbf{Q} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 119.4 \\ 157.1 \end{pmatrix} \end{matrix}$$

Answer $\mathbf{Q} =$ [2]

(f) State what the elements of $\mathbf{Q}$ represent.

The **average amount** received from the sales on Saturday and Sunday of all
……
sizes (S, M, L) of **Raisins and Chunkies** flavoured ice-cream **respectively**.
…… [1]

(g) Evaluate the matrix $\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{PC}$.

$$\mathbf{PC} = \begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \end{matrix}$$

$$\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{PC}$$

$$\mathbf{R} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{matrix} \begin{matrix} \$ \end{matrix} \\ \begin{pmatrix} 238.8 \\ 314.2 \end{pmatrix} \end{matrix}\begin{matrix} \\ \text{R} \\ \text{C} \end{matrix}$$

$$\mathbf{R} = \begin{pmatrix} 238.8 + 314.2 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 553 \end{pmatrix}$$

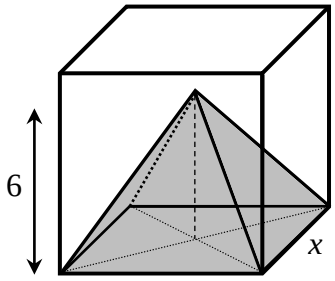
Answer $\mathbf{R} =$ [2]

(h) State what the elements of $\mathbf{R}$ represent.

The **total amount** received from the sales on Saturday and Sunday of all
……
sizes (S, M, L) of **Raisins and Chunkies** flavoured ice-cream.
…… [1]

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6 The following diagram shows a solid which is made up of a cube whose edge is x cm with a right square pyramid removed from the base of the cube.
The pyramid has the same square base as the cube and its vertical height is 6 cm.



(a) Show that the volume of the remaining solid, y, is $x^3 - 2x^2$.

Answer

Volume of solid = Volume of $(\text{cube} - \text{pyramid})$

$$y = (L \times B \times H) - \left(\frac{1}{3} \times \text{base area} \times h\right) = (x \times x \times x) - \left(\frac{1}{3} \times x \times x \times 6\right)$$

$$y = x^3 - 2x^2$$

[1]

(b) Complete the table of values for $y = x^3 - 2x^2$.
Values are given to one decimal place where appropriate.

x	0	1	1.5	2	3	4	5	6	6.5
y	0	– 1	– 1.1	0	9	32	75	144	190.1

[1]

(c) On the grid opposite, draw the graph of $y = x^3 - 2x^2$ for $0 \le x \le 6.5$. [3]

(d) Use your graph to estimate the volume of the remaining solid when the pyramid has a base area of 24 cm².

Base Area $= 24$ cm^2

$x^2 = 24$

$x = \sqrt{24} = 4.898979\ldots = 4.9$

From graph, $x = 4.9$

$y = (70 \pm 2)$ cm^3

[Actual value of y = 69.6]

Answer Volume = ... cm³ [2]

(e) **(i)** Use your graph, for $0 < x < 2$, to give a possible deduction on the volume of the remaining solid, y.

For $0 < x < 2$, graph is below y-axis, to indicate volume is negative, it shows that the volume of pyramid > volume of cube and hence removing the pyramid is not even possible. [1]

(ii) Suggest a suitable range of x such that it is possible to remove a full right square pyramid from the cube.

Answer $x \ge 6$ [1]

(f) By drawing the line $y = ax + b$, solve the equation $-x^3 + 2x^2 + 15x + 20 = 0$.

$-x^3 + 2x^2 + 15x + 20 = 0$

$15x + 20 = x^3 - 2x^2$

$15x + 20 = y$

Draw $y = 15x + 20$

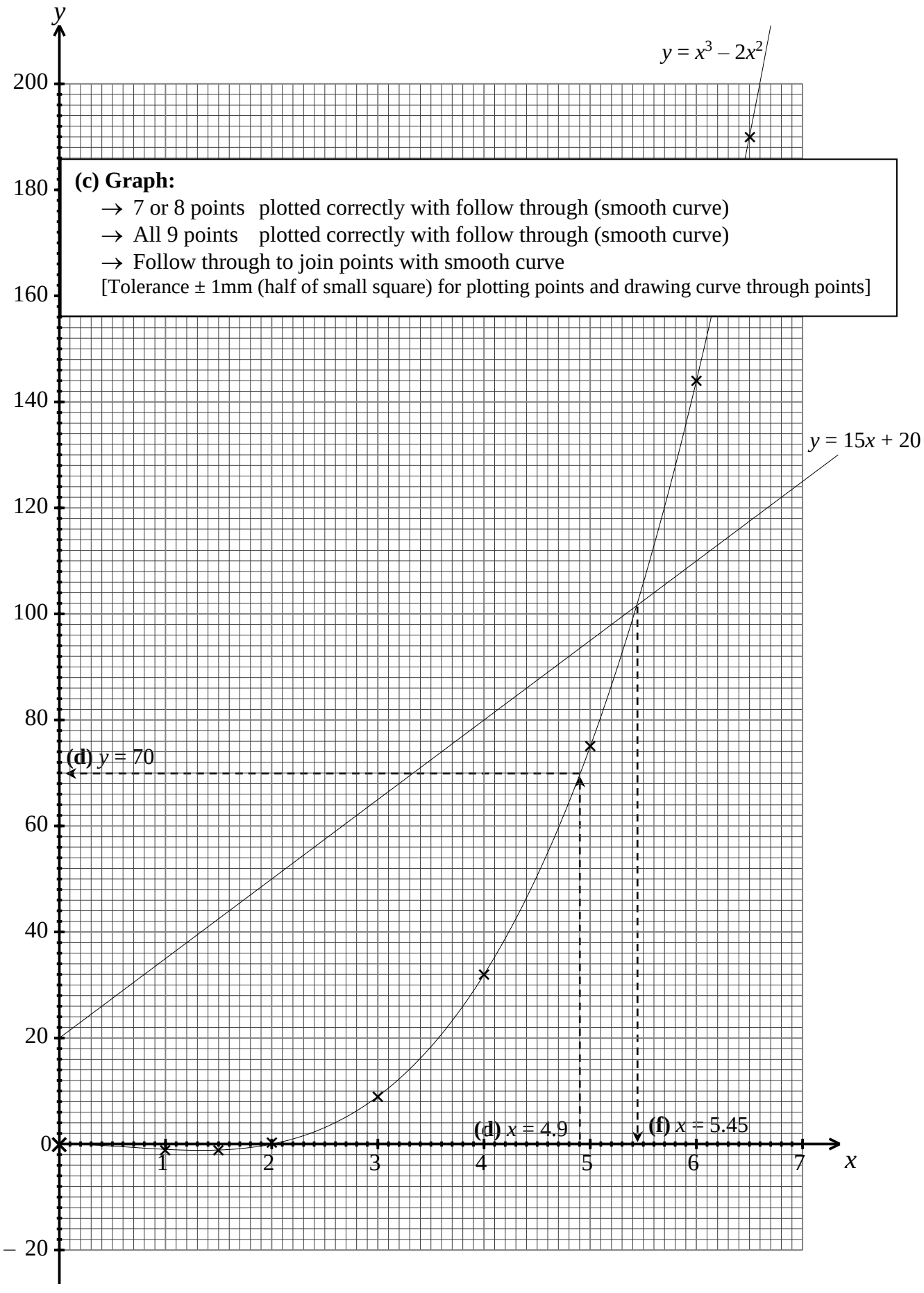
From graph, point of intersection: $x = (5.45 \pm 0.1)$

[Actual value of x = 5.45]

Answer $x =$ ..... [2]

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6 (c)



(c) Graph:

→ 7 or 8 points plotted correctly with follow through (smooth curve)

→ All 9 points plotted correctly with follow through (smooth curve)

→ Follow through to join points with smooth curve

[Tolerance ± 1mm (half of small square) for plotting points and drawing curve through points]

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7 **(a)** The point P is (5, – 8) and the point Q is (– 4, 7).

The point L is such that $\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$ and O is the origin.

Find $\left|\overrightarrow{OL}\right|$.

$\overrightarrow{OP} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$, $\overrightarrow{QP} = \frac{1}{2}\overrightarrow{PL}$

$\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP} = -\begin{pmatrix} -4 \\ 7 \end{pmatrix} + \begin{pmatrix} 5 \\ -8 \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix} + \begin{pmatrix} 5 \\ -8 \end{pmatrix} = \begin{pmatrix} 9 \\ -15 \end{pmatrix}$

$\frac{1}{2}\overrightarrow{PL} = \frac{1}{2}\left(\overrightarrow{PO} + \overrightarrow{OL}\right)$, $\overrightarrow{QP} = \frac{1}{2}\left(\overrightarrow{PO} + \overrightarrow{OL}\right)$

$\begin{pmatrix} 9 \\ -15 \end{pmatrix} = \frac{1}{2}\left[-\begin{pmatrix} 5 \\ -8 \end{pmatrix} + \overrightarrow{OL}\right]$

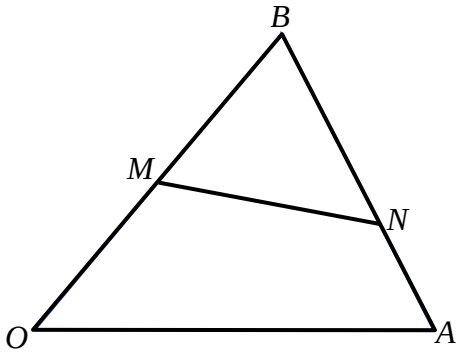
$\begin{pmatrix} 18 \\ -30 \end{pmatrix} = \begin{pmatrix} -5 \\ 8 \end{pmatrix} + \overrightarrow{OL}$

$\overrightarrow{OL} = \begin{pmatrix} 18 \\ -30 \end{pmatrix} - \begin{pmatrix} -5 \\ 8 \end{pmatrix} = \begin{pmatrix} 23 \\ -38 \end{pmatrix}$

$\left|\overrightarrow{OL}\right| = \sqrt{23^2 + (-38)^2} = \sqrt{1973} = 44.418 \approx 44.4$ units (3sf)

Answer ..... [4]

(b) In the diagram $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
M is a point on OB where $OB = 2MB$ and the point N lies on AB such that $3BN = 2BA$.



(i) Express $\overrightarrow{BN}$, as simply as possible, in terms of **a** and **b**.

$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\mathbf{b} + \mathbf{a}$

$\overrightarrow{BN} = \frac{2}{3}\overrightarrow{BA} = \frac{2}{3}(-\mathbf{b} + \mathbf{a}) = \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$

Answer $\overrightarrow{BN}$ = ... [1]

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7 (b) (ii) Express $\overrightarrow{MN}$, as simply as possible, in terms of **a** and **b**.

$\overrightarrow{MN} = \overrightarrow{MB} + \overrightarrow{BN}$

$= \frac{1}{2}\mathbf{b} + \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$

$\overrightarrow{MN} = \frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b}$

Answer $\overrightarrow{MN}$ = ... [2]

(iii) The point P lies on OA produced such that $OA : OP = 1 : 2$.
Determine whether the points *M*, *N* and *P* are collinear.
Justify your answer.

$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP} = -\frac{1}{2}\mathbf{b} + 2\mathbf{a}$, $\overrightarrow{MN} = \frac{2}{3}\mathbf{a} - \frac{1}{6}\mathbf{b} = \frac{1}{3}\left(2\mathbf{a} - \frac{1}{2}\mathbf{b}\right) = \frac{1}{3}\overrightarrow{MP}$

Since both vectors $\overrightarrow{MP}$ and $\overrightarrow{MN}$ have the same direction components of $2\mathbf{a} - \frac{1}{2}\mathbf{b}$, and *N* is connecting point, they are collinear.

.....

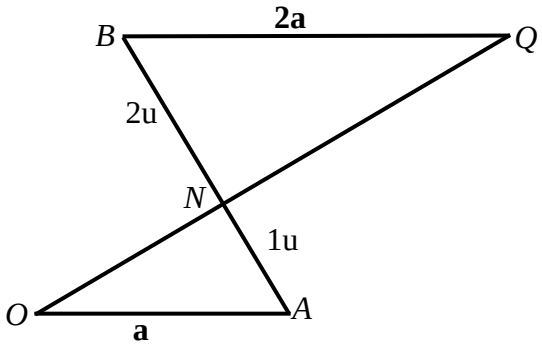
[includes explanation on the same direction component which makes the line collinear]

..... [2]

(iv) The point Q is such that $k\overrightarrow{BQ} = \mathbf{a}$ and ONQ is a straight line.
Write down the value of k.

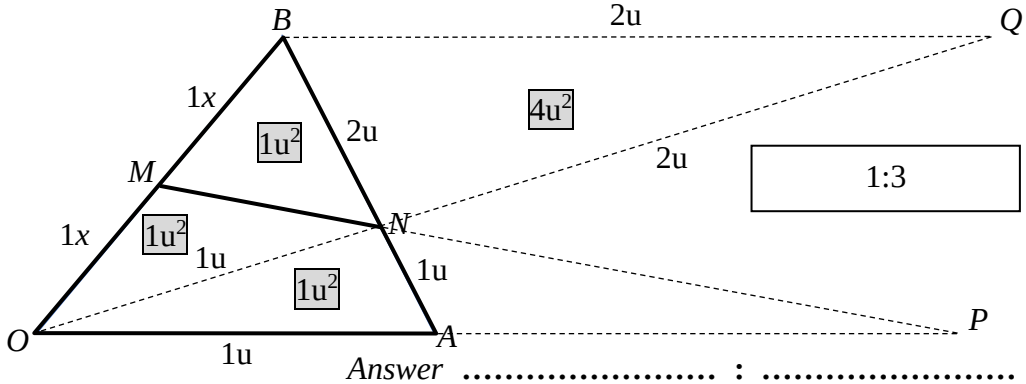
Able to identify the location of Q and that BQ is parallel to OA.

$\overrightarrow{BQ} = 2\mathbf{a}$, $k = \frac{1}{2}$



Answer k = ... [2]

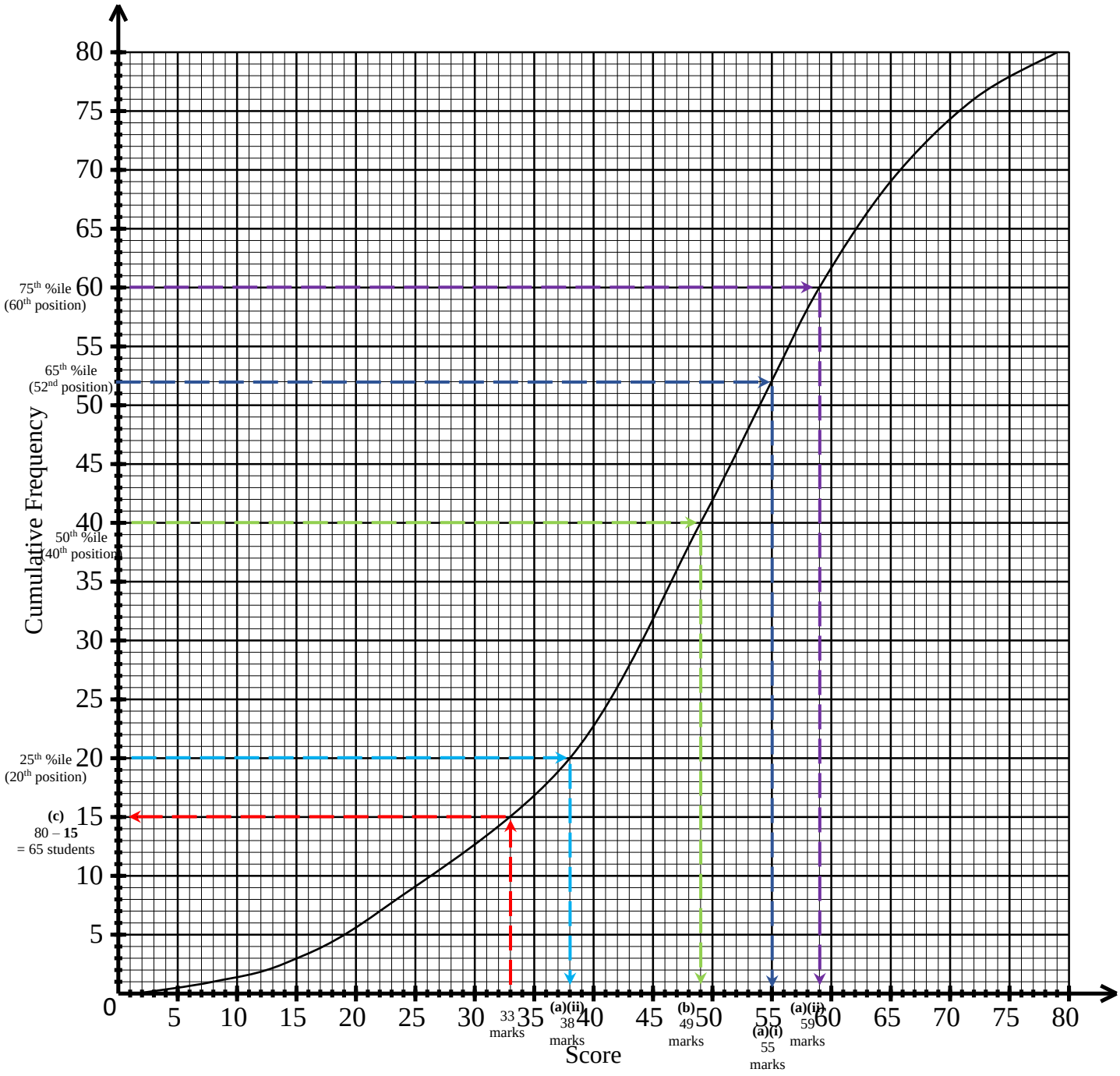
(v) Find the ratio area of triangle *BMN* : area of triangle *OBA*.



Answer ..... : ..... [1]

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8 The cumulative frequency curve shows the distribution of the test scores of a group of 80 students.



(a) Use the curve to estimate

(i) the 65th percentile,

$\frac{65}{100} \times 80 = 52^{\text{nd}}$ student

$\therefore$ From graph: 65^{th} percentile $\rightarrow 55$ marks

Answer ..... 55 ..... [1]

(ii) the interquartile range.

$\frac{25}{100} \times 80 = 20^{\text{th}}$ student

$\frac{75}{100} \times 80 = 60^{\text{th}}$ student

From graph:

75^{th} %ile (UQ) = 59.0 marks

25^{th} %ile (LQ) = 38.0 marks

IQ Range $= \text{UQ} - \text{LQ}$

$= 59.0 - 38.0$

$= 21.0$

Answer ..... 21 ..... [2]

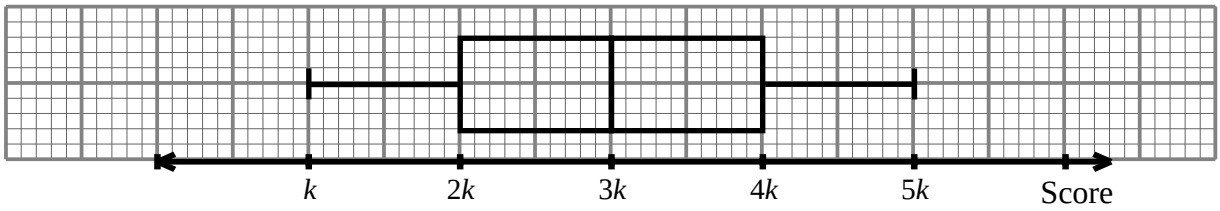
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8 (b) (i) Complete the table by using the estimations from the curve.

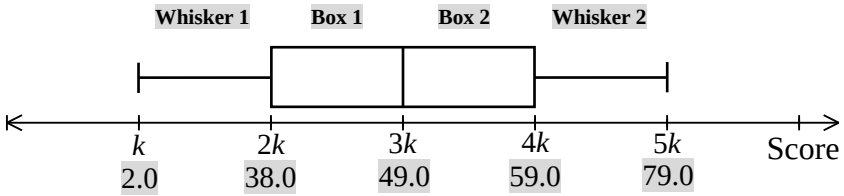
Lowest Score	**2.0 (3.0)**
Lower Quartile	**38.0**
Median	**49.0**
Upper Quartile	**59.0**
Highest Score	**79.0**

[2]

(ii) The distribution of the scores of the 80 students is represented in a box-and-whisker plot below, where k is a constant.
Do you agree with the representation?
Provide justifications by using your answer from **part (b)(i)**.



Answer



Disagree. Each box and whisker **should not be equally spaced**.
The box-and-whisker plot shows that k, $2k$, $3k$, $4k$, $5k$ are all equally spaced
(Increase by k each time from Lowest, LQ, Median, UQ, Highest).
.....
From the actual values, Length of: Whisker 1 > Whisker 2 > Box 1 > Box 2.
(36 > 20 > 11 > 10)
For any comment substantiated with logical reasons.
..... [2]

(c) Students who scored 33 marks and below are required to attend remedial.
A student is selected at random from the group.
Find the probability that the student selected is not required to attend remedial.
From graph: 33 marks $\rightarrow 15^{th}$ student
Number of the students who are surfaced for remedial
$= 15$ students
Number of the students who are not surfaced for remedial
$= 80 - 15 = 65$ students
Probability that student selected is not required to attend remedial
$= \frac{65}{80} = \frac{13}{16}$ students

Answer $\frac{13}{16}$ ..... [2]

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9 Ann was planning to buy a car and chanced upon the following while doing her research.



Adapted from: https://blog.seedly.sg/factors-cost-car-price-singapore/

Her annual salary is $108 000. When calculating her budget, she planned to set aside 27% of her monthly salary for the expenses incurred if she finally decided to buy a car.

(a) Based on the information that she chanced upon in her research, deduce if Ann decided to buy a car. Calculate the amount she budgeted monthly for the expenses incurred.

(a) Ann's monthly salary $= \frac{108000}{12} = \$9000 \left(> \$8729, \text{ from research}\right)$

27% of Ann's monthly salary $= \frac{27}{100} \times \$9000 = \boxed{\$2430}$

Answer Buys a car: **YES** / **NO** (Circle the response)

$ ..... [1]

In Singapore, a Certificate of Entitlement (COE) is required for vehicle ownership, which represents the right to register and use a vehicle in Singapore for a period of 10 years.

Types of vehicle	Current COE	Previous COE
A Cars up to 1600cc	**$88,200**	$92,700
B Cars above 1600cc	**$100,607**	$105,689
C Goods Vehicle & Bus	**$70,589**	$70,620
D Motorcycle	**$8,989**	$9,649
E Open- All except Motorcycle	**$101,600**	$105,002

Age of vehicle	Certificate of Entitlement (COE)
Brand new vehicle	**Based on the table shown above, included in the cost of the brand new vehicle.**
Less than 10 years	**Buyer takes over the remaining COE, included in the cost of the used vehicle.**
More than 10 years	**Buyer needs to renew the COE for the used vehicle in 5 or 10 year blocks.**

Adapted from: https://onemotoring.lta.gov.sg/content/onemotoring/home/buying/coe-open-bidding.html

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9 Ann approached a car dealer who recommended two different packages for a used and a new car based on Ann's needs and preferences. Ann decided to take a loan for the purchase of the car after placing a downpayment from her savings. She planned to repay the loan monthly.

Details of package	**Used car**	**New car**
Engine Capacity	1800 cc	1500 cc
Total Cost	S$98 000	S$150 000
Downpayment	40%	30%
Intended loan period	5 years	5 years
Type of interest (per annum)	Years 1 and 2: Simple interest 2.28% Year 3 onwards: Simple interest 3%	Simple interest at 2.88%

She collated all the other expenses that come with owning a car:

Other expenses	**Used car**	**New car**
Monthly parking fees	\$110	\$110
Monthly petrol expenditure	\$340	\$270
Annual Electronic Road Pricing (ERP)	\$360	\$360
Annual Insurance	\$1800	\$1600
Car maintenance (every 6 months)	\$650 per maintenance	\$500 per maintenance

The road tax of the car is calculated in dollars based on its engine capacity:

Engine Capacity (EC) (in cc)	**Road Tax Formula (per annum)**
$EC \leq 600$	$\$200 \times 0.782$
$600 < EC \leq 1000$	$[\$200 + \$0.125 \times (EC - 600)] \times 0.782$
$1000 < EC \leq 1600$	$[\$250 + \$0.375 \times (EC - 1000)] \times 0.782$
$1600 < EC \leq 3000$	$[\$475 + \$0.75 \times (EC - 1600)] \times 0.782$
$EC > 3000$	$[\$1525 + \$1.00 \times (EC - 3000)] \times 0.782$

(b) Calculate the annual road tax for the

(i) used car,
1800 cc:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$1600 < EC \leq 3000$	$[\$475 + \$0.75 \times (EC - 1600)] \times 0.782$

Annual road tax (Used Car)

$= \left[\$475 + \$0.75 \times (\mathbf{1800} - 1600)\right] \times 0.782 = \boxed{\$488.75}$

Answer \$ ..... [1]

(ii) new car.
1500 cc:

Engine Capacity (EC) (in cc)	Road Tax Formula (per annum)
$1000 < EC \leq 1600$	$[\$250 + \$0.375 \times (EC - 1000)] \times 0.782$

Annual road tax (New Car)

$= \left[\$250 + \$0.375 \times (\mathbf{1500} - 1000)\right] \times 0.782 = \$342.125 = \boxed{\$342.13}$

Answer \$ ..... [1]

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9 **(c)** Determine which car Ann should purchase based on her monthly budget for all the expenses incurred in owning a car (including road tax).
Justify the decision you make and show your calculations clearly.

[Reasonable and correct working preferably with accuracy of values]

Used car

Loan amount (after making downpayment) $= \frac{(100-\mathbf{40})}{100} \times 98000 = \mathbf{60} \times 980 = \$58800$

Simple Interest incurred (Years 1 and 2) $= \frac{P \times R \times T}{100} = \frac{58800 \times 2.28 \times 2}{100} = \$2681.28$

Simple Interest incurred (Years 3, 4 and 5) $= \frac{P \times R \times T}{100} = \frac{58800 \times 3 \times 3}{100} = \$5292.00$

Monthly (Bank Loan Repayment) $= \frac{58800 + 2681.28 + 5292}{(5 \times 12)} = \frac{66773.28}{60} = \$1112.888$

Monthly (other expenses) = $\underset{\text{Parking Fees}}{110} + \underset{\text{Petrol Expense}}{340} + \underset{\text{ERP}}{(360 \div 12)} + \underset{\text{Insurance}}{(1800 \div 12)} + \underset{\text{Maintenance}}{(650 \div 6)} = \$738\frac{1}{3}$

Monthly (Road Tax) = $\frac{488.75}{12} = \$40\frac{35}{48}$

TOTAL Monthly (All Expenses Incurred)

TOTAL Monthly (Bank Loan Repayment + Others + Road Tax)

$= \$1112.888 + \$738\frac{1}{3} + \$40\frac{35}{48} = \$1891.9505$

= \$1891.95 **(USED Car, Total monthly expense)** < \$2430 **(Budget of 27% monthly salary)**

New car

Loan amount (after making downpayment) $= \frac{(100-\mathbf{30})}{100} \times 150000 = \mathbf{70} \times 1500 = \$105000$

Simple Interest incurred (Years 1 to 5) $= \frac{P \times R \times T}{100} = \frac{105000 \times 2.88 \times 5}{100} = \$15120$

Monthly (Bank Loan Repayment) $= \frac{105000 + 15120}{(5 \times 12)} = \frac{120120}{60} = \$2002$

Monthly (other expenses) = $\underset{\text{Parking Fees}}{110} + \underset{\text{Petrol Expense}}{270} + \underset{\text{ERP}}{(360 \div 12)} + \underset{\text{Insurance}}{(1600 \div 12)} + \underset{\text{Maintenance}}{(500 \div 6)} = \$626\frac{2}{3}$

Monthly (Road Tax) = $\frac{342.125}{12} = \$28\frac{49}{96}$

TOTAL Monthly (All Expenses Incurred)

TOTAL Monthly (Bank Loan Repayment + Others + Road Tax)

$= \$2002 + \$626\frac{2}{3} + \$28\frac{49}{96} = \$2657.177083$

= \$2657.18 **(NEW Car, Total monthly expense)** > \$2430 **(Budget of 27% monthly salary)**

Ann should purchase the **Used car**, as the total expenses incurred is **within her budget of 27%** of monthly salary earned. Purchase of new car will incur higher monthly expenses. [7]

End of Paper

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