Name:	Class:	Class Register Number:

中正中学
CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2024
SECONDARY 4

MATHEMATICS **4052/01**

Paper 1 **Friday 23 August 2024**

2 hours 15 minutes

Candidates answer on the Question Paper.

MARKS SCHEME

This document consists of **20** printed pages.

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Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

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Answer **all** the questions.

1 The length of a court is 0.028 km and the width of the court is 16 m. Express the ratio of the length to the width of the court in its simplest form.

$28:16$
$7:4$

Answer ..... **:** ..... [1]

---

2 Given that $0 < x < 180$, find the possible values of x for $\sin x° = 0.985$.
Give your answers correct to 1 decimal place.

$\sin x° = 0.985$
$x = \sin^{-1} 0.985$ or $180 - \sin^{-1} 0.985$
$= 80.1$ or 99.9

Answer x = ...or..... [2]

---

3 **(a)** Simplify $\dfrac{\left(4a^4b\right)^3}{10a^3b^7}$, leaving your answer in positive index form.

$$\frac{\left(4a^4b\right)^3}{10a^3b^7} = \frac{64a^{12}b^3}{10a^3b^7}$$
$$= \frac{32a^9}{5b^4}$$

Answer ..... [2]

(b) Given that $2^x = 3$, $2^y = 7$ and $2^z = \dfrac{9}{49}$, use the laws of indices to find the value of $2^{3x-y+\frac{1}{2}z}$.

$$2^{3x-y+\frac{1}{2}z} = \left(2^x\right)^3 \div 2^y \times \left(2^z\right)^{\frac{1}{2}}$$
$$= 3^3 \div 7 \times \sqrt{\frac{9}{49}}$$
$$= \frac{81}{49}$$

Law of indices:
1: $\left(a^m\right)^n = a^{mn}$
2: $a^m \times a^n = a^{m+n}$
3: $a^m \div a^n = a^{m-n}$
4: $a^{\frac{1}{n}} = \sqrt[n]{a}$

to award M1 if student show understanding of either law of indices

Answer ..... [2]

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4 A group of students won \$226 in a competition and shared the amount equally, leaving \$4 leftover. Competing for a second time, the same group of students won \$296. They shared the \$296 along with the \$4 left over from the first competition equally, with no amount remaining.

Assuming that each student received a whole number of dollars in both distributions, find the greatest possible number of students in the group.

	222	300
2	111	150
3	37	50

HCF is 6. There are 6 students.

Answer ……students [2]

---

5 **(a)** On 6 June 2020, Elijah invested some money in a bank which pays a simple interest at a rate of 3.5% per annum. He received a total interest of \$680.40 on 6 June 2023.

Find the amount of money Elijah invested in the bank.

$$\text{Amount of interest earn per year} = \$680.40 \div 3$$
$$= \$226.80$$
$$\text{Amount of money invested} = \frac{100}{3.5} \times \$226.80$$
$$= \$6480$$

Answer \$…….... [2]

(b) Freddy invested \$8000 in another bank that paid compound interest at a rate of 1.75% per annum, compounded quarterly.

Find the total amount Freddy received at the end of 5 years. Give your answer correct to the nearest cent.

$$\text{Total amount} = 8000\left(1 + \frac{1.75 \div 4}{100}\right)^{5\times 4}$$
$$= \$8729.87 \text{ (nearest cent)}$$

Answer \$…….... [2]

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6

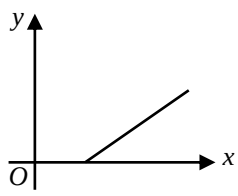


Diagram 1

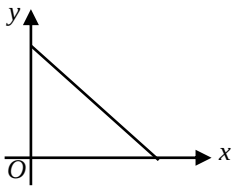


Diagram 2

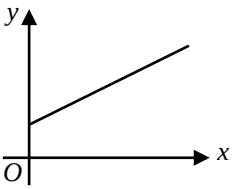


Diagram 3

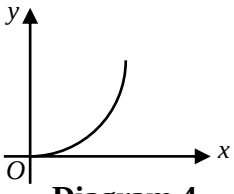


Diagram 4

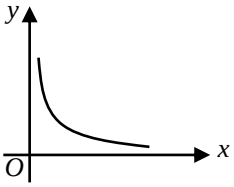


Diagram 5

From the diagrams above, select one of them which best illustrates each of the following statements.

(a) $y = 4\pi x^3$.

(b) The cost of a project \$$y$ is a linear function of x, where x is the number of man-hours required to complete the project. The project has a fixed cost of \$100.

(c) An object is travelling at a constant speed towards a fixed point O. The distance y (in metres) represents how far the object is from point O at time x (in minutes).

Answer **(a)** Diagram …4…… [1]

(b) Diagram …3…… [1]

(c) Diagram …2…… [1]

---

7 Write as a single fraction in its simplest form $\dfrac{4-x}{x+2}-\dfrac{2}{3-x}$.

$$\frac{4-x}{x+2}-\frac{2}{3-x}=\frac{(4-x)(3-x)-2(x+2)}{(x+2)(3-x)}$$

$$=\frac{12-7x+x^2-2x-4}{(x+2)(3-x)}$$

$$=\frac{x^2-9x+8}{(x+2)(3-x)}$$

$$=\frac{(x-1)(x-8)}{(x+2)(3-x)}$$

Answer ……..... [2]

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8 The table shows the distribution of the weights of 40 students.

Weight (W kg)	$40 < W \leq 50$	$50 < W \leq 60$	$60 < W \leq 70$	$70 < W \leq 80$	$80 < W \leq 90$
Frequency	6	8	a	11	3

(a) Find the value of a.

Answer $a =$ 12 [1]

(b) Calculate an estimate for

(i) the mean weight of the students,

Answer 64.25 kg [1]

(ii) the standard deviation of the weights.

Answer 11.7 kg [1]

(c) An error in the weighing machine caused the students' weights to be recorded 2 kg more than their actual values.

Explain how the mean and standard deviation will change after the error is rectified.

Answer

The mean will decrease by 2kg to 62.25kg while the standard deviation remains unchanged. [1]

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9 Andy bought a limited-edition watch from an online website based in Thailand for 10 460 baht. Andy also paid \$15 in Singapore dollars for shipping and GST of 9% on the cost of the watch. The exchange rate between Thai baht (฿) and Singapore dollars (S\$) was ฿100=S\$$P$. Andy spent a total of \$436 in Singapore dollars. Find the value of P, giving your answer correct to 3 significant figures.

$$\text{Cost of watch in Singapore dollars (without GST)} = \frac{10460}{100} \times P$$
$$= 104.6P$$
$$\text{Cost of watch in Singapore dollars (with GST)} = 1.09 \times 104.6P$$
$$= 114.014P$$
$$114.014P + 15 = 436$$
$$P = 3.69 \text{(3sf)}$$

Answer $P =$..... [3]

---

10 The gradient of the line joining the points $(-3+2a, 7)$ and $(a+1, 2)$ is $-\frac{2}{3}$. Find the value of a.

$$\text{gradient} = \frac{2-7}{a+1-(-3+2a)}$$
$$= \frac{-5}{-a+4}$$
$$\text{Given gradient } = -\frac{2}{3}$$
$$\frac{-5}{-a+4} = -\frac{2}{3}$$
$$-15 = -2(-a+4)$$
$$a = -\frac{7}{2}$$

Answer $a =$ ..... [2]

11 Given that $a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$, find the value of ab.

$$a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$$
$$a^2 - 2a\left(\frac{3}{b}\right) + \frac{9}{b^2} = 0$$
$$\left(a - \frac{3}{b}\right)^2 = 0$$
$$a - \frac{3}{b} = 0$$
$$a = \frac{3}{b}$$
$$ab = 3$$

Answer $ab =$..... [2]

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12 Factorise completely $4mn-16n^2-4m^2n+64n^3$.

$$\begin{aligned}4mn-16n^2-4m^2n+64n^3 &= 4n(m-4n)-4n(m^2-16n^2)\\ &= 4n(m-4n)-4n(m-4n)(m+4n)\\ &= 4n(m-4n)(1-m-4n)\end{aligned}$$

Answer ……..... [3]

---

13 A bag contains 4 black coins, 7 red coins and 11 white coins. Two coins are drawn from the bag at random, one after another without replacement.

(a) Find the probability that a white coin will be chosen on the second draw.

$$\begin{aligned}\text{Total number of coins} &= 4+7+11\\ &= 22\\ \text{P}(\text{second coin is white}) &= \text{P}(\text{1st coin of any colours, 2nd coin is white})\\ &= 1\times\frac{11}{22}\\ &= \frac{1}{2}\end{aligned}$$

Answer ……..... [1]

(b) x yellow coins are added to the bag. The probability of picking a black coin in **both** draws is $\frac{1}{50}$. Find the value of x.

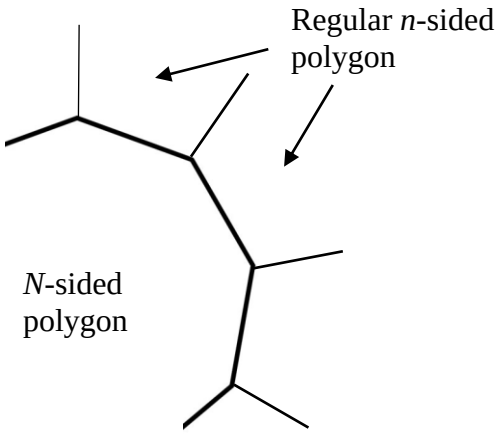
$$\begin{aligned}\text{Total number of coins} &= 22+x\\ \text{P}(\text{both coins are black}) &= \frac{4}{22+x}\times\frac{3}{21+x}\\ \frac{12}{(22+x)(21+x)} &= \frac{1}{50}\\ 600 &= (22+x)(21+x)\\ 600 &= 462+22x+21x+x^2\\ x^2+43x-138 &= 0\\ (x+46)(x-3) &= 0\\ x &= -46 \text{ or } \quad x=3\end{aligned}$$

(rejected)

Answer x = ……..... [3]

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14 A number of regular *n*-sided polygons are placed together in a ring to form a regular *N*-sided polygon as shown in the diagram below.



(a) Show that $N = \frac{2n}{n-4}$.

Answer

$$\frac{180(n-2)}{n} \times 2 + \frac{180(N-2)}{N} = 360$$
$$2N(n-2) + n(N-2) = 2nN$$
$$2nN - 4N + nN - 2n = 2nN$$
$$2n = nN - 4N$$
$$N(n-4) = 2n$$
$$N = \frac{2n}{n-4}$$

[3]

(b) Hence, explain why a regular octagon cannot be formed by placing smaller *n*-sided regular polygons in a ring.

Answer

$$\frac{2n}{n-4} = 8$$
$$2n = 8n - 32$$
$$6n = 32$$
$$n = \frac{16}{3}$$

It is not a octagon as *n* is not an integer.

[2]

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15 **(a)** Express x^2+5x+7 in the form $(x+a)^2+b$.

$$x^2+5x+7=\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+7$$
$$=\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$$

Answer ……..... [2]

(b) Hence, explain why the expression will never be negative.

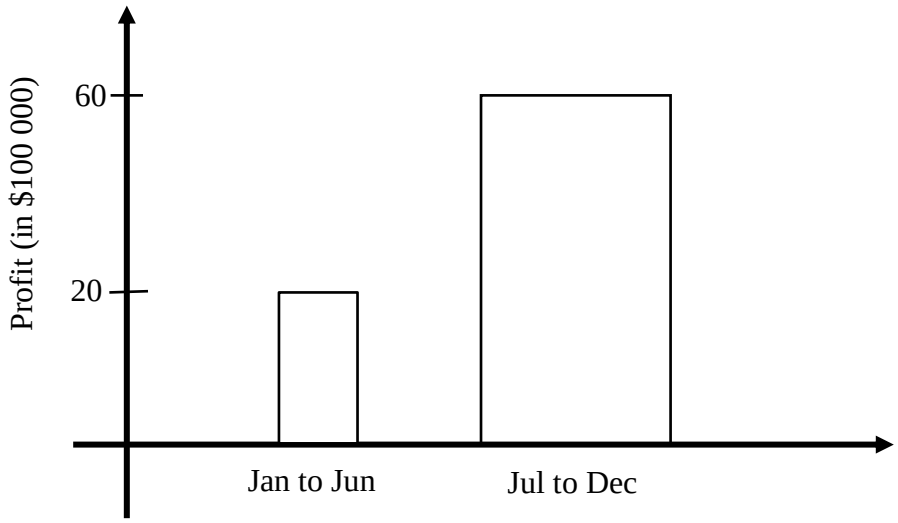
Answer

Since $\left(x+\frac{5}{2}\right)^2 \geq 0$ for all x, $\left(x+\frac{5}{2}\right)^2+\frac{3}{4} \geq \frac{3}{4}$. Therefore the expression will never be negative.

[1]

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16 A company presented their 2023 financial report in this graph.



State one aspect of the graph that may be misleading and explain how they may lead to a misinterpretation.

Answer

Each bar in the graph should be of the same width. The different widths of each bar will lead readers to misinterpretation such as interpreting a wider bar to mean a profit of a larger proportion that it actually is.

[2]

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17 $\varepsilon = \{\text{integer } x:\ 0 < x \leq 12\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{numbers that have at least 2 distinct factors}\}$

(a) Explain why A is a proper subset of B without listing down the elements.

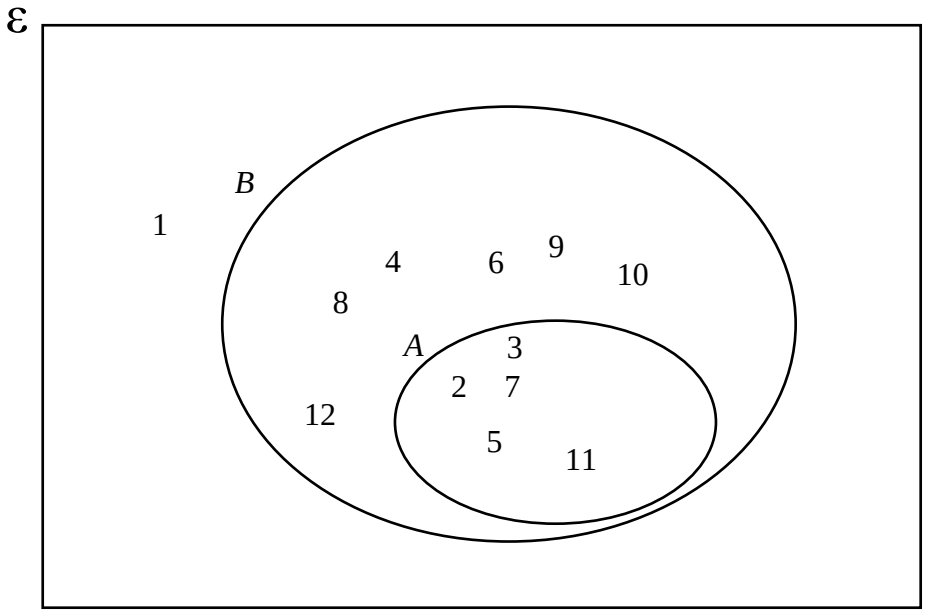
Answer

Prime numbers have 2 distinct factors, therefore every element of set A must also be an element of set B. However, some elements in B are composites and therefore not in Set A. Thus A is a proper subset of B.

[2]

(b) Draw a Venn diagram and list down the elements to illustrate the above information.

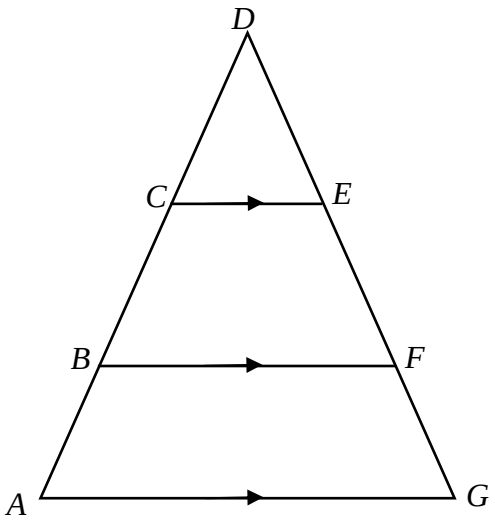
Answer



[2]

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18 The diagram shows a triangle AGD with the points C and B lying on AD and points E and F lying on GD.



It is given that $CE//BF//AG$, $BF = 2CE$ and $AG = 3CE$.

(a) Show that triangle DCE is similar to triangle DBF.

Answer

$\angle DCE = \angle DBF$ (corresponding angles)
$\angle CDE = \angle BDE$ (common angle)
$\therefore$ Triangle DCE is similar to triangle DBF

[2]

(b) Given that the area of trapezium $BFEC = 15\ \text{cm}^2$, find the area of trapezium $AGFB$.

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DBF} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DAG} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{\text{area of trapezium } AGFB}{\text{area of trapezium } BFEC} = \frac{9-4}{4-1} = \frac{5}{3}$$

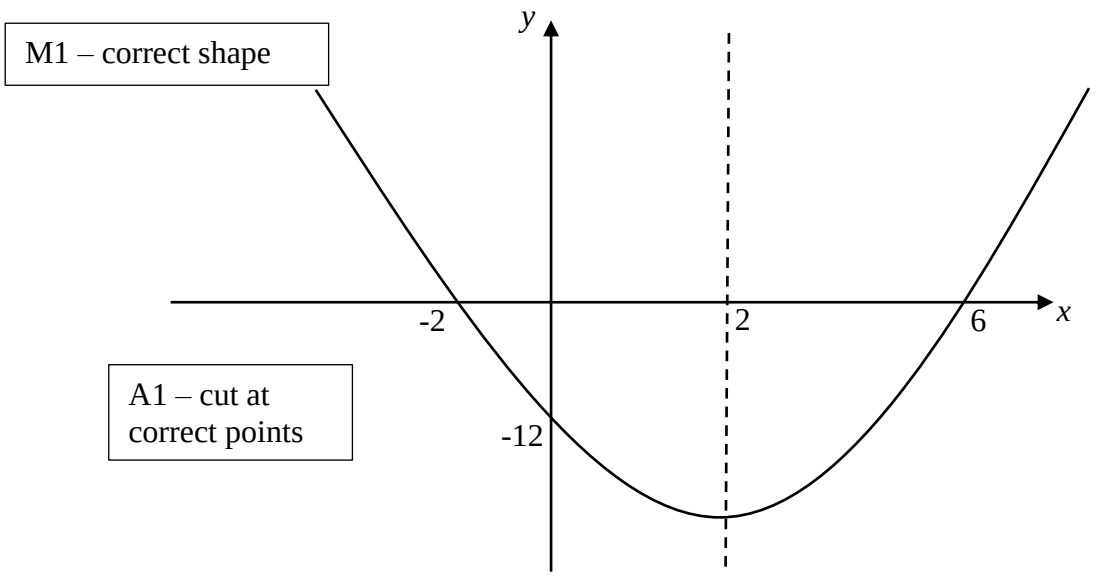
$$\frac{\text{area of trapezium } AGFB}{15} = \frac{5}{3}$$

area of trapezium $AGFB = 25\ \text{cm}^2$

Answer …….....cm^2 [3]

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19 Sketch the graph of $y=(2+x)(x-6)$ on the axes below. Indicate clearly the values where the graph crosses the axes and write down the equation of the line of symmetry.



M1 – correct shape

A1 – cut at correct points

Answer line of symmetry: ……. $x=2$

---

20 It is given that y is inversely proportional to the square root of x.
Find the percentage change in x when the value of y decreases by 50%.

$$y=\frac{k}{\sqrt{x}}$$

$$\sqrt{x}=\frac{k}{y}$$

$$x=\frac{k^2}{y^2}$$

$$\text{new } x=\frac{k^2}{(0.5y^2)}$$

$$=\frac{4k^2}{y^2}$$

$$\text{Percentage change in } x=\frac{\frac{4k^2}{y^2}-\frac{k^2}{y^2}}{\frac{k^2}{y^2}}\times 100\%$$

$$=300\%$$

Answer ……..... [2]

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21 The table shows the prices of movie tickets categorised by different days of the week and the various age groups.

	Monday to Thursday	Friday	Saturday and Sunday
Child	\$6.00	\$8.50	\$9.00
Adult	\$7.00	\$10.00	\$12.50
Senior Citizen	\$6.50	\$9.00	\$10.00

(a) Write down a 3×3 matrix **P** to represent the above information.

$$\begin{pmatrix} 6 & 8.5 & 9 \\ 7 & 10 & 12.5 \\ 6.5 & 9 & 10 \end{pmatrix}$$

Answer **P**=……..... [1]

(b) (i) The Ng family comprises a child, two adults and a senior citizen while the Tan family comprises three children and two adults. Represent this information as a 2×3 matrix **Q**.

$$\mathbf{Q}=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 0 \end{pmatrix}$$

Answer **Q** =…….....[1]

(ii) Evaluate **QP** and explain what the elements represent.

Answer

$$\mathbf{QP}=\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 0 \end{pmatrix}\begin{pmatrix} 6 & 8.5 & 9 \\ 7 & 10 & 12.5 \\ 6.5 & 9 & 10 \end{pmatrix}$$

$$=\begin{pmatrix} 26.5 & 37.5 & 44 \\ 32 & 45.5 & 52 \end{pmatrix}$$

……

The elements in **QP** represent the respective total cost of movie tickets for the Ng family of 1 child, 2 adults and 1 senior citizen and the Tan Family of 3 children and 2 adults on the different days of the week.

…… [2]

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22 Sam's daily morning routine involves getting breakfast before going to work. He travels by car. On a particular day, Sam leaves home at 8:00 am and arrives at the breakfast place at 8:15 am. He leaves the breakfast place at 8:30 am. He arrives at his office at 9:00 am. Sam's average speed for the whole journey is 42 km/h.

His average speed from home to the breakfast place is 10 km/h faster than his average speed from the breakfast place to the office.

Find the distance between the breakfast place and office.

Total time taken = 1 hour

Total distance travelled = 42 km

Let x be the distance between the breakfast place to office.

Distance between home to breakfast place = $42 - x$

$$V_1 = \frac{42 - x}{\left(\frac{15}{60}\right)} = 4(42 - x)$$

$$V_2 = \frac{x}{\left(\frac{1}{2}\right)} = 2x$$

$$4(42 - x) - 2x = 10$$

$$168 - 4x - 2x = 10$$

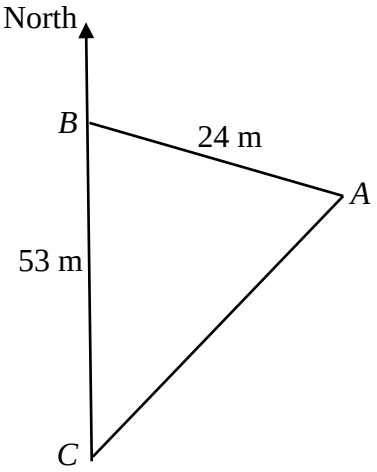
$$6x = 158$$

$$x = 26\frac{1}{3}$$

Answer …….....km [4]

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23 Three points *A*, *B* and *C* lie on a horizontal ground are such that $AB = 24$ m and $BC = 53$ m. Point *B* is due north of *C*. The bearing of *A* from *B* is $122°$.



(a) Find the distance *AC*.

$$AC = \sqrt{53^2 + 24^2 - 2(53)(24)\cos 58}$$
$$= 45.13186...$$
$$= 45.1 \text{ (3sf)}$$

Answer …….....m [2]

(b) Find the bearing of *C* from *A*.

By sine rule,

$$\frac{\sin \angle BAC}{53} = \frac{\sin \angle ABC}{45.13186...}$$
$$\sin \angle BAC = 53 \times \frac{\sin 58°}{45.13186...}$$
$$\angle BAC = 84.8060...$$
Bearing of C from $A = 360° - 58° - 84.8060...°$
$$= 217.2°$$

Answer …….....° [3]

(c) Find the area of the triangular plot *ABC*.

$$\text{Area of } \Delta ABC = \frac{1}{2}(53)(24)\sin 58°$$
$$= 539.35...$$
$$= 539$$

Answer ……..m^2 [2]

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24 The first four terms in a sequence of numbers are

$$3+k, 1+k, -1+k, -3+k, ...$$

where k is a constant.

(a) Find an expression in terms of n and k, for the nth term in this sequence.

$$\begin{aligned} T_n &= 3+k+(n-1)(-2) \\ &= 3+k-2n+2 \\ &= 5-2n+k \end{aligned}$$

Answer ……..... [1]

(a) State two conditions on k such that 39 is a term of the sequence.

Answer

$5-2n+k=39$

$k-2n=34$

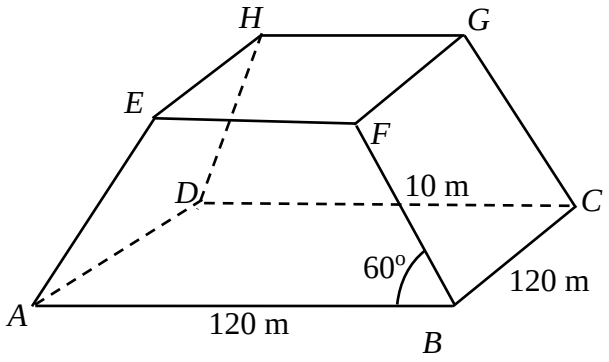
$k=34+2n$

Since $n \geq 1$, $34+2n \geq 36$, therefore $k \geq 36$ and k must be an even number.

[2]

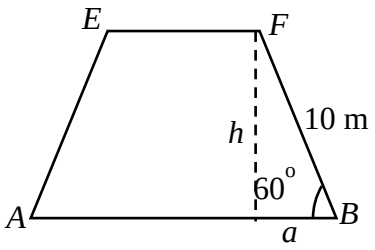
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25 The diagram below shows an indoor adventure park in the shape of a trapezoidal prism with a square base *ABCD*. The indoor adventure park is positioned on horizontal ground and the walls *ADEH* and *BCFG* are slanted while the walls *ABFE* and *DCGH* are vertically upright.



The top of the prism, *EFGH*, is the ceiling of the adventure park which is also horizontal. *EFGH* is a square and the centre of *EFGH* lies vertically above the centre of *ABCD*.
$AB = 120$ m, $AE = BF = CG = DH = 10$ m and $\angle ABF = 60°$.

(a) Find the area of *ABFE*.



Since $AE = BF$, *ABFE* is an isosceles trapezium.

$$\sin 60° = \frac{h}{10}$$
$$h = 10 \sin 60°$$
$$= 8.660254...$$
$$\cos 60° = \frac{a}{10}$$
$$a = 10 \cos 60°$$
$$= 5$$

Thus $EF = 12 - 5 - 5 = 2$

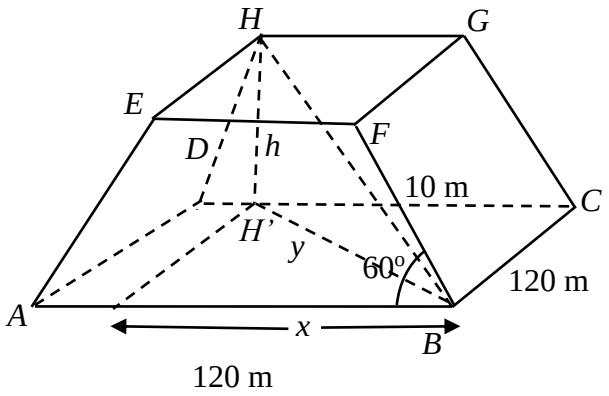
$$\text{Area of trapezium } ABFE = \frac{1}{2}(120 + 110)(8.660254...)$$
$$= 996$$

Answer ……m^2 [4]

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(b) The owner of the indoor park wants to build a flying fox feature from point H to point B. The angle of depression from point H to point B must not exceed $5°$, in order to meet the safety requirements. Explain with mathematical workings whether the flying fox feature can be built.

Answer



$x = 120 - 5$
$= 115$

$y = \sqrt{120^2 + 115^2}$
$= \sqrt{27625}$

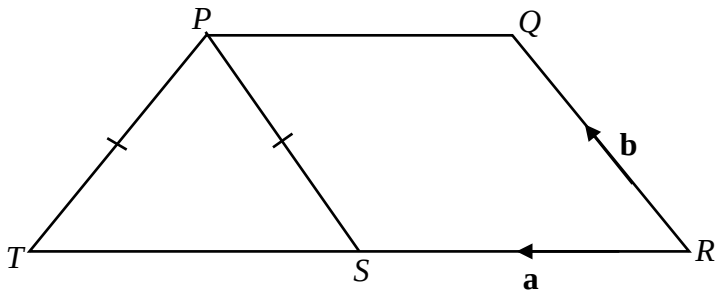
$\tan \angle HBH' = \dfrac{8.660254...}{\sqrt{27625}}$
$\angle HBH' = 2.9826...°$
$= 3.0°$

By alternate angle, the angle of depression is $3.0°$ which is less than $5°$, therefore the flying fox feature can be built.

[3]

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26 In the diagram below, *PQRST* is a trapezium made up of a parallelogram *PQRS* and an isosceles triangle *PST* where $PT = PS$. It is given that $\overrightarrow{RS} = \mathbf{a}, \overrightarrow{RQ} = \mathbf{b}$ and $SR = ST$.



(a) Show that $\overrightarrow{QS} = \overrightarrow{PT}$.

Answer

$$\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$$
$$= -\mathbf{b} + \mathbf{a}$$
$$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$$
$$= -\mathbf{b} + \mathbf{a}$$

Therefore $\overrightarrow{QS} = \overrightarrow{PT}$.

[2]

(b) Hence or otherwise, prove that the trapezium is made up of three isosceles triangles *PST*.

Answer

Since *PQRS* is a parallelogram,
$PS // QR, PQ // SR$

$PS = QR$ and $PQ = SR$

From (a), since $\overrightarrow{QS} = \overrightarrow{PT}$, thus $PT = QS$.

$PT = QS = PS = QR$ and $PQ = SR = TS$.

By SSS congruency test, triangle *PTS*, triangle *PQS* and triangle *QSR* are congruent, therefore the trapezium is made up of three isosceles triangles *PST*.

[3]

End of Paper

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Alternative solutions

9	Total cost excluding shipping in Singapore dollars $= \$436 - \$15$ $= \$421$ Cost of watch excluding GST in Singapore dollars $= \frac{100}{109} \times \$421$ $= \$386.238532$ $\frac{386.2385321}{P} \times 100 = 10460$ $386.2385321 = 104.60P$ $P = 3.69$ (3sf)
11	$a^2b^2 - 6ab + 9 = 0$ $(ab)^2 - 2(ab)(3) + 3^2 = 0$ $(ab-3)^2 = 0$ $ab - 3 = 0$ $ab = 3$
14(a)	Exterior $\angle$ of N-sided polygon $= \frac{360}{N}$ Interior $\angle$ of n-sided polygon $= \frac{180(n-2)}{n}$ $180 + \frac{360}{N} = \frac{2 \times 180(n-2)}{n}$ $1 + \frac{2}{N} = \frac{2(n-2)}{n}$ $\frac{2}{N} = \frac{2n-4}{n} - 1$ $\frac{2}{N} = \frac{2n-4-n}{n}$ $\frac{N}{2} = \frac{n}{n-4}$ $N = \frac{2n}{n-4}$

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