Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
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PRELIMINARY EXAMINATION 2024
SECONDARY 4

MATHEMATICS

4052/01

Paper 1

Friday 23 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use	
Total	**/ 90**

This document consists of **20** printed pages.

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Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

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Answer **all** the questions.

1 The length of a court is 0.028 km and the width of the court is 16 m. Express the ratio of the length to the width of the court in its simplest form.

Answer ..... : ..... [1]

---

2 Given that $0 < x < 180$, find the possible values of x for $\sin x° = 0.985$.
Give your answers correct to 1 decimal place.

Answer $x =$ .....or..... [2]

---

3 **(a)** Simplify $\dfrac{(4a^4b)^3}{10a^3b^7}$, leaving your answer in positive index form.

Answer ..... [2]

(b) Given that $2^x = 3$, $2^y = 7$ and $2^z = \dfrac{9}{49}$, use the laws of indices to find the value of $2^{3x-y+\frac{1}{2}z}$.

Answer ..... [2]

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4 A group of students won \$226 in a competition and shared the amount equally, leaving \$4 leftover. Competing for a second time, the same group of students won \$296. They shared the \$296 along with the \$4 left over from the first competition equally, with no amount remaining.

Assuming that each student received a whole number of dollars in both distributions, find the greatest possible number of students in the group.

Answer .....students [2]

---

5 **(a)** On 6 June 2020, Elijah invested some money in a bank which pays a simple interest at a rate of 3.5% per annum. He received a total interest of \$680.40 on 6 June 2023.

Find the amount of money Elijah invested in the bank.

Answer \$..... [2]

(b) Freddy invested \$8000 in another bank that paid compound interest at a rate of 1.75% per annum, compounded quarterly.

Find the total amount Freddy received at the end of 5 years. Give your answer correct to the nearest cent.

Answer \$..... [2]

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6

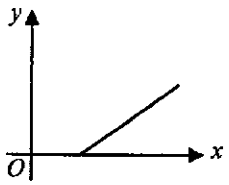


Diagram 1

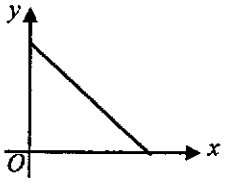


Diagram 2

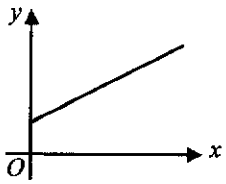


Diagram 3

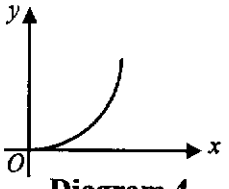


Diagram 4

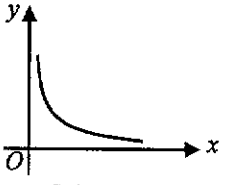


Diagram 5

From the diagrams above, select one of them which best illustrates each of the following statements.

(a) $y = 4\pi x^3$.

(b) The cost of a project \$$y$ is a linear function of x, where x is the number of man-hours required to complete the project. The project has a fixed cost of \$100.

(c) An object is travelling at a constant speed towards a fixed point O. The distance y (in metres) represents how far the object is from point O at time x (in minutes).

Answer **(a)** Diagram ..... [1]

(b) Diagram ..... [1]

(c) Diagram ..... [1]

---

7 Write as a single fraction in its simplest form $\frac{4-x}{x+2} - \frac{2}{3-x}$.

Answer ..... [2]

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8 The table shows the distribution of the weights of 40 students.

Weight (W kg)	$40 < W \leq 50$	$50 < W \leq 60$	$60 < W \leq 70$	$70 < W \leq 80$	$80 < W \leq 90$
Frequency	6	8	a	11	3

(a) Find the value of a.

Answer $a =$ .....[1]

(b) Calculate an estimate for

(i) the mean weight of the students,

Answer .....kg [1]

(ii) the standard deviation of the weights.

Answer .....kg [1]

(c) An error in the weighing machine caused the students' weights to be recorded 2 kg more than their actual values.

Explain how the mean and standard deviation will change after the error is rectified.

Answer

.....

.....

.....[1]

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9 Andy bought a limited-edition watch from an online website based in Thailand for 10 460 baht.
Andy also paid \$15 in Singapore dollars for shipping and GST of 9% on the cost of the watch.
The exchange rate between Thai baht (฿) and Singapore dollars (S\$) was ฿100=S\$$P$.
Andy spent a total of \$436 in Singapore dollars.
Find the value of P, giving your answer correct to 3 significant figures.

Answer $P=$..... [3]

---

10 The gradient of the line joining the points $(-3+2a,7)$ and $(a+1,2)$ is $-\frac{2}{3}$. Find the value of a.

Answer $a=$ ..... [2]

---

11 Given that $a^2-\frac{6a}{b}+\frac{9}{b^2}=0$, find the value of ab.

Answer $ab=$..... [2]

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12 Factorise completely $4mn-16n^2-4m^2n+64n^3$.

Answer ..... [3]

---

13 A bag contains 4 black coins, 7 red coins and 11 white coins. Two coins are drawn from the bag at random, one after another without replacement.

(a) Find the probability that a white coin will be chosen on the second draw.

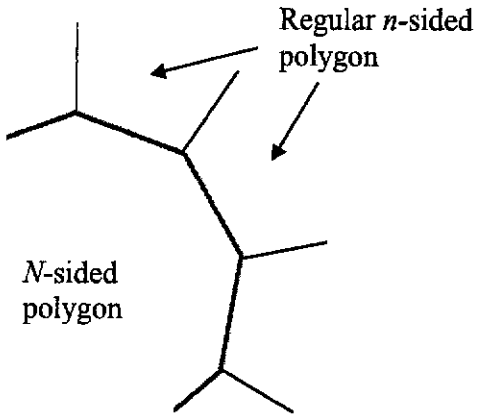
Answer ..... [1]

(b) x yellow coins are added to the bag. The probability of picking a black coin in **both** draws is $\frac{1}{50}$. Find the value of x.

Answer x = ... [3]

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14 A number of regular n-sided polygons are placed together in a ring to form a regular N-sided polygon as shown in the diagram below.



(a) Show that $N = \dfrac{2n}{n-4}$.

Answer

[3]

(b) Hence, explain why a regular octagon cannot be formed by placing smaller n-sided regular polygons in a ring.

Answer

[2]

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15 **(a)** Express x^2+5x+7 in the form $(x+a)^2+b$.

Answer ..... [2]

(b) Hence, explain why the expression will never be negative.

Answer

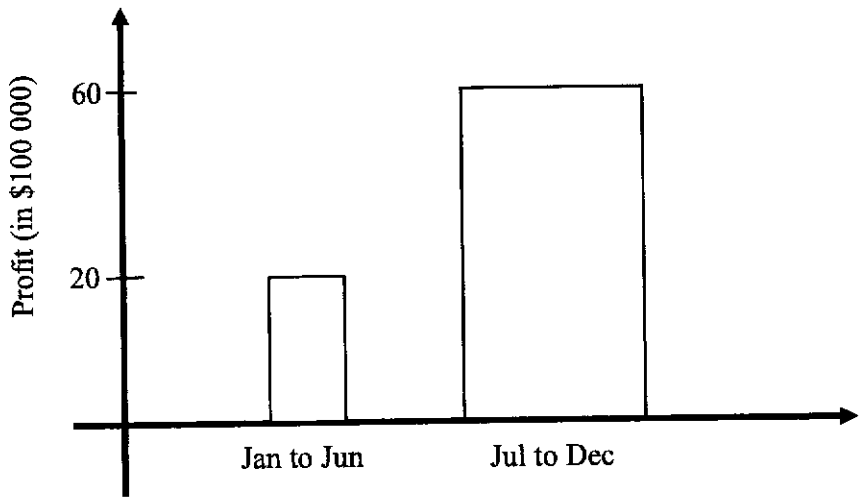
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..... [1]

---

16 A company presented their 2023 financial report in this graph.



State one aspect of the graph that may be misleading and explain how they may lead to a misinterpretation.

Answer

.....

.....

..... [2]

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17 $\varepsilon = \{\text{integer } x:\ 0 < x \leq 12\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{numbers that have at least 2 distinct factors}\}$

(a) Explain why A is a proper subset of B without listing down the elements.

Answer

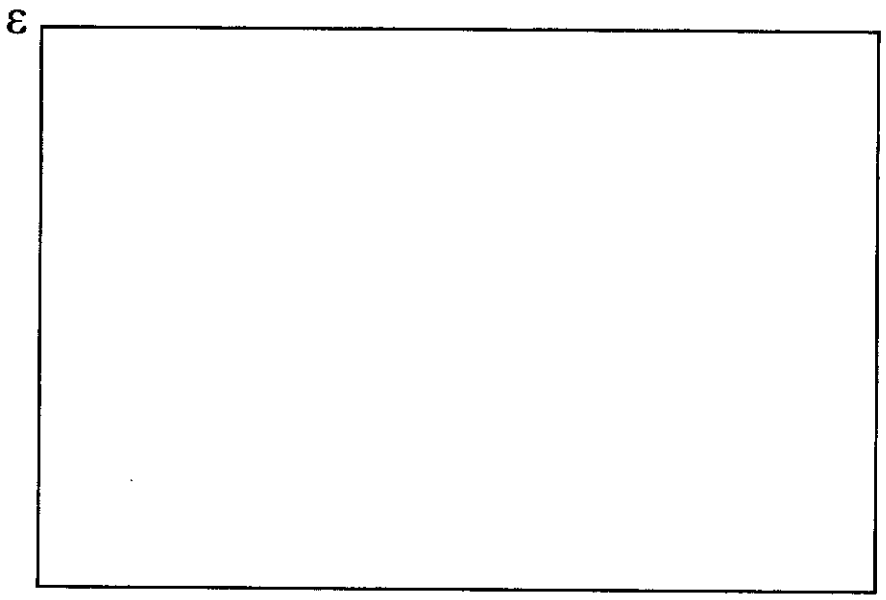
.....

.....

..... [2]

(b) Draw a Venn diagram and list down the elements to illustrate the above information.

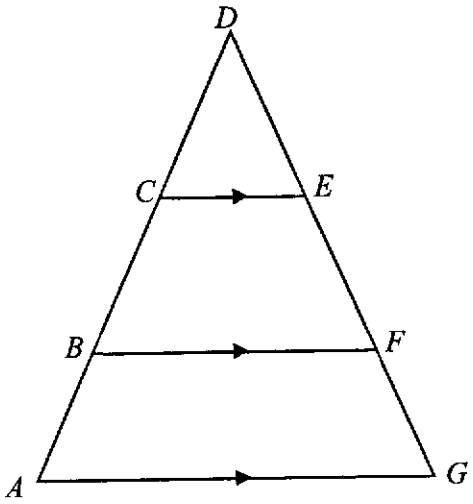
Answer



[2]

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18 The diagram shows a triangle AGD with the points C and B lying on AD and points E and F lying on GD.



It is given that $CE//BF//AG$, $BF = 2CE$ and $AG = 3CE$.

(a) Show that triangle DCE is similar to triangle DBF.

Answer

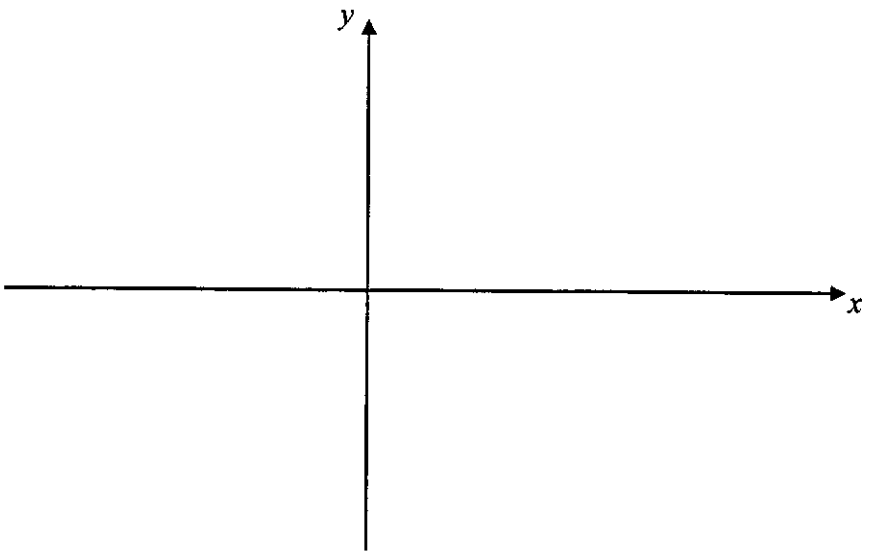
[2]

(b) Given that the area of trapezium $BFEC = 15\ \text{cm}^2$, find the area of trapezium $AGFB$.

Answer .....cm^2 [3]

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19 Sketch the graph of $y=(2+x)(x-6)$ on the axes below. Indicate clearly the values where the graph crosses the axes and write down the equation of the line of symmetry.



Answer line of symmetry: ..... [3]

---

20 It is given that y is inversely proportional to the square root of x.
Find the percentage change in x when the value of y decreases by 50%.

Answer ..... [2]

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21 The table shows the prices of movie tickets categorised by different days of the week and the various age groups.

	Monday to Thursday	Friday	Saturday and Sunday
Child	\$6.00	\$8.50	\$9.00
Adult	\$7.00	\$10.00	\$12.50
Senior Citizen	\$6.50	\$9.00	\$10.00

(a) Write down a 3×3 matrix **P** to represent the above information.

Answer **P**=..... [1]

(b) (i) The Ng family comprises a child, two adults and a senior citizen while the Tan family comprises three children and two adults. Represent this information as a 2×3 matrix **Q**.

Answer **Q** =.....[1]

(ii) Evaluate **QP** and explain what the elements represent.

Answer

.....

.....

.....

.....

..... [2]

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22 Sam's daily morning routine involves getting breakfast before going to work. He travels by car. On a particular day, Sam leaves home at 8:00 am and arrives at the breakfast place at 8:15 am. He leaves the breakfast place at 8:30 am. He arrives at his office at 9:00 am. Sam's average speed for the whole journey is 42 km/h.

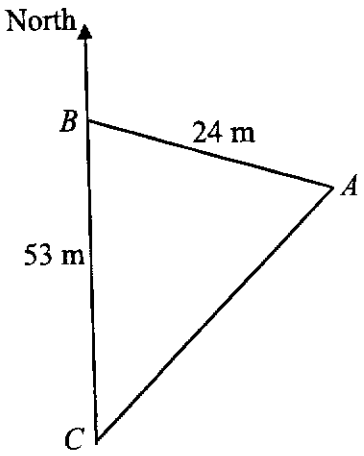
His average speed from home to the breakfast place is 10 km/h faster than his average speed from the breakfast place to the office.

Find the distance between the breakfast place and Sam's office.

Answer .....km [4]

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23 Three points A, B and C lie on a horizontal ground are such that $AB = 24$ m and $BC = 53$ m. Point B is due north of C. The bearing of A from B is $122°$.



(a) Find the distance AC.

Answer .....m [2]

(b) Find the bearing of C from A.

Answer .....° [3]

(c) Find the area of the triangular plot ABC.

Answer .....m^2 [2]

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24 The first four terms in a sequence of numbers are

$$3+k, 1+k, -1+k, -3+k, \ldots$$

where k is a constant.

(a) Find an expression in terms of n and k, for the nth term in this sequence.

Answer ..... [1]

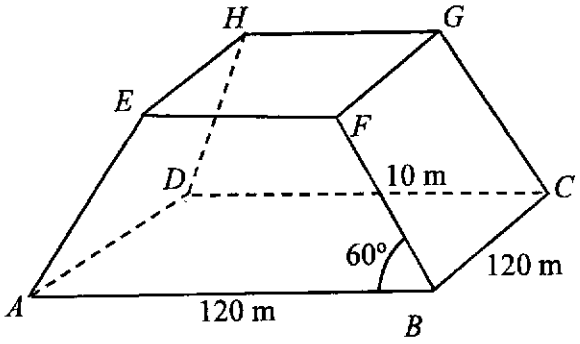
(b) State two conditions on k such that 39 is a term of the sequence.

Answer

[2]

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25 The diagram below shows an indoor adventure park in the shape of a trapezoidal prism with a square base *ABCD*. The indoor adventure park is positioned on horizontal ground and the walls *ADHE* and *BCGF* are slanted.



The top of the prism, *EFGH*, is the ceiling of the adventure park which is also horizontal. *EFGH* is a square and the centre of *EFGH* lies vertically above the centre of *ABCD*.
$AB = 120$ m, $AE = BF = CG = DH = 10$ m and $\angle ABF = 60°$.

(a) Find the area of *ABFE*.

Answer .....m^2 [4]

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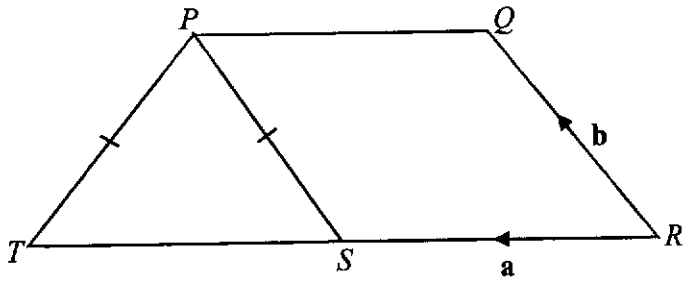
(b) The owner of the indoor park wants to build a flying fox feature from point H to point B. The angle of depression from point H to point B must not exceed $5°$, in order to meet the safety requirements. Explain, with mathematical working, whether the flying fox feature can be built.

Answer

[3]

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26 In the diagram below, $PQRST$ is a trapezium made up of a parallelogram $PQRS$ and an isosceles triangle PST where $PT = PS$. It is given that $\overrightarrow{RS} = \mathbf{a}, \overrightarrow{RQ} = \mathbf{b}$ and $SR = ST$.



(a) Show that $\overrightarrow{QS} = \overrightarrow{PT}$.

Answer

[2]

(b) Hence or otherwise, prove that the trapezium is made up of three isosceles triangles PST.

Answer

[3]

End of Paper

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PRELIMINARY EXAMINATION 2024
SECONDARY 4

MATHEMATICS

4052/02

Paper 2

Tuesday 20 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper

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For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total Marks	

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TURN OVER FOR QUESTION 1

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1 **(a)** The table shows the total electricity consumption in Singapore over three years.

Year	2020	2021	2022
Total Electricity Consumption in Gigawatt Hours (GWh)	50 779	53 483	54 884

(i) In 2021, the electricity consumed by households took up 15.5% of the total electricity consumption. Calculate the amount of electricity consumed by households in 2021, correct to two significant figures.

Answer ..... GWh [1]

(ii) Calculate the percentage increase in the total electricity consumption from 2020 to 2022.

Answer .....% [2]

(iii) Express the 2020 electricity consumption in kilowatt hours (kWh), leaving your answer in standard form, correct to two significant figures.

Answer ..... kWh [1]

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(b) A microprocessor is in the shape of a cube where the sides are 5 mm in length.

(i) Find the maximum number of microprocessors that can be placed into a container with dimensions 10 cm by 2 cm by 8 cm.

Answer ..... [2]

(ii) A model of the microprocessor was made to a scale of 10 : 1. Given that the surface area of microprocessor is 150 mm^2, find the surface area of the model microprocessor in square centimetres.

Answer ..... cm^2 [2]

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2 **(a)** Solve $2x-7=3(1-3x)$.

Answer $x =$ ..... [2]

(b) Solve the inequalities $7x-1<13$ and $\frac{x+1}{2} \geq -2(x-2)$.

Answer ..... [3]

(c) Rearrange the formula $y=\frac{x^2+5}{7x^2}$ to make x the subject.

Answer $x =$ ..... [3]

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(d) Solve the equation $\frac{2x}{1-x}-3=\frac{1}{2x-3}$.

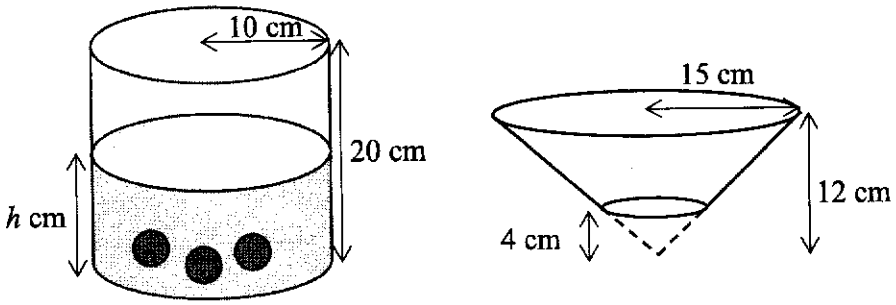
Give your solutions correct to two decimal places.

Answer $x =$ ..... or ..... [5]

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3 The diagram shows two open containers, a right cylinder and an inverted frustum. The right cylinder has a base radius of 10 cm and a height of 20 cm. The frustum is formed by cutting a smaller cone off the bottom of a larger cone. The smaller cone that was cut off has a height of 4 cm. The larger cone has a base radius of 15 cm and a height of 12 cm.

Three identical spherical marbles, each of radius 3 cm, are placed into the cylindrical container and water is poured in to a depth of h cm.



(a) Find the volume of each marble, leaving your answer in terms of π.

Answer ..... cm^3 [1]

(b) Find the volume of the frustum, leaving your answer in terms of π.

Answer ..... cm^3 [3]

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(c) All the water from the cylinder, without the marbles, is then poured into the empty frustum, filling it completely without any overflow.

Find the value of h, correct to two decimal places.

Answer h = ... [3]

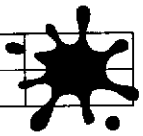
(d) The exterior surface area of each container is painted.
Find the total area painted.

Answer ..... cm^2 [6]

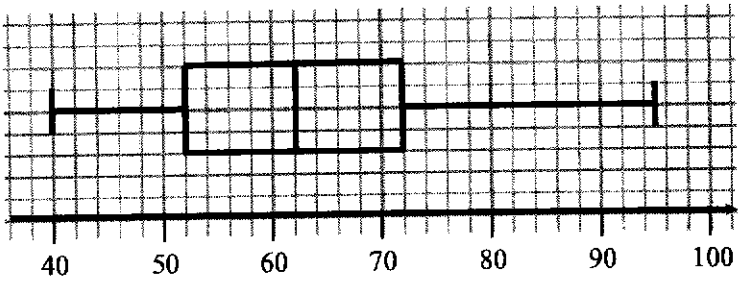
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4 (a) 12 students from class *A* took a Mathematics test.
The table below shows the test marks of the students. However, **two** of the students' marks are covered with ink.

40	95	84	47	63
52	63	55	52	61



(i) Given that the box-and-whisker plot below shows the distribution of the results, explain why the information may not be sufficient to find the missing marks of the two students.



Answer

.....

.....

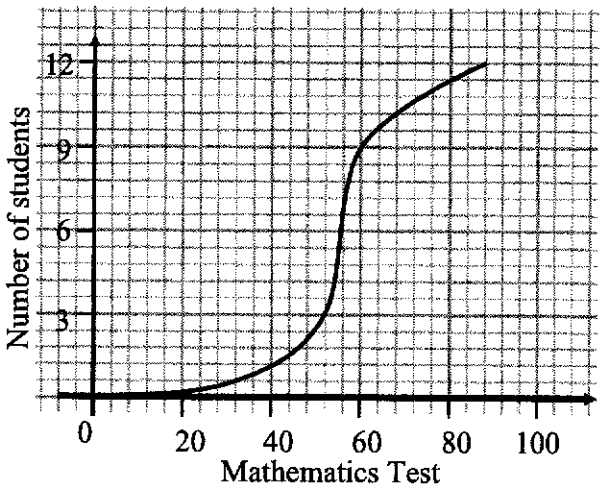
..... [1]

(ii) Given further that the modal mark is 63, find the two missing marks.

Answer ..... and ..... [2]

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(iii) 12 students from class *B* also took the same Mathematics test. The distribution of their test marks is shown on the cumulative frequency graph.



Make a comment comparing the averages and a comment comparing the distribution of the Mathematics Test marks between the two classes.
Use figures to support your answers.

Answer

.....

.....

.....

..... [3]

(b) Alice rolled a six-sided die, *X*, 100 times. Bala rolled another six-sided die, *Y*, 80 times. The number of times they each obtained a '6' is recorded in the table.

Die	Number of rolls	Number of times '6' is obtained
X	100	16
Y	80	18

(i) Find the probability of rolling a '6' by Alice and Bala respectively.

Answer P(obtaining a '6' by Alice) = ... [1]

P(obtaining a '6' by Bala) = ... [1]

(ii) One of the dice is biased. Using your answers in **(b)(i)**, determine which die, *X* or *Y*, is likely to be the unbiased 6-sided die. Explain your answer.
Answer

.....

..... [1]

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5 **(a)** Complete the table of values for $y = \frac{2}{x^2} - 2x$.

Values are given to one decimal place where appropriate.

x	–3	–2	–1	–0.5	–0.4	0.5	1	2	3
y		4.5	4	9	13.3	7	0	–3.5	–5.8

[1]

(b) On the grid opposite, draw the graph of $y = \frac{2}{x^2} - 2x$ for $-3 \le x \le 3$. [3]

(c) **(i)** On the same grid, draw the graph of $y + x = 6$ for $-3 \le x \le 3$.

[1]

(ii) Write down the x-coordinates of the points where the line intersects the curve.

Answer $x =$ ..... and ..... [2]

(iii) These values of x are solutions of the equation $x^3 + Ax^2 + B = 0$.

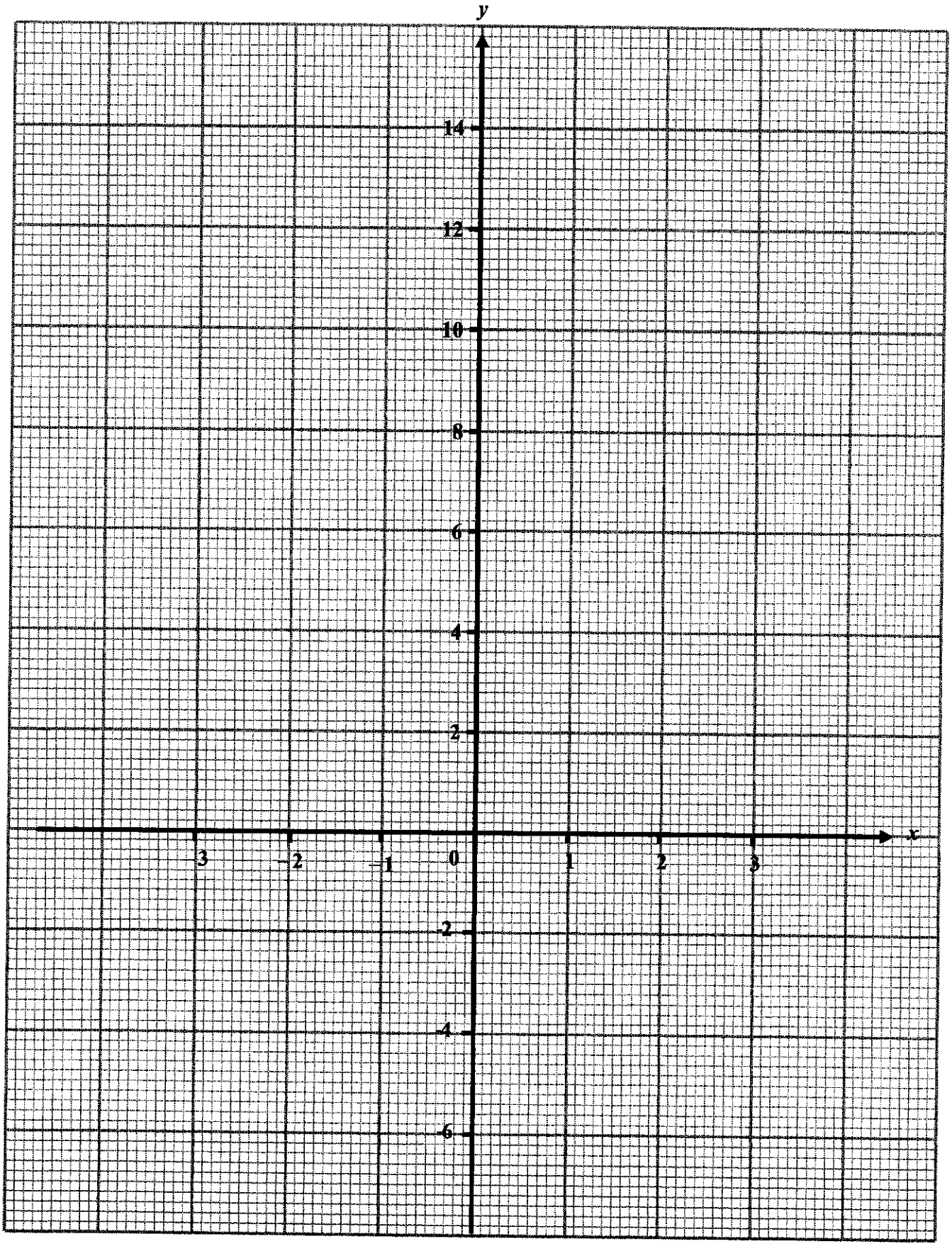
Find the value of A and of B.

Answer $A =$ .....

$B =$ .....

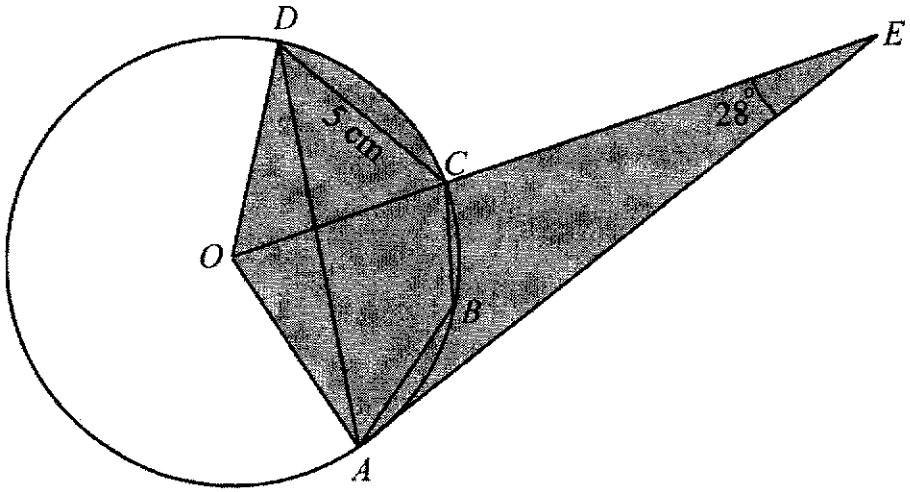
[3]

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6



A, B, C and D are points on the circle with centre O and radius 5 cm.
AE is a tangent to the circle at A and OE is a straight line that passes through C.
Angle $OEA = 28°$ and $CD = 5$ cm.

(a) Find, giving a reason for each step of your working,

(i) angle OAD,

Answer Angle OAD = ... ° [3]

(ii) angle ABC.

Answer Angle ABC = ... ° [2]

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(b) Find the area of the shaded region.

Answer ..... cm^2 [4]

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7 **(a)** A, B, C and D are four points such that the coordinates of A and C are $(4,2)$ and $(10,-34)$ respectively. $\overrightarrow{AB} = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 24 \\ -12 \end{pmatrix}$.

(i) Find the coordinates of B.

Answer (..... , ...) [1]

(ii) Find $\left|\overrightarrow{AB}\right|$.

Answer $\left|\overrightarrow{AB}\right|$ = ... units [1]

It is given that E is a point on BD such that $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BD}$.

(iii) Show that A, C and E are collinear.

Answer

[4]

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(b) A is the point $(2,6)$, B is the point $(-4,-2)$ and C is the point $(6,-2)$. A line L passes through point C and is parallel to AB.

(i) Find the equation of line L.

Answer ..... [2]

Point D lies on line L such that $AD // BC$.

(ii) Find the coordinates of point D.

Answer D (..... , ...) [2]

(iii) Show that $ABCD$ is a rhombus.

Answer

[2]

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8 **(a)** The figure shows the first three figures of a sequence formed by sticks of the same size.

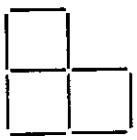


Figure 1

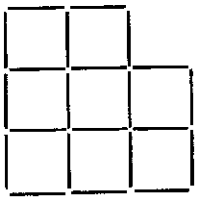


Figure 2

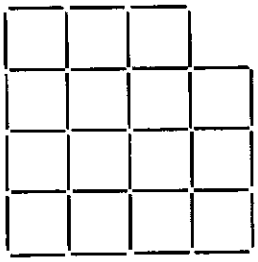


Figure 3

The number of small squares formed in each of the figures is recorded in the table.

Figure Number	Number of Squares
1	$(1+1)^2-1$
2	$(2+1)^2-1$
3	$(3+1)^2-1$

(i) Find the number of small squares formed in Figure 5.

Answer ..... [1]

(ii) Find, in terms of n, an expression for the number of squares in Figure n.

Answer ..... [1]

(iii) Explain why the sum of the number of squares in two consecutive figures is always odd.

Answer

.....

..... [2]

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(b) The table below shows the n^{th} terms of 4 sequences.

Sequence number	n^{th} term
1	$4n+3$
2	$7n+1$
3	$14n$
4	$8n-1$

For each sequence, are the numbers in the sequence always multiples of 7, sometimes multiples of 7 or never multiples of 7?

Write down the letter 'A', 'S' or 'N' to represent your answer.

A Always multiples of 7

S Sometimes multiples of 7

N Never multiples of 7

Answer

Sequence 1 .....

Sequence 2 .....

Sequence 3 .....

Sequence 4 ..... [2]

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9 Sam is a 40-year-old man who earns a gross salary of $8000 a month. He is a Singaporean and an employee of a Singapore firm. Sam is required to put a certain percentage of his monthly gross salary into his Central Provident Fund (CPF) account.

CPF is a mandatory (social security) savings scheme funded by contributions from employers and employees. The table below shows the CPF contribution rates by employers and employees.

Employee's age (Years)	**CPF Contribution Rates from 1 January 2024 (Monthly gross salary > $750)**		
	Total (% of gross salary)	**By Employer (% of gross salary)**	**By Employee (% of gross salary)**
55 and below	37	17	20
Above 55 to 60	31	15	16
Above 60 to 65	22	11.5	10.5
Above 65 to 70	16.5	9	7.5
Above 70	12.5	7.5	5

(a) Find the amount of money Sam's employer must contribute to his CPF monthly.

Answer $..... [1]

Sam wants to plan his monthly savings and has tabulated his monthly expenditure as shown in the table. He hopes to save at least 20% of his salary each month after CPF deductions.

Expenditure	**Amount ($)**
Food and Groceries	820
Transportation	90
Insurance and Healthcare	1000
Phone and Internet Subscriptions	80
Utilities	300
Housing Loan	1000
Leisure and Entertainment (movies, sports, books subscription fees, dining etc)	1580

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(b) Determine whether Sam is able to achieve his saving goals by clearly showing your calculations.

.....

..... [4]

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(c) After a year of saving, Sam decides to invest \$15 000 of his savings for a period of 5 years. He comes up with 2 investment plans.

Plan A

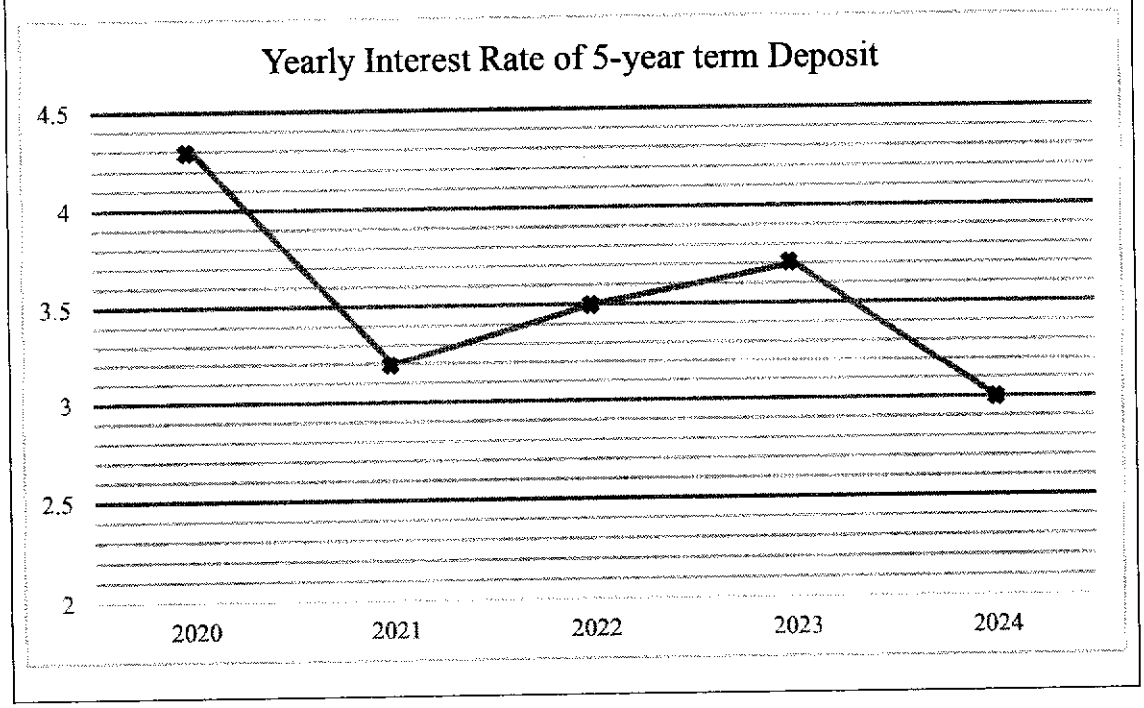
Sam can invest his money in a 5-year savings bond at the beginning of 2025. The interest rate for the 5-year term is shown in the table. The savings bond compounds annually.

Year from issue date	1	2	3	4	5
Interest per year %	3.19	3.19	3.20	3.28	3.31

Plan B

Sam can invest his money at the beginning of 2025 into an insurance company's fixed deposit account for a period of 5 years, compounded yearly. The interest rate is fixed over the duration of the investment and is determined by the year of issuance.
For example: If Sam had invested his money in 2020, over a period of 5 years, his investment plan would be at an interest of 4.3% compounded yearly over the period of 5 years.

However, the interest rate for 2025 is not yet available. Sam finds the information about the interest rates offered by the insurance company for the last 5 years.



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Determine which plan Sam should choose.

Justify the decision you make and show your calculations clearly.

.....

.....[5]

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中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Chung Cheng High School (Main) Chung Cheng High School (Main) Chung Cheng High School (Main)

PRELIMINARY EXAMINATION 2024

SECONDARY 4

MATHEMATICS **4052/01**

Paper 1 **Friday 23 August 2024**

2 hours 15 minutes

Candidates answer on the Question Paper.

MARKS SCHEME

This document consists of **20** printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

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BP~245

BP~246

Answer **all** the questions.

1 The length of a court is 0.028 km and the width of the court is 16 m. Express the ratio of the length to the width of the court in its simplest form.

28 : 16
7 : 4

Answer ..... : ..... [1]

---

2 Given that $0 < x < 180$, find the possible values of x for $\sin x^\circ = 0.985$.
Give your answers correct to 1 decimal place.

$\sin x^\circ = 0.985$
$x = \sin^{-1} 0.985$ or $180 - \sin^{-1} 0.985$
$= 80.1$ or 99.9

Answer x = ...or..... [2]

---

3 **(a)** Simplify $\dfrac{(4a^4b)^3}{10a^3b^7}$, leaving your answer in positive index form.

$$\frac{(4a^4b)^3}{10a^3b^7} = \frac{64a^{12}b^3}{10a^3b^7}$$
$$= \frac{32a^9}{5b^4}$$

Answer ..... [2]

(b) Given that $2^x = 3$, $2^y = 7$ and $2^z = \dfrac{9}{49}$, use the laws of indices to find the value of $2^{3x-y+\frac{1}{2}z}$.

$$2^{3x-y+\frac{1}{2}z} = (2^x)^3 \div 2^y \times (2^z)^{\frac{1}{2}}$$
$$= 3^3 \div 7 \times \sqrt{\frac{9}{49}}$$
$$= \frac{81}{49}$$

Law of indices:	
1: $(a^m)^n = a^{mn}$	
2: $a^m \times a^n = a^{m+n}$	to award M1 if student
3: $a^m \div a^n = a^{m-n}$	show understanding of
4: $a^{\frac{1}{n}} = \sqrt[n]{a}$	either law of indices

Answer ..... [2]

[Turn over

4 A group of students won \$226 in a competition and shared the amount equally, leaving \$4 leftover. Competing for a second time, the same group of students won \$296. They shared the \$296 along with the \$4 left over from the first competition equally, with no amount remaining.

Assuming that each student received a whole number of dollars in both distributions, find the greatest possible number of students in the group.

	222	300
2	111	150
3	37	50

HCF is 6. There are 6 students.

Answer .....students [2]

---

5 **(a)** On 6 June 2020, Elijah invested some money in a bank which pays a simple interest at a rate of 3.5% per annum. He received a total interest of \$680.40 on 6 June 2023.

Find the amount of money Elijah invested in the bank.

Amount of interest earn per year $= \$680.40 \div 3$
$= \$226.80$

Amount of money invested $= \dfrac{100}{3.5} \times \$226.80$
$= \$6480$

Answer \$..... [2]

(b) Freddy invested \$8000 in another bank that paid compound interest at a rate of 1.75% per annum, compounded quarterly.

Find the total amount Freddy received at the end of 5 years. Give your answer correct to the nearest cent.

Total amount $= 8000\left(1 + \dfrac{1.75 \div 4}{100}\right)^{5\times4}$
$= \$8729.87$ (nearest cent)

Answer \$..... [2]

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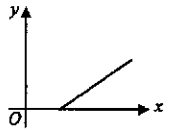


Diagram 1

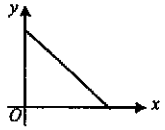


Diagram 2

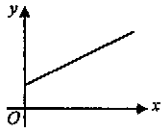


Diagram 3

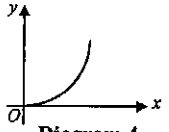


Diagram 4

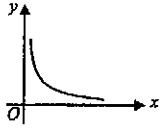


Diagram 5

From the diagrams above, select one of them which best illustrates each of the following statements.

(a) $y = 4\pi x^3$.

(b) The cost of a project \$$y$ is a linear function of x, where x is the number of man-hours required to complete the project. The project has a fixed cost of \$100.

(c) An object is travelling at a constant speed towards a fixed point O. The distance y (in metres) represents how far the object is from point O at time x (in minutes).

Answer **(a)** Diagram ...4..... [1]

(b) Diagram ...3..... [1]

(c) Diagram ...2..... [1]

---

7 Write as a single fraction in its simplest form $\frac{4-x}{x+2} - \frac{2}{3-x}$.

$$\frac{4-x}{x+2} - \frac{2}{3-x} = \frac{(4-x)(3-x)-2(x+2)}{(x+2)(3-x)}$$

$$= \frac{12-7x+x^2-2x-4}{(x+2)(3-x)}$$

$$= \frac{x^2-9x+8}{(x+2)(3-x)}$$

$$= \frac{(x-1)(x-8)}{(x+2)(3-x)}$$

Answer ..... [2]

8 The table shows the distribution of the weights of 40 students.

Weight (W kg)	$40 < W \leq 50$	$50 < W \leq 60$	$60 < W \leq 70$	$70 < W \leq 80$	$80 < W \leq 90$
Frequency	6	8	a	11	3

(a) Find the value of a.

Answer $a =$ ...12..... [1]

(b) Calculate an estimate for

(i) the mean weight of the students,

Answer .....64.25.....kg [1]

(ii) the standard deviation of the weights.

Answer .....11.7.....kg [1]

(c) An error in the weighing machine caused the students' weights to be recorded 2 kg more than their actual values.

Explain how the mean and standard deviation will change after the error is rectified.

Answer

The mean will decrease by 2kg to 62.25kg while the standard deviation remains unchanged.

[1]

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BP~248

9 Andy bought a limited-edition watch from an online website based in Thailand for 10 460 baht. Andy also paid \$15 in Singapore dollars for shipping and GST of 9% on the cost of the watch. The exchange rate between Thai baht (฿) and Singapore dollars (S\$) was ฿100=S\$$P$. Andy spent a total of \$436 in Singapore dollars. Find the value of P, giving your answer correct to 3 significant figures.

Cost of watch in Singapore dollars (without GST) $= \frac{10460}{100} \times P$

$= 104.6P$

Cost of watch in Singapore dollars (with GST) $= 1.09 \times 104.6P$

$= 114.014P$

$114.014P + 15 = 436$

$P = 3.69$(3sf)

Answer $P =$ ..... [3]

---

10 The gradient of the line joining the points $(-3+2a, 7)$ and $(a+1, 2)$ is $-\frac{2}{3}$. Find the value of a.

gradient $= \frac{2-7}{a+1-(-3+2a)}$

$= \frac{-5}{-a+4}$

Given gradient $= -\frac{2}{3}$

$\frac{-5}{-a+4} = -\frac{2}{3}$

$-15 = -2(-a+4)$

$a = -\frac{7}{2}$

Answer $a =$ ..... [2]

11 Given that $a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$, find the value of ab.

$a^2 - \frac{6a}{b} + \frac{9}{b^2} = 0$

$a^2 - 2a\left(\frac{3}{b}\right) + \frac{9}{b^2} = 0$

$\left(a - \frac{3}{b}\right)^2 = 0$

$a - \frac{3}{b} = 0$

$a = \frac{3}{b}$

$ab = 3$

Answer $ab =$ ..... [2]

12 Factorise completely $4mn - 16n^2 - 4m^2n + 64n^3$.

$4mn - 16n^2 - 4m^2n + 64n^3 = 4n(m-4n) - 4n(m^2 - 16n^2)$

$= 4n(m-4n) - 4n(m-4n)(m+4n)$

$= 4n(m-4n)(1-m-4n)$

Answer ..... [3]

---

13 A bag contains 4 black coins, 7 red coins and 11 white coins. Two coins are drawn from the bag at random, one after another without replacement.

(a) Find the probability that a white coin will be chosen on the second draw.

Total number of coins $= 4+7+11$

$= 22$

P(second coin is white) = P(1st coin of any colours, 2nd coin is white)

$= 1 \times \frac{11}{22}$

$= \frac{1}{2}$

Answer ..... [1]

(b) x yellow coins are added to the bag. The probability of picking a black coin in **both** draws is $\frac{1}{50}$. Find the value of x.

Total number of coins $= 22 + x$

P(both coins are black) $= \frac{4}{22+x} \times \frac{3}{21+x}$

$\frac{12}{(22+x)(21+x)} = \frac{1}{50}$

$600 = (22+x)(21+x)$

$600 = 462 + 22x + 21x + x^2$

$x^2 + 43x - 138 = 0$

$(x+46)(x-3) = 0$

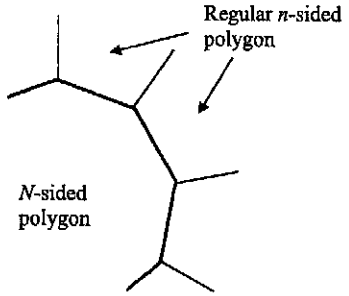
$x = -46$ or $x = 3$

(rejected)

Answer $x =$ ..... [3]

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14 A number of regular n-sided polygons are placed together in a ring to form a regular N-sided polygon as shown in the diagram below.



(a) Show that $N=\dfrac{2n}{n-4}$.

Answer

$$\frac{180(n-2)}{n}\times 2+\frac{180(N-2)}{N}=360$$
$$2N(n-2)+n(N-2)=2nN$$
$$2nN-4N+nN-2n=2nN$$
$$2n=nN-4N$$
$$N(n-4)=2n$$
$$N=\frac{2n}{n-4}$$

[3]

(b) Hence, explain why a regular octagon cannot be formed by placing smaller n-sided regular polygons in a ring.

Answer

$$\frac{2n}{n-4}=8$$
$$2n=8n-32$$
$$6n=32$$
$$n=\frac{16}{3}$$

It is not a octagon as n is not an integer.

[2]

[Turn over

15 **(a)** Express x^2+5x+7 in the form $(x+a)^2+b$.

$$x^2+5x+7=\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2+7$$
$$=\left(x+\frac{5}{2}\right)^2+\frac{3}{4}$$

Answer ..... [2]

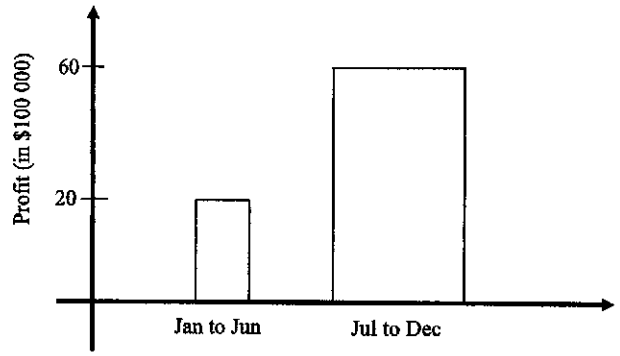
(b) Hence, explain why the expression will never be negative.

Answer

Since $\left(x+\frac{5}{2}\right)^2\geq 0$ for all x, $\left(x+\frac{5}{2}\right)^2+\frac{3}{4}\geq\frac{3}{4}$. Therefore the expression will never be negative.

..... [1]

16 A company presented their 2023 financial report in this graph.



State one aspect of the graph that may be misleading and explain how they may lead to a misinterpretation.

Answer

Each bar in the graph should be of the same width. The different widths of each bar will lead readers to misinterpretation such as interpreting a wider bar to mean a profit of a larger proportion that it actually is.

..... [2]

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BP~249

BP~250

17 $\varepsilon = \{\text{integer } x:\ 0 < x \leq 12\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{numbers that have at least 2 distinct factors}\}$

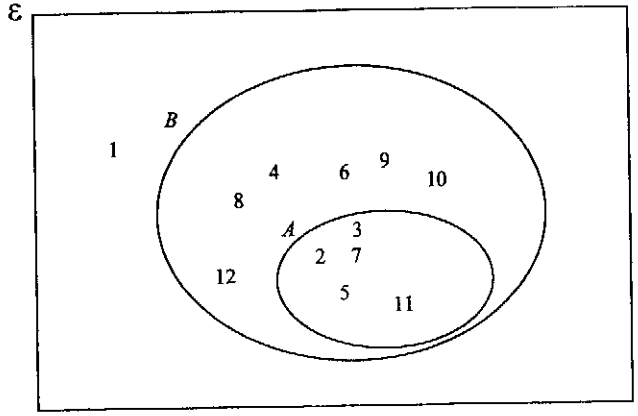
(a) Explain why A is a proper subset of B without listing down the elements.

Answer

Prime numbers have 2 distinct factors, therefore every element of set A must also be an element of set B. However, some elements in B are composites and therefore not in Set A. Thus A is a proper subset of B. [2]

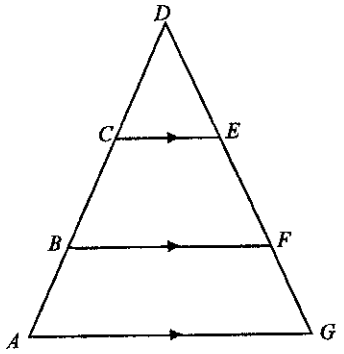
(b) Draw a Venn diagram and list down the elements to illustrate the above information.

Answer



[2]

18 The diagram shows a triangle AGD with the points C and B lying on AD and points E and F lying on GD.



It is given that $CE//BF//AG$, $BF = 2CE$ and $AG = 3CE$.

(a) Show that triangle DCE is similar to triangle DBF.

Answer

$\angle DCE = \angle DBF$ (corresponding angles)

$\angle CDE = \angle BDE$ (common angle)

$\therefore$ Triangle DCE is similar to triangle DBF

[2]

(b) Given that the area of trapezium $BFEC = 15\ \text{cm}^2$, find the area of trapezium $AGFB$.

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DBF} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{\text{Area of } \Delta DCE}{\text{Area of } \Delta DAG} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{\text{area of trapezium } AGFB}{\text{area of trapezium } BFEC} = \frac{9-4}{4-1} = \frac{5}{3}$$

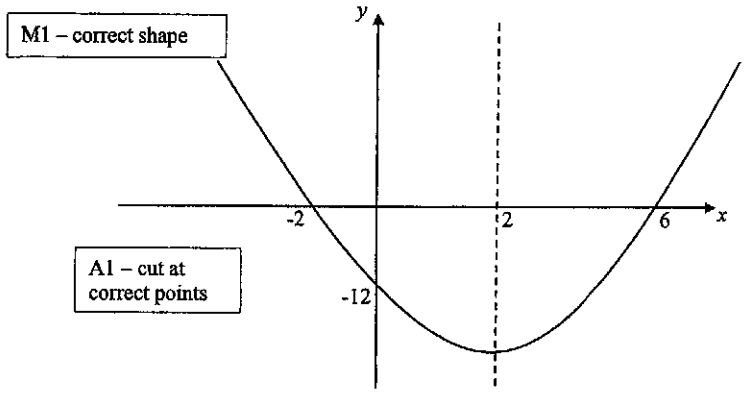
$$\frac{\text{area of trapezium } AGFB}{15} = \frac{5}{3}$$

area of trapezium $AGFB = 25\ \text{cm}^2$

Answer .....cm^2 [3]

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19 Sketch the graph of $y=(2+x)(x-6)$ on the axes below. Indicate clearly the values where the graph crosses the axes and write down the equation of the line of symmetry.



M1 – correct shape

A1 – cut at correct points

Answer line of symmetry: ..... $x=2$

20 It is given that y is inversely proportional to the square root of x.
Find the percentage change in x when the value of y decreases by 50%.

$$y=\frac{k}{\sqrt{x}}$$

$$\sqrt{x}=\frac{k}{y}$$

$$x=\frac{k^2}{y^2}$$

$$\text{new } x=\frac{k^2}{(0.5y^2)}$$

$$=\frac{4k^2}{y^2}$$

$$\text{Percentage change in } x=\frac{\frac{4k^2}{y^2}-\frac{k^2}{y^2}}{\frac{k^2}{y^2}}\times 100\%$$

$$=300\%$$

Answer ..... [2]

21 The table shows the prices of movie tickets categorised by different days of the week and the various age groups.

	Monday to Thursday	Friday	Saturday and Sunday
Child	\$6.00	\$8.50	\$9.00
Adult	\$7.00	\$10.00	\$12.50
Senior Citizen	\$6.50	\$9.00	\$10.00

(a) Write down a 3×3 matrix **P** to represent the above information.

$$\begin{pmatrix}6 & 8.5 & 9\\ 7 & 10 & 12.5\\ 6.5 & 9 & 10\end{pmatrix}$$

Answer **P**=..... [1]

(b) (i) The Ng family comprises a child, two adults and a senior citizen while the Tan family comprises three children and two adults. Represent this information as a 2×3 matrix **Q**.

$$\mathbf{Q}=\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 0\end{pmatrix}$$

Answer **Q** =.....[1]

(ii) Evaluate **QP** and explain what the elements represent.

Answer

$$\mathbf{QP}=\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 0\end{pmatrix}\begin{pmatrix}6 & 8.5 & 9\\ 7 & 10 & 12.5\\ 6.5 & 9 & 10\end{pmatrix}$$

$$=\begin{pmatrix}26.5 & 37.5 & 44\\ 32 & 45.5 & 52\end{pmatrix}$$

The elements in **QP** represent the respective total cost of movie tickets for the Ng family of 1 child, 2 adults and 1 senior citizen and the Tan Family of 3 children and 2 adults on the different days of the week.

[2]

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BP~252

22 Sam's daily morning routine involves getting breakfast before going to work. He travels by car. On a particular day, Sam leaves home at 8:00 am and arrives at the breakfast place at 8:15 am. He leaves the breakfast place at 8:30 am. He arrives at his office at 9:00 am. Sam's average speed for the whole journey is 42 km/h.
His average speed from home to the breakfast place is 10 km/h faster than his average speed from the breakfast place to the office.
Find the distance between the breakfast place and office.

Total time taken = 1 hour
Total distance travelled = 42 km
Let x be the distance between the breakfast place to office.
Distance between home to breakfast place = $42-x$

$$V_1 = \frac{42-x}{\left(\frac{15}{60}\right)} = 4(42-x)$$

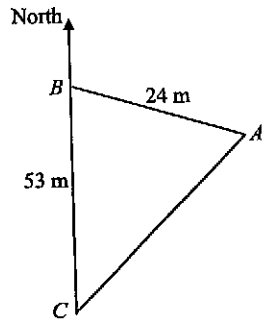
$$V_2 = \frac{x}{\left(\frac{1}{2}\right)} = 2x$$

$$4(42-x)-2x=10$$
$$168-4x-2x=10$$
$$6x=158$$
$$x=26\frac{1}{3}$$

Answer .....km [4]

[Turn over

23 Three points A, B and C lie on a horizontal ground are such that $AB = 24$ m and $BC = 53$ m. Point B is due north of C. The bearing of A from B is 122°.



(a) Find the distance AC.

$$AC = \sqrt{53^2+24^2-2(53)(24)\cos 58}$$
$$= 45.13186...$$
$$= 45.1 \text{ (3sf)}$$

Answer .....m [2]

(b) Find the bearing of C from A.

By sine rule,
$$\frac{\sin \angle BAC}{53} = \frac{\sin \angle ABC}{45.13186...}$$
$$\sin \angle BAC = 53 \times \frac{\sin 58°}{45.13186...}$$
$$\angle BAC = 84.8060...$$
Bearing of C from $A = 360° - 58° - 84.8060...°$
$$= 217.2°$$

Answer .....° [3]

(c) Find the area of the triangular plot ABC.

$$\text{Area of } \Delta ABC = \frac{1}{2}(53)(24)\sin 58°$$
$$= 539.35...$$
$$= 539$$

Answer .....m^2 [2]

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24 The first four terms in a sequence of numbers are

$$3+k, 1+k, -1+k, -3+k, \ldots$$

where k is a constant.

(a) Find an expression in terms of n and k, for the nth term in this sequence.

$T_n = 3+k+(n-1)(-2)$
$= 3+k-2n+2$
$= 5-2n+k$

Answer ..... [1]

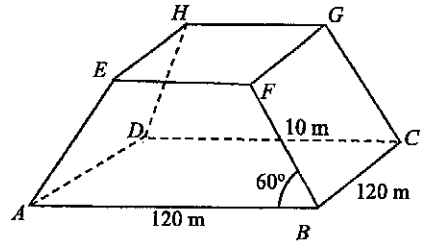
(a) State two conditions on k such that 39 is a term of the sequence.

Answer

$5-2n+k=39$
$k-2n=34$
$k=34+2n$
Since $n \geq 1$, $34+2n \geq 36$, therefore $k \geq 36$ and k must be an even number.

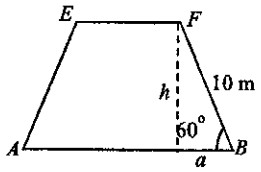
[2]

25 The diagram below shows an indoor adventure park in the shape of a trapezoidal prism with a square base $ABCD$. The indoor adventure park is positioned on horizontal ground and the walls $ADEH$ and $BCFG$ are slanted while the walls $ABFE$ and $DCGH$ are vertically upright.



The top of the prism, $EFGH$, is the ceiling of the adventure park which is also horizontal. $EFGH$ is a square and the centre of $EFGH$ lies vertically above the centre of $ABCD$.
$AB = 120$ m, $AE = BF = CG = DH = 10$ m and $\angle ABF = 60°$.

(a) Find the area of $ABFE$.



Since $AE = BF$, $ABFE$ is an isosceles trapezium.
$\sin 60° = \frac{h}{10}$
$h = 10\sin 60°$
$= 8.660254...$
$\cos 60° = \frac{a}{10}$
$a = 10\cos 60°$
$= 5$
Thus $EF = 12-5-5 = 2$
Area of trapezium $ABFE = \frac{1}{2}(120+110)(8.660254...)$
$= 996$

Answer .....m^2 [4]

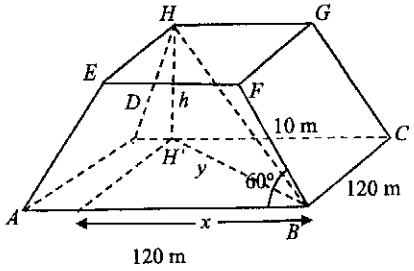
BP~253

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BP~254

(b) The owner of the indoor park wants to build a flying fox feature from point H to point B. The angle of depression from point H to point B must not exceed $5°$, in order to meet the safety requirements. Explain with mathematical workings whether the flying fox feature can be built.

Answer



$x = 120 - 5$
$= 115$

$y = \sqrt{120^2 + 115^2}$
$= \sqrt{27625}$

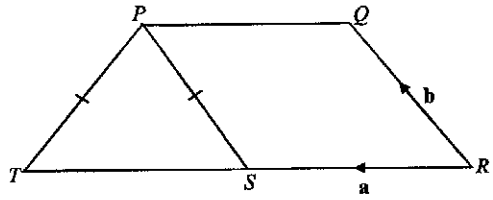
$\tan \angle HBH' = \dfrac{8.660254...}{\sqrt{27625}}$
$\angle HBH' = 2.9826...°$
$= 3.0°$

By alternate angle, the angle of depression is $3.0°$ which is less than $5°$, therefore the flying fox feature can be built.

[3]

[Turn over

26 In the diagram below, $PQRST$ is a trapezium made up of a parallelogram $PQRS$ and an isosceles triangle PST where $PT = PS$. It is given that $\overrightarrow{RS} = \mathbf{a}, \overrightarrow{RQ} = \mathbf{b}$ and $SR = ST$.



(a) Show that $\overrightarrow{QS} = \overrightarrow{PT}$.

Answer

$\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$
$= -\mathbf{b} + \mathbf{a}$
$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$
$= -\mathbf{b} + \mathbf{a}$

Therefore $\overrightarrow{QS} = \overrightarrow{PT}$.

[2]

(b) Hence or otherwise, prove that the trapezium is made up of three isosceles triangles PST.

Answer

Since $PQRS$ is a parallelogram,
$PS // QR, PQ // SR$
$PS = QR$ and $PQ = SR$
From (a), since $\overrightarrow{QS} = \overrightarrow{PT}$, thus $PT = QS$.
$PT = QS = PS = QR$ and $PQ = SR = TS$.
By SSS congruency test, triangle PTS, triangle PQS and triangle QSR are congruent, therefore the trapezium is made up of three isosceles triangles PST.

[3]

End of Paper

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Alternative solutions

9	Total cost excluding shipping in Singapore dollars $= \$436 - \$15$ $= \$421$ Cost of watch excluding GST in Singapore dollars $= \frac{100}{109} \times \$421$ $= \$386.238532$ $\frac{386.2385321}{P} \times 100 = 10460$ $386.2385321 = 104.60P$ $P = 3.69$ (3sf)
11	$a^2b^2 - 6ab + 9 = 0$ $(ab)^2 - 2(ab)(3) + 3^2 = 0$ $(ab-3)^2 = 0$ $ab - 3 = 0$ $ab = 3$
14(a)	Exterior $\angle$ of N-sided polygon $= \frac{360}{N}$ Interior $\angle$ of n-sided polygon $= \frac{180(n-2)}{n}$ $180 + \frac{360}{N} = \frac{2 \times 180(n-2)}{n}$ $1 + \frac{2}{N} = \frac{2(n-2)}{n}$ $\frac{2}{N} = \frac{2n-4}{n} - 1$ $\frac{2}{N} = \frac{2n-4-n}{n}$ $\frac{N}{2} = \frac{n}{n-4}$ $N = \frac{2n}{n-4}$

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Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
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PRELIMINARY EXAMINATION 2024
SECONDARY 4

MATHEMATICS **4052/02**

Paper 2 **Tuesday 20 August 2024**

2 hours 15 minutes

Candidates answer on the Question Paper

MARKS SCHEME

This document consists of 23 printed pages and 1 blank page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radian

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radian

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

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BP~257

BP~258

TURN OVER FOR QUESTION 1

1 **(a)** The table shows the total electricity consumption in Singapore over three years.

Year	2020	2021	2022
Total Electricity Consumption in Gigawatt Hours (GWh)	50 779	53 483	54 884

(i) In 2021, the electricity consumed by households took up 15.5% of the total electricity consumption. Calculate the amount of electricity consumed by households in 2021, correct to two significant figures.

Electricity consumed by household $= 15.5\% \times 53483$
$= 8289.865$
$= 8300$ GWh (2 s.f.)

Answer ..... GWh [1]

(ii) Calculate the percentage increase in the total electricity consumption from 2020 to 2022.

$$\text{Percentage increase} = \frac{54884 - 50779}{50779} \times 100\%$$
$$= 8.08\%$$

Answer .....% [2]

(iii) Express the 2020 electricity consumption in kilowatt hours (kWh), leaving your answer in standard form, correct to two significant figures.

$50779 \times 10^9 \div 10^3 = 5.0779 \times 10^{10}$
$= 5.1 \times 10^{10}$ kWh (2 s.f.)

Answer ..... kWh [1]

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(b) A microprocessor is in the shape of a cube where the sides are 5 mm in length.

(i) Find the maximum number of microprocessors that can be placed into a container with dimensions 10 cm by 2 cm by 8 cm.

$$\frac{10}{0.5}=20$$
$$\frac{2}{0.5}=4$$
$$\frac{8}{0.5}=16$$
$$\therefore 20\times 4\times 16$$
$$=1280$$

Answer ..... [2]

(ii) A model of the microprocessor was made to a scale of 10 : 1. Given that the surface area of microprocessor is 150 mm^2, find the surface area of the model microprocessor in square centimetres.

	Model : Actual
(Linear Scale)	10 : 1
(Area Scale)	100 : 1

The surface area of the model is 100 times of the actual object.

$$\text{Surface Area of Model} = 100\times 150$$
$$=15000\ mm^2$$
$$=\frac{15000}{100}cm^2$$
$$=150\ cm^2$$

Answer ..... cm^2 [2]

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2 (a) Solve $2x-7=3(1-3x)$.

$$2x-7=3(1-3x)$$
$$2x-7=3-9x$$
$$11x=10$$
$$x=\frac{10}{11}$$

Answer $x=$ ..... [2]

(b) Solve the inequalities $7x-1<13$ and $\frac{x+1}{2}\geq -2(x-2)$.

$$7x-1<13 \quad \text{and} \quad \frac{x+1}{2}\geq -2(x-2)$$
$$7x<14 \quad \text{and} \quad (x+1)\geq -4(x-2)$$
$$x<2 \quad \text{and} \quad x+1\geq -4x+8$$
$$5x\geq 7$$
$$x\geq 1.4$$
$$\therefore 1.4\leq x<2$$

Answer ..... [3]

(c) Rearrange the formula $y=\frac{x^2+5}{7x^2}$ to make x the subject.

$$y=\frac{x^2+5}{7x^2}$$
$$7x^2y=x^2+5$$
$$7x^2y-x^2=5$$
$$x^2(7y-1)=5$$
$$x^2=\frac{5}{7y-1}$$
$$x=\pm\sqrt{\frac{5}{7y-1}}$$

Answer $x=$ ..... [3]

BP~259

BP~260

(d) Solve the equation $\frac{2x}{1-x}-3=\frac{1}{2x-3}$.

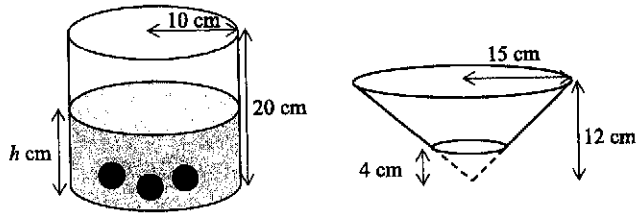
Give your solutions correct to two decimal places.

$$\frac{2x}{1-x}-3=\frac{1}{2x-3}$$
$$2x(2x-3)-3(1-x)(2x-3)=1-x$$
$$4x^2-6x-3(2x-3-2x^2+3x)=1-x$$
$$4x^2-6x-6x+9+6x^2-9x=1-x$$
$$10x^2-20x+8=0$$
$$5x^2-10x+4=0$$
$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(5)(4)}}{2(5)}$$
$x=1.4472...$ or $0.552786...$
$x=1.45$ or 0.55 (2 s.f.)

Answer $x=$ ..... or ..... [5]

3 The diagram shows two open containers, a right cylinder and an inverted frustum. The right cylinder has a base radius of 10 cm and a height of 20 cm. The frustum is formed by cutting a smaller cone off the bottom of a larger cone. The smaller cone that was cut off has a height of 4 cm. The larger cone has a base radius of 15 cm and a height of 12 cm.

Three identical spherical marbles, each of radius 3 cm, are placed into the cylindrical container and water is poured in to a depth of h cm.



(a) Find the volume of each marble, leaving your answer in terms of π.

$$\text{Volume of each marble}=\frac{4}{3}\pi(3)^3$$
$$=36\pi \text{ cm}^3$$

Answer ..... cm^3 [1]

(b) Find the volume of the frustum, leaving your answer in terms of π.

$$\text{Volume of big cone}=\frac{1}{3}\pi(15)^2(12)$$
$$=900\pi \text{ cm}^3$$
$$\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3$$
$$\frac{V_1}{900\pi}=\left(\frac{4}{12}\right)^3$$
$$V_1=\frac{100}{3}\pi \text{ cm}^3$$
$$\text{Volume of frustum}=900\pi-\frac{25}{6}\pi$$
$$=\frac{2600}{3}\pi \text{ cm}^3$$

Answer ..... cm^3 [3]

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(c) All the water from the cylinder, without the marbles, is then poured into the empty frustum, filling it completely without any overflow.

Find the value of h, correct to two decimal places.

$$\text{Volume of water and marbles} = \frac{2600}{3}\pi + 3(36\pi)$$
$$= \frac{2924}{3}\pi \text{ cm}^3$$
$$\pi(10)^2 h = \frac{2924}{3}\pi$$
$$h = \frac{\frac{2924}{3}\pi}{\pi(10)^2}$$
$$h = 9.75$$

Answer $h =$ ..... [3]

(d) The exterior surface area of each container is painted.
Find the total area painted.

Using Pythagoras' Theorem,

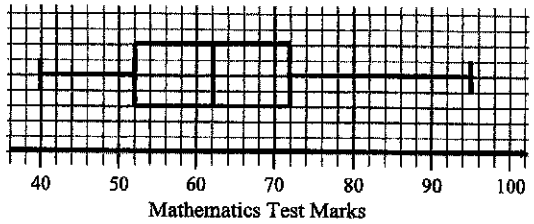
$$l^2 = 12^2 + 15^2$$
$$l = \sqrt{369} \text{ cm}$$
$$\text{Surface area of cone} = \pi(15)\left(\sqrt{369}\right)$$
$$= 905.22036.. \text{ cm}^2$$
$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$
$$\frac{A_1}{\pi(15)\left(\sqrt{369}\right)} = \left(\frac{4}{12}\right)^2$$
$$A_1 = 100.58004.. \text{ cm}^2$$
$$\frac{l_1}{15} = \frac{4}{12}$$
$$l_1 = 5 \text{ cm}$$
$$\text{Surface Area of frustum} = (905.22036.. - 100.58004..) + \pi(5)^2$$
$$= 804.6403.. + 78.53981$$
$$= 883.18013...$$
$$\text{Total Surface area} = 883.18013... + (2 \times 10 \times \pi \times 20) + \pi(10)^2$$
$$= 883.18013... + 1256.63706.. + 314.1592..$$
$$= 2453.9764..$$
$$= 2450 \text{ cm}^2 \text{ (3s.f)}$$

Answer ..... cm^2 [6]

4 **(a)** 12 students from class A took a Mathematics test.
The table below shows the test marks of the students. However, **two** of the students' marks are covered with ink.

40	95	84	47	63
52	63	55	52	61

(i) Given that the box-and-whisker plot below shows the distribution of the results, explain why the information may not be sufficient to find the missing marks of the two students.



Answer

The box-and-whisker plot is not able to provide individual data point. It can only provide the minimum, maximum mark, 25th, 50th and 75th percentile and these data are insufficient to calculate the missing marks.

(ii) Given further that the modal mark is 63, find the two missing marks.

Since mode = 63, one of the missing marks = 63

40 47 52 52 55 61 63 63 63 _ 84 _ 95

Since upper quartile = 72,

$$\frac{63 + x}{2} = 72$$
$$63 + x = 144$$
$$x = 81$$

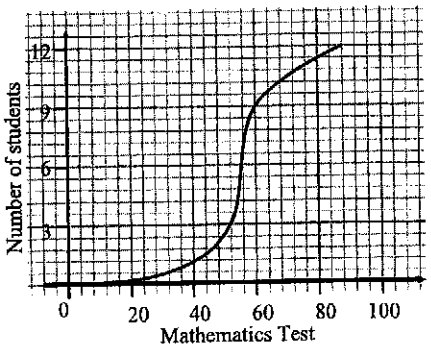
Answer ..... and ..... [2]

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BP~261

BP~262

(iii) 12 students from class B also took the same Mathematics test. The distribution of their test marks is shown on the cumulative frequency graph.



Make a comment comparing the averages and a comment comparing the distribution of the Mathematics Test marks between the two classes.
Use figures to support your answers.

Generally, Class A did better. The median of Class A is 62 marks which is higher than the median of Class B which is 56 marks.
Interquartile range of Class $A = 72 - 52 = 20$
Interquartile range of Class $B = 60 - 52 = 8$
Since the interquartile range of Class $A = 20 >$ the interquartile range of Class $B = 8$, the distribution of Class A's marks is more spread out than Class B's.
[3]

(b) Alice rolled a six-sided die, X, 100 times. Bala rolled another six-sided die, Y, 80 times. The number of times they each obtained a '6' is recorded in the table.

Die	Number of rolls	Number of times '6' is obtained
X	100	16
Y	80	18

(i) Find the probability of rolling a '6' by Alice and Bala respectively.

Answer P(obtaining a '6' by Alice) = $\frac{4}{25}$ or 0.16 [1]

P(obtaining a '6' by Bala) = $\frac{9}{40}$ or 0.225 [1]

(ii) One of the dice is biased. Using your answers in **(b)(i)**, determine which die, X or Y, is likely to be the unbiased 6-sided die. Explain your answer.

Answer

Alice's die is more likely to be an unbiased die as the probability of getting a '6' is closer to that of an unbiased die. [1]

[Turn over

5 (a) Complete the table of values for $y = \frac{2}{x^2} - 2x$.

Values are given to one decimal place where appropriate.

x	−3	−2	−1	−0.5	−0.4	0.5	1	2	3
y	6.2	4.5	4	9	13.3	7	0	−3.5	−5.8

[1]

(b) On the grid opposite, draw the graph of $y = \frac{2}{x^2} - 2x$ for $-3 \le x \le 3$. [3]

(c) (i) On the same grid, draw the graph of $y + x = 6$ for $-3 \le x \le 3$.

x	−3	3
y	9	3

[1]

(ii) Write down the x-coordinates of the points where the line intersects the curve.

Answer $x =$ −0.6 and 0.55 [2]

(iii) These values of x are solutions of the equation $x^3 + Ax^2 + B = 0$.

Find the value of A and of B.

$6 - x = \frac{2}{x^2} - 2x$

Multiply x^2 throughout,

$6x^2 - x^3 = 2 - 2x^3$

$6x^2 - 2 = -x^3$

$x^3 + 6x^2 - 2 = 0$

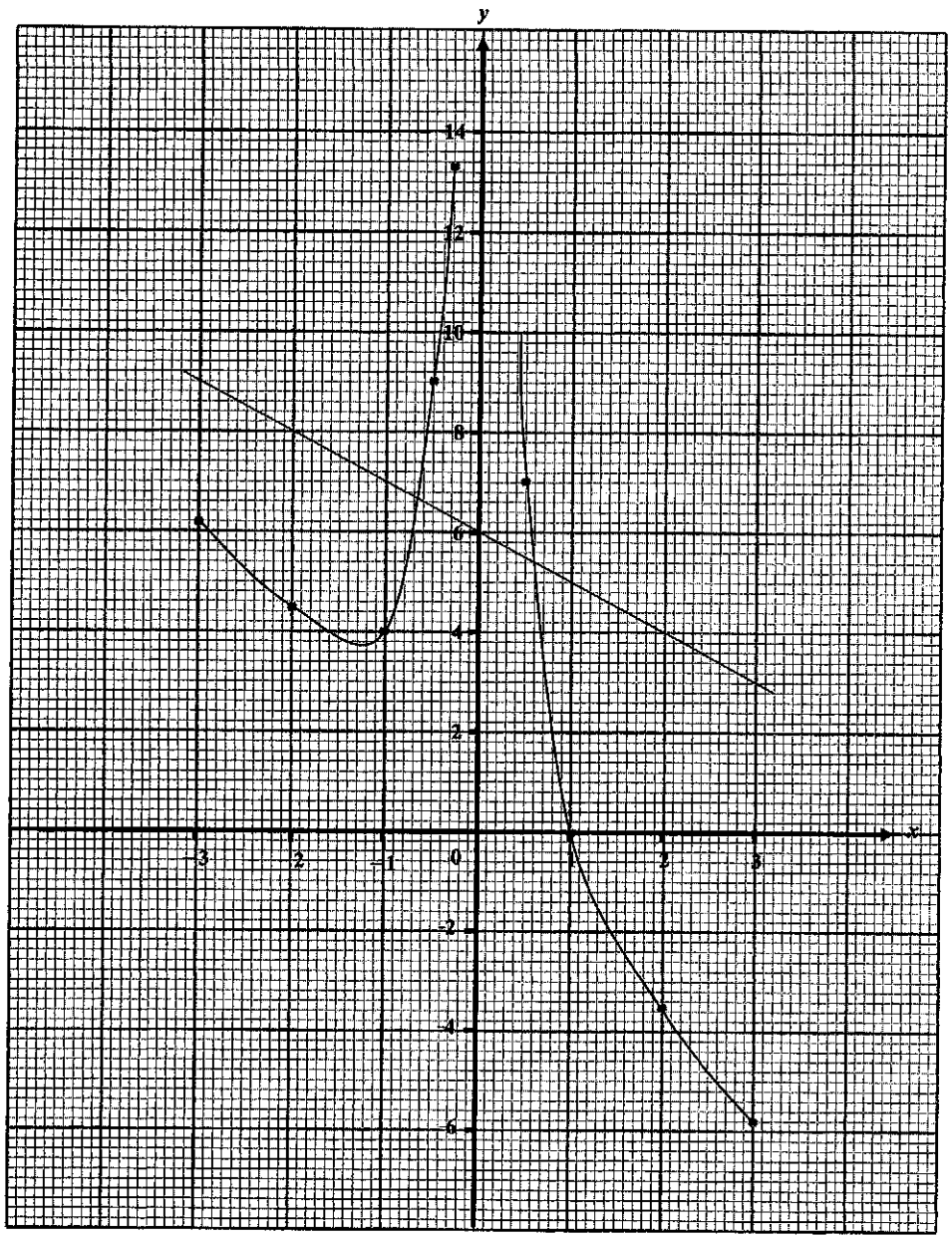
$A = 6, B = -2$

Answer $A =$ .....

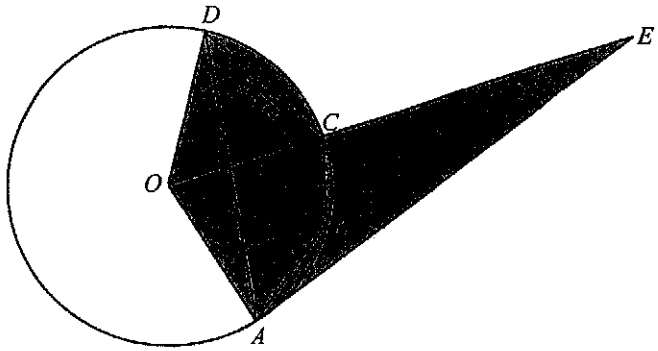
$B =$ .....

[3]

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6



A, B, C and D are points on the circle with centre O and radius 5 cm.
AE is a tangent to the circle at A and OE is a straight line that passes through C.
Angle $OEA = 28°$ and $CD = 5$ cm.

(a) Find, giving a reason for each step of your working,

(i) angle OAD,

$\angle OAE = 90°$ (tangent perpendicular radius)
$\angle COA = 180° - 90° - 28°$ (angle sum of triangle)
$= 62°$
$\angle COD = 60°$ (angle in an equilateral triangle)
$\angle AOD = 60° + 62°$
$= 122°$
$\angle OAD = \frac{180° - 122°}{2}$ (base angles of an isosceles triangle)
$= 29°$

Answer Angle OAD = ... ° [3]

(ii) angle ABC.

reflex $\angle COA = 360° - 62°$ (angles at a point)
$= 298°$
$\angle ABC = \frac{298°}{2}$ (angle at center = twice angle at circumference)
$= 149°$

Answer Angle ABC = ... ° [2]

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BP~263

BP~264

(b) Find the area of the shaded region.

$$\text{Area of sector } OCD = \frac{60°}{360°}(\pi)(5)^2$$
$$= \frac{25}{6}\pi = 13.089969..\text{ cm}^2$$
$$\tan 28° = \frac{5}{AE}$$
$$AE = \frac{5}{\tan 28°}$$
$$\text{Area of triangle} = \frac{1}{2} \times \frac{5}{\tan 28°} \times 5$$
$$= \frac{25}{2\tan 28°}$$
$$= 23.50908..\text{ cm}^2$$
$$\text{Area of shaded region} = 13.089969.. + 23.50908..$$
$$= 36.6\text{ cm}^2 \text{ (3s.f)}$$

Answer ..... cm^2 [4]

[Turn over

7 **(a)** A, B, C and D are four points such that the coordinates of A and C are $(4,2)$ and $(10,-34)$ respectively. $\overrightarrow{AB} = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 24 \\ -12 \end{pmatrix}$.

(i) Find the coordinates of B.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\begin{pmatrix} -9 \\ -12 \end{pmatrix} = \overrightarrow{OB} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$\overrightarrow{OB} = \begin{pmatrix} -5 \\ -10 \end{pmatrix}$$
$$\therefore B(-5,-10)$$

Answer (..... , ...) [1]

(ii) Find $\left|\overrightarrow{AB}\right|$.

$$\left|\overrightarrow{AB}\right| = \sqrt{9^2 + 12^2}$$
$$= 15$$

Answer $\left|\overrightarrow{AB}\right|$ = ... units [1]

It is given that E is a point on BD such that $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BD}$.

(iii) Show that A, C and E are collinear.

Answer

$$\overrightarrow{BE} = \frac{1}{3}\begin{pmatrix} 33 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$$
$$\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$$
$$\begin{pmatrix} 11 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + \overrightarrow{AE}$$
$$\overrightarrow{AE} = \begin{pmatrix} 2 \\ -12 \end{pmatrix}$$
$$\overrightarrow{AC} = \begin{pmatrix} 10 \\ -34 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ -36 \end{pmatrix}$$
$$\therefore \overrightarrow{AC} = 3\begin{pmatrix} 2 \\ -12 \end{pmatrix} = 3\overrightarrow{AE}$$

Since $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AC}$ and with a common point A, therefore A, C and E are collinear.

[4]

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(b) A is the point $(2,6)$, B is the point $(-4,-2)$ and C is the point $(6,-2)$. A line L passes through point C and is parallel to AB.

(i) Find the equation of line L.

$$m_{AB} = \frac{6-(-2)}{2-(-4)}$$
$$= \frac{8}{6} = \frac{4}{3}$$

equation of line L:

$$y-(-2) = \frac{4}{3}(x-6)$$
$$y+2 = \frac{4}{3}x-8$$
$$y = \frac{4}{3}x-10$$

Answer ..... [2]

Point D lies on line L such that $AD // BC$.

(ii) Find the coordinates of point D.

$AD = BC = 10$ units

Since $AB // CD$ and $AD = AC$, $ABCD$ is a parallelogram.

thus $AD // BC$.

$$x_D = 2+10 = 12$$
$$y_D = \frac{4}{3}(12)-10$$
$$= 6$$
$$\therefore D(12,6)$$

Answer D (..... , ...) [2]

(iii) Show that $ABCD$ is a rhombus.

Answer

$$AB = \sqrt{(2-(-4))^2+(6-(-2))^2}$$
$$= \sqrt{100} = 10 \text{ units}$$
$$CD = \sqrt{(6-(12))^2+(-2-6)^2}$$
$$= \sqrt{100} = 10 \text{ units}$$
$$BC = \sqrt{(6-(-4))^2+(-2-(-2))^2}$$
$$= \sqrt{100} = 10 \text{ units}$$

Since $AD = BC = CD = AB = 10$ units, $\therefore ABCD$ is a rhombus.

[2]

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8 **(a)** The figure shows the first three figures of a sequence formed by sticks of the same size.

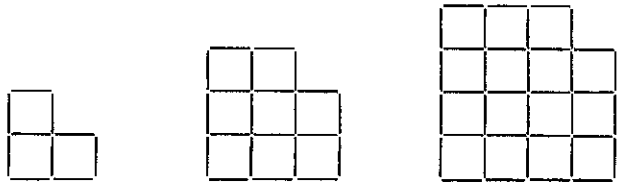


Figure 1 Figure 2 Figure 3

The number of small squares formed in each of the figures is recorded in the table.

Figure Number	Number of Squares
1	$(1+1)^2-1$
2	$(2+1)^2-1$
3	$(3+1)^2-1$

(i) Find the number of small squares formed in Figure 5.

number of small squares formed in Figure 5 = 35

Answer ..... [1]

(ii) Find, in terms of n, an expression for the number of squares in Figure n.

expression for the number of squares = $(n+1)^2-1$

Answer ..... [1]

(iii) Explain why the sum of the number of squares in two consecutive figures is always odd.

Answer

$$\left((n+1)^2-1\right)+\left((n+1+1)^2-1\right)$$
$$= n^2+2n+1-1+n^2+4n+4-1$$
$$= 2n^2+6n+3$$
$$= 2(n^2+3n)+3$$

Since $2(n^2+3n)$ is a multiple of 2 which is an even number, and that 3 is an odd number therefore $2(n^2+3n)+3$ will always be odd.

[2]

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(b) The table below shows the n^{th} terms of 4 sequences.

Sequence number	n^{th} term
1	$4n+3$
2	$7n+1$
3	$14n$
4	$8n-1$

For each sequence, are the numbers in the sequence always multiples of 7, sometimes multiples of 7 or never multiples of 7?

Write down the letter 'A', 'S' or 'N' to represent your answer.

A Always multiples of 7

S Sometimes multiples of 7

N Never multiples of 7

Answer

Sequence 1 ..... S

Sequence 2 ..... N

Sequence 3 ..... A

Sequence 4 ..... S [2]

[Turn over

9 Sam is a 40-year-old man who earns a gross salary of $8000 a month. He is a Singaporean and an employee of a Singapore firm. Sam is required to put a certain percentage of his monthly gross salary into his Central Provident Fund (CPF) account.

CPF is a mandatory (social security) savings scheme funded by contributions from employers and employees. The table below shows the CPF contribution rates by employers and employees.

Employee's age (Years)	**CPF Contribution Rates from 1 January 2024 (Monthly gross salary > $750)**		
	Total (% of gross salary)	**By Employer (% of gross salary)**	**By Employee (% of gross salary)**
55 and below	37	17	20
Above 55 to 60	31	15	16
Above 60 to 65	22	11.5	10.5
Above 65 to 70	16.5	9	7.5
Above 70	12.5	7.5	5

(a) Find the amount of money Sam's employer must contribute to his CPF monthly.

$$\text{amount of money his employer must contribute} = \frac{17}{100} \times \$8000$$
$$= \$1360$$

Answer $..... [1]

Sam wants to plan his monthly savings and has tabulated his monthly expenditure as shown in the table. He hopes to save at least 20% of his salary each month after CPF deductions.

Expenditure	**Amount ($)**
Food and Groceries	820
Transportation	90
Insurance and Healthcare	1000
Phone and Internet Subscriptions	80
Utilities	300
Housing Loan	1000
Leisure and Entertainment (movies, sports, books subscription fees, dining etc)	1580

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(b) Determine whether Sam is able to achieve his saving goals by clearly showing your calculations.

$$\text{amount of money he must contribute to CPF} = \frac{20}{100} \times \$8000$$
$$= \$1600$$

$$\text{total expenditure} = 820+90+1000+80+300+1000+1580$$
$$= \$4870$$

$$\text{Savings} = 8000-1600-4870$$
$$= \$1530$$

$$\text{Savings percentage} = \frac{1530}{8000-1600} \times 100\%$$
$$= 23.90625..\%$$
$$= 23.9\% \text{ (3s.f.)}$$

Since 23.9% > 20%, therefore Sam is able to save the amount he hopes to.

.....

..... [4]

(c) After a year of saving, Sam decides to invest $15 000 of his savings for a period of 5 years. He comes up with 2 investment plans.

Plan A

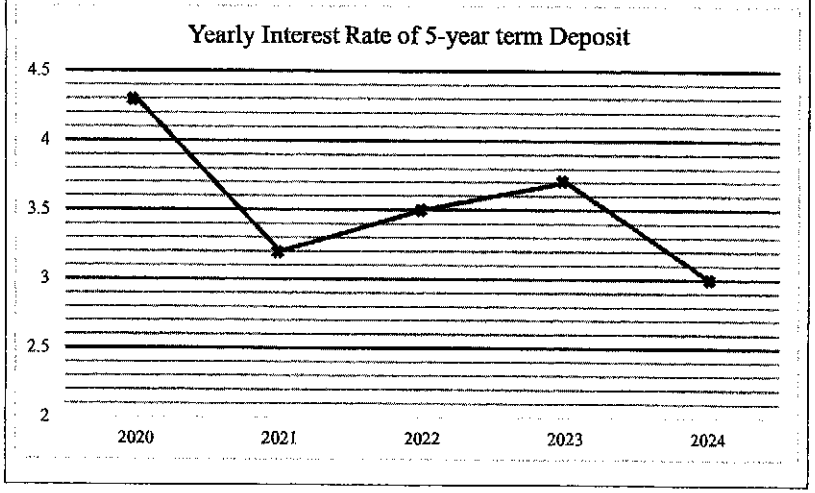
Sam can invest his money in a 5-year savings bond at the beginning of 2025. The interest rate for the 5-year term is shown in the table. The savings bond compounds annually.

Year from issue date	1	2	3	4	5
Interest per year %	3.19	3.19	3.20	3.28	3.31

Plan B

Sam can invest his money at the beginning of 2025 into an insurance company's fixed deposit account for a period of 5 years, compounded yearly. The interest rate is fixed over the duration of the investment and is determined by the year of issuance.
For example: If Sam had invested his money in 2020, over a period of 5 years, his investment plan would be at an interest of 4.3% compounded yearly over the period of 5 years.

However, the interest rate for 2025 is not yet available. Sam finds the information about the interest rates offered by the insurance company for the last 5 years.



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Determine which plan Sam should choose.

Justify the decision you make and show your calculations clearly.

Answer

Plan A

After first 2 years	$A = 15000\left(1+\frac{3.19}{100}\right)^2$ $= \$15972.26415...$
After 3rd year	$A = (15972.26415...)\left(1+\frac{3.2}{100}\right)$ $= \$16483.3766...$
After 4th year	$A = (16483.3766...)\left(1+\frac{3.28}{100}\right)$ $= \$17024.03136...$
After 5th year	$A = (17024.03136...)\left(1+\frac{3.31}{100}\right)$ $= \$17587.52679..$ $= \$17587.53$ (nearest cent)

Plan B

$$\text{Average interest rate} = \frac{4.3+3.2+3.5+3.7+3}{5}$$
$$= 3.54$$

$$\text{Amount after 5 years} = 15000\left(1+\frac{3.54}{100}\right)^5$$
$$= \$17849.75 \text{ (nearest cents)}$$

Since \$17849.75 > \$17587.53, Sam should consider Plan B.

.....

.....[5]

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