S4E/5N EM Prelim 2024 Paper 2 Marking Scheme

Answer all the questions.

1. **(a)** Factorise completely $x^2-4x-xy+4y$.

$x^2-4x-xy+4y$
$= x(x-4)-y(x-4)$ ------ M1 (1st level)
$= (x-y)(x-4)$ ------ A1

Answer …… [2]

(b) Given that $4^{-\frac{1}{2}}=8^{\frac{1}{4}}\div 2^{x+1}$, find the exact value of x.

$4^{-\frac{1}{2}}=8^{\frac{1}{4}}\div 2^{x+1}$
$2^{2(-\frac{1}{2})}=2^{3(\frac{1}{4})}\div 2^{x+1}$ ------ M1 (base 2 for at least one term)
$2^{-1}=2^{\frac{3}{4}-(x+1)}$
$\therefore -1=\frac{3}{4}-x-1$ ------ M1 (compare power, no ECF)
$x=\frac{3}{4}$ ------ A1

Answer x = …… [3]

(c) Given that $k=\frac{2h+1}{3h-1}$, express h in terms of k.

$k=\frac{2h+1}{3h-1}$
$k(3h-1)=2h+1$
$3kh-k=2h+1$ ------ M1 (linear and expand correctly)
$3kh-2h=k+1$
$h(3k-2)=k+1$
$h=\frac{k+1}{3k-2}$ ------ A1 accept $h=\frac{-k-1}{-3k+2}$

Answer h = …… [2]

(d) Solve $\frac{2x}{3}<\frac{2x+1}{2}\le\frac{3x+7}{4}$.

$\frac{2x}{3}<\frac{2x+1}{2}$ and $\frac{2x+1}{2}\le\frac{3x+7}{4}$

$4x<6x+3$ ------- M1 (linear)
$-2x<3$
$x>-\frac{3}{2}$

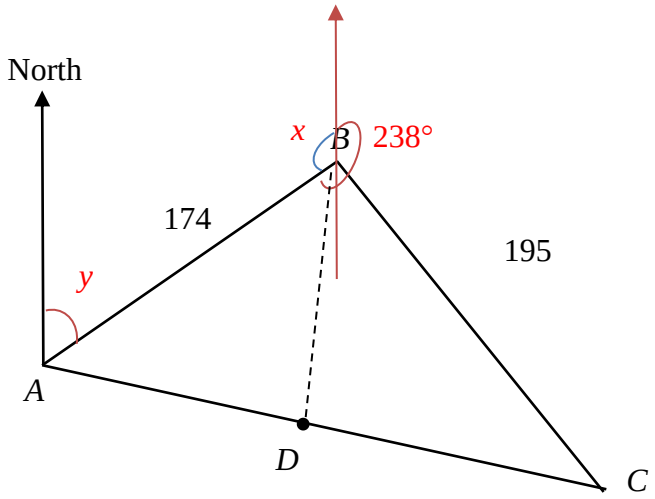
$8x+4\le 6x+14$ ------- M1 (linear)
$2x\le 10$
$x\le 5$

$\therefore\ -\frac{3}{2}<x\le 5$ ------- A1

Answer …… [3]

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2. The diagram shows three points, *A*, *B* and *C* on the ground.
AB = 174 m and *BC* = 195 m.
The bearing of *A* from *B* is 238°.The bearing of *C* from *A* is 108°.
A point *D* lies on the path *AC* such that it is equidistant to *A* and to *B*



(a) Show that angle *BAC* = 50°.

$\angle x = 360 - 238$ (angles at a point) ---------- M1 (with reason)
= 122°

$\angle y$ = 180 – 122 (int angles) ---------- M1 (with reason)
= 58°

$\angle BAC = 108 - 58$ ---------- A1 (shown)
= 50°

(MUST **label** angles clearly to be awarded full marks)

[3]

(b) Find the angle *BCA*.

$\frac{\sin BCA}{174} = \frac{\sin 50}{195}$ ---------- M1

$\angle BCA = \sin^{-1}(0.683547)$
= 43.12147
= **43.1°** ---------- A1

Answer ……..° [2]

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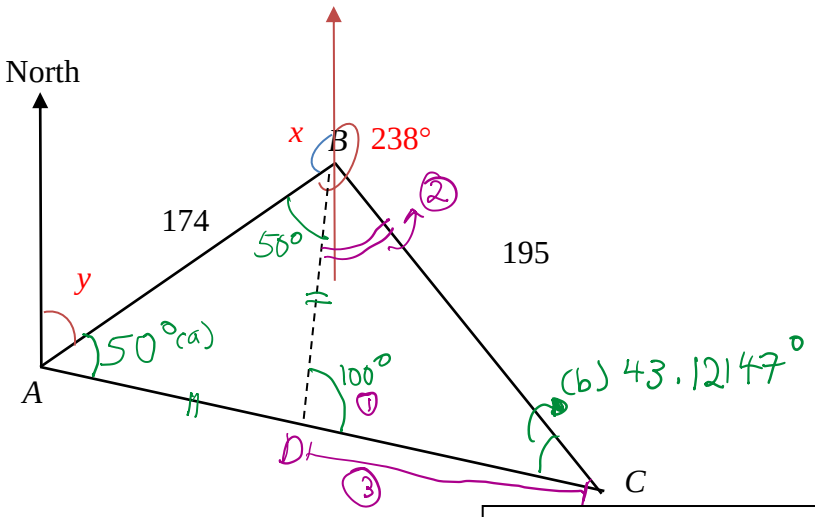
(c) Find the area of triangle *ABC*.

$\angle ABC = 180 - 43.12147 - 50$ ---------- M1
$= 86.87853°$

Area of triangle $= \frac{1}{2} \times 174 \times 195 \times \sin 86.87853$ ---------- M1 (using 'their' $\angle ABC$)
$= 16939.82$
$=$ **16900 m²** ---------- A1 (or any accurate higher approximation)

Answer …… m² [3]

(d) Find distance from *C* to *D*.



Since *AD* = *BD*, *ABD* is isosceles triangle.

$\angle BDC$ = 50 + 50 (ext ∠ of Δ) ---------- M1
$= 100°$

$\angle DBC = 180 - 100 - 43.12147$ ---------- M1
$=$ **36.87853°**

Using sine rule
$\frac{CD}{\sin 36.87853} = \frac{195}{\sin 100°}$ ---------- M1 (using their ∠D
$CD = 118.8287782$
$=$ **119 m** (3 sf) ---------- A1 (accept more accurate approximations)

or 180 – 50 – 50 – 43.12147 = 36.87853
gets 2 marks

Alternative

Using cosine rule AC= -- M2

AD = M1

CD = AC – AD ---- A1

Answer …… m [4]

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3. The variables x and y are connected by the equation $y = \frac{x^3}{2} - 4x + 3$.

Some corresponding values of x and y are given in the table below

x	–3.5	–3	–2	–1	0	1	2	2.5	3
y	–4.4	1.5	p	6.5	3	–0.5	–1	0.8	4.5

(a) Find the value of p.

$p = 7$ ------- B1

Answer p = ... [1]

(b) On the graph paper found in the next page, draw the graph of $y = \frac{x^3}{2} - 4x + 3$ for $-3.5 \le x \le 3$. [3]

Points – 2 M smooth curve 1 M

(c) By drawing a tangent, find the gradient of the curve at x = 2.5.

Tangent line --- M1

Gradient = 5.4 ------- A1 (accept 4.9 to 5.8)

Answer ..... [2]

(d) (i) On the same axes, draw the line $y = -x - 1$ for $-3.5 \le x \le 3$.

Answer [on graph] [1]

(ii) Write down the x-coordinate of the point where this line intersects the curve.

–2.9 ------- B1 (accept –2.95 to –2.8)

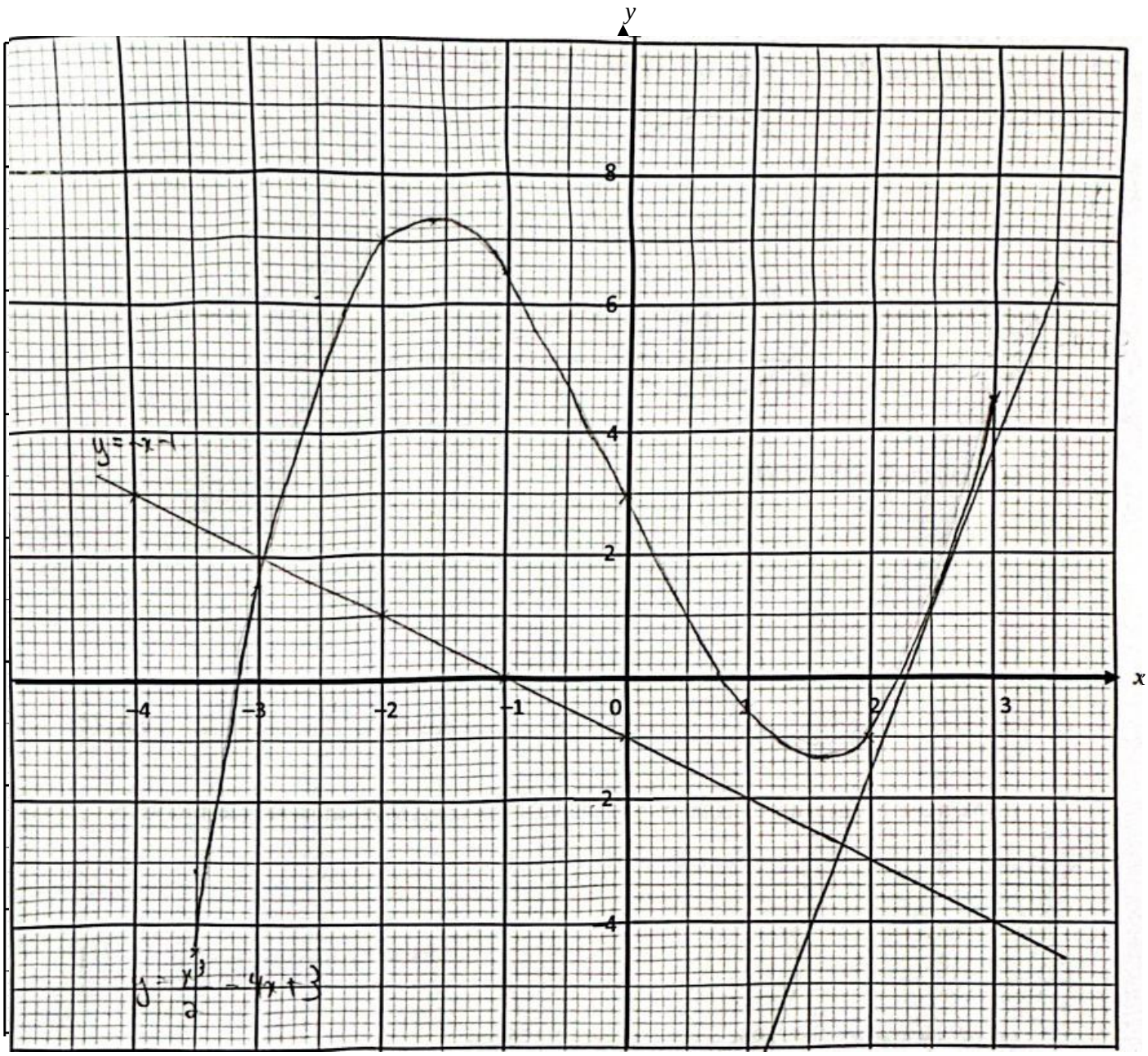
Answer x = ... [1]

(e) State the minimum value of $y = \frac{x^3}{2} - 4x + 3$ for $0 \le x \le 3$.

–1.35 ------- B1 (accept –1.5 to –1.2)

Answer ..... [1]

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(f) The equation $\frac{x^3}{2} - 4x + 5 = 0$ has only one solution.

Explain how this can be seen from your graph.

Answer :

$\frac{x^3}{2} - 4x + 5 = 0$

$\frac{x^3}{2} - 4x + 5 - 2 = 0 - 2$

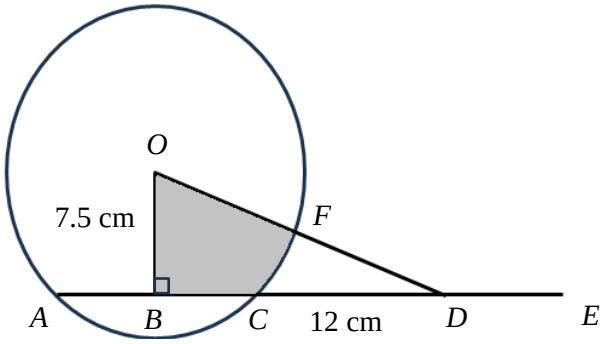
$\frac{x^3}{2} - 4x + 3 = -2$ ------ M1

By **drawing the line $y = -2$, there is one point of intersection**. ------ A1 (no need to draw, but must mention y = -2)

[2]

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4. The diagram shows a circle with centre O.
$ABCDE$ is a straight line. $AC = CD$ and line OD meets the circle at F.
It is given that $OB = 7.5$ cm, $CD = 12$ cm and $\angle OBD = 90°$.



(a) Find the length of OD.

$OD^2 = 7.5^2 + 18^2$
OD = 19.5 cm ---------- B1

Answer …… cm [1]

(b) Without the use of a calculator, find the value of $\cos \angle ODE$ in its simplest form.

$\cos \angle ODE = -\frac{18}{19.5} = -\frac{12}{13}$ ---------- B1

Answer …… [1]

(c) Show that angle COD is approximately 0.501 rad.

Answer :

$\tan \angle BOD = \frac{18}{7.5}$ ---------- M1
= 1.176005
= 1.176 rad
$\tan \angle BOC = \frac{6}{7.5}$ ---------- M1
= 0.67474 rad

$\angle COD = 1.176005 - 0.67474$
= 0.501264
= 0.501 rad ---------- A1

[3]

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(d) Find the perimeter of shaded region.

Radius OC = $\sqrt{7.5^2 + 6^2}$ = 9.60468 cm ---------- M1

Length arc *CE* = 9.60468 × 0.501264 ---------- M1 (using '*their*' radius)
= 4.81448 cm

Perimeter = 7.5 + 6 + 9.60458 + 4.81448
= 27.919
= 27.9 cm ---------- A1

Answer ..... cm [3]

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5. A lead technician working with his trainee, can repair a machine together.
If each of them worked alone, the lead technician would take x hours, while the trainee will need 3.5 hours more.

(a) Find in terms of x,

(i) the fraction of work done by the lead technician in one hour,

$\frac{1}{x}$ ------- B1

Answer ..... [1]

(ii) the fraction of work done by the trainee in one hour.

$\frac{1}{x+3.5}$ ------- B1

Answer ..... [1]

(b) In one hour, both the lead technician and his trainee will complete $\frac{5}{21}$ of the repairs for the machine.
Form an equation and show that it reduces to $10x^2 - 49x - 147 = 0$.

$\frac{1}{x} + \frac{1}{x+3.5} = \frac{5}{21}$ ------- M1 accept $\frac{1}{x} + \frac{1}{x+3.5} = \frac{1}{4.2}$

Or $\frac{42}{x} + \frac{42}{x+3.5} = 1$

$\frac{x+3.5+x}{x^2+3.5x} = \frac{5}{21}$ ------- M1 (single fraction from their first equ of same difficulty level)

$42x + 73.5 = 5x^2 + 17.5x$
$0 = 5x^2 - 24.5x - 73.5$ ------- A1
$10x^2 - 49x - 147 = 0$

[3]

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(c) Solve $10x^2 - 49x - 147 = 0$.

$(10x + 21)(x - 7) = 0$ ------- M1 (must write '=0')

$x = -2.1$ or $x = 7$ ---------- A2

or

$x = \dfrac{-(-49) \pm \sqrt{(-49)^2 - 4(10)(-147)}}{2(10)}$ ---- M1 (must write 'x=')

$x = \dfrac{49 \pm \sqrt{8281}}{20}$

$x = -2.1$ or $x = 7$ ---------- A2

Answer x = ... or x = ... [3]

(d) Hence, find the number of hours needed to repair the machine by **two trainees**.

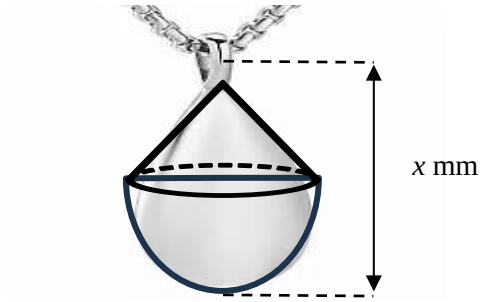
One trainee need $7 + 3.5$ = 10.5 hours

Two trainees will need **5.25** hours ------- B1

Answer ..... h [1]

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6. A gold pendant is made by joining a hemisphere and a cone.



(a) The radius of the hemisphere is 6 mm.
The volume of the pendant is 0.2π cm^3.

Find x, the height of the pendant in mm.

$\frac{2}{3}\pi(0.6^3)+\frac{1}{3}\pi(0.6^2)\left(\frac{x}{10}-0.6\right)=0.2\pi$ ------ M1 conversion to '0.6 cm'

------ M1 (cone application **and/or** hemisphere)

$0.144\pi+0.12\pi\left(\frac{x}{10}-0.6\right)=0.2\pi$

$0.12\pi\left(\frac{x}{10}-0.6\right)=0.056\pi$ -------- M1 (accept $0.37699\left(\frac{x}{10}-0.6\right)=0.175929$) or better

$\frac{x}{10}-0.6=\frac{7}{15}$ or M1 $h=\frac{7}{15}$

$x=10\frac{2}{3}$ mm ------ A1 (accept 10.7/ 10.67)

Answer x =……..…. [4]

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(b) The mass of the pendant is 1.8π grams.

The gold pendant is priced at $101.60 per gram.

(i) Mrs Tan bought is at a discount and paid $528.60.

Calculate the percentage discount given, correct to the nearest whole number.

Price of gold pendant = $101.60 × 1.8π
= $574.53 ------ M1

$$\% \text{ discount} = \frac{574.53 - 528.60}{574.53} \times 100$$ ------ M1

= 7.995
= **8%** (nearest whole number) ------ A1

Answer …… % [3]

(ii) A week later, the shop removed the discount offer.
Mr Pang bought the pendant along with the gold chain priced at $810.
He paid a downpayment of $200 and paid the balance with a fixed interest rate of 5.3% per annum for a period of 4 months.

Calculate the amount of each monthly instalment, correct to the nearest 10 cents.

Total = $574.53 + 810 = $1384.5344

Balance = $1384.5344 – 200
= $1184.5344 ------ M1 (no ecf)

$$\text{Interest} = \frac{1184.5344 \times 5.3 \times \frac{4}{12}}{100}$$ ------ M1 (SI application, no ecf)

= $20.92677

$$\text{Monthly installment} = \frac{1184.5344 + 20.92677}{4}$$ ------ M1 ('their' balance + interest divide by 4)

= $301.365
= **$301.40** (nearest 10 cent) ------ A1

Answer $ …… [4]

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7. A class of 40 Secondary Four students were asked how much time they have spent on social media in the last week.

The results are shown in the table.

Time spent (x hours)	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$
Frequency	p	6	12	q	3

The lower quartile of the time spent on social media was 20 hours.

(i) Show that $p = 4$.

Since **lower quartile** = 20 hours,

¼ of 40 = 10 students spent 20 hours of less ------ M1 (must show in some form for full 2 marks)

P = 10 – 6 ------ A1/B1 only if 10 – 6 is seen
= 4

[2]

(ii) Hence state the value of q.

q = 40 – 4 – 6 – 12 – 3 = **15** ------- B1

Answer q = …… [1]

(iii) Estimate the mean number of hours spent by the students.

$$\text{Mean} = \frac{(4\times5)+(6\times15)+(12\times25)+(15\times35)+(3\times45)}{40}$$ ------- M1 (or sum below)

$$= \frac{1070}{40}$$

= **26.75** ---- A1/B1

Answer …… [2]

(iv) Estimate the standard deviation.

10.9287
=10.9 ------- B1

Answer …… [1]

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A class of 40 Secondary Three students were asked how much time (in hours) they have spent on social media in the last week.

The results are summarised in the table.

Mean	15.8
Standard Deviation	10.9

(i) Make two comparisons between the number of hours spent on social media by the Secondary Three students and by the Secondary Four students. Use the data to support your answers.

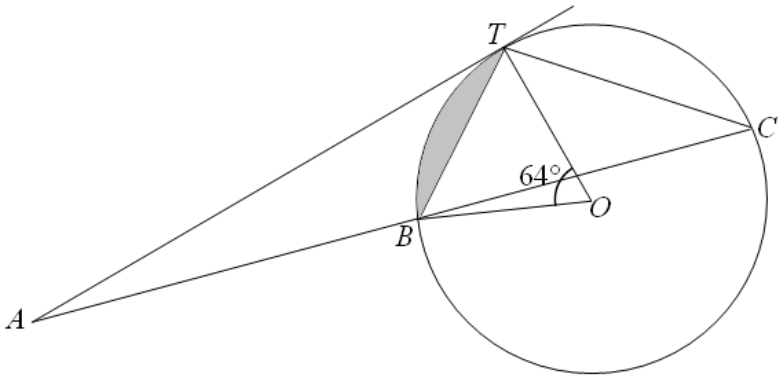
Answer

The Sec 4 students have a **higher mean time** spent on social media, this indicates that the **Sec 4 students generally spend more time of social media** compared to Sec 3 students. ------- B1 (must use mean for comparison)

Both cohorts have the **same standard deviation**, this means their time spent is **equally spread out/equal(same) consistency**. ------- B1 [2]

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8.



In the diagram, AT is a tangent to the circle, centre O.

A line through A meets the circle at B and C.

It is given that angle $TOB = 64°$.

(a) Find each of the following angles, stating your reason(s) clearly.

(i) $\angle BCT =$ 32° …….. °.

Reason : Angle at centre is twice angle at circumference ----- B1 (accept abbreviations) …….. [1]

(ii) $\angle BTO =$ 58° …….. °.

Reason : Base angle <u>isosceles triangle</u> ----- B1 (accept abbreviations) …….. [1]

(b) Hence or otherwise, show that triangle ATB and triangle ACT are similar.

Answer :

$\angle ATB = 90 - 58$ (tan ⊥ rad) ----- M1 (accept only with reason)
$= 32°$
$= \angle TCB$

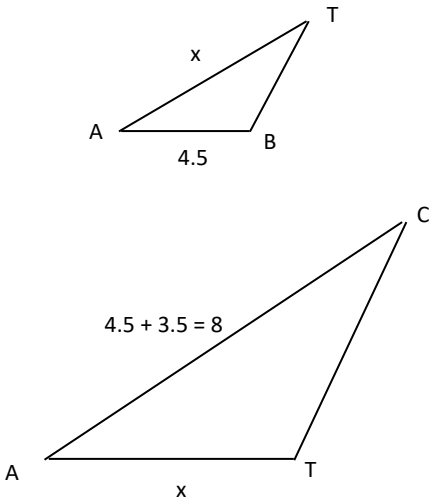
$\angle TAC$ is common/shared angle. ----- M1

$\therefore$ triangle ATB and triangle ACT are similar (AA similarity test) ----- A1
(give A1 if 1st M1 had no reason)

[3]

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(c) Given that $AB = 4.5$ cm and $BC = 3.5$ cm, find the length of AT.



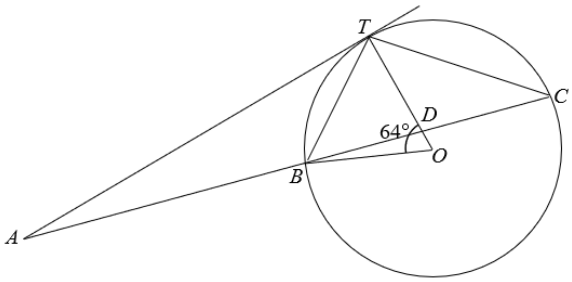
$\frac{x}{8} = \frac{4.5}{x}$ ------ M1

$x^2 = 36$

$x = 6$ cm ------ A1

Answer …… cm [2]

(d) Find the value of $\frac{\text{Area } \Delta BCT}{\text{Area } \Delta ABT}$.



$\frac{\text{Area } \Delta BCT}{\text{Area } \Delta ABT} = \frac{9}{7}$ ------ B1

Answer …… [1]

(e) Given that the radius of the circle is 1.8 cm, find the area of the shaded segment

$\text{Area} = \left(\frac{64}{360} \times \pi \times 1.8^2\right) - \left(\frac{1}{2} \times 1.8 \times 1.8 \times \sin 64\right)$ ------- M1, M1

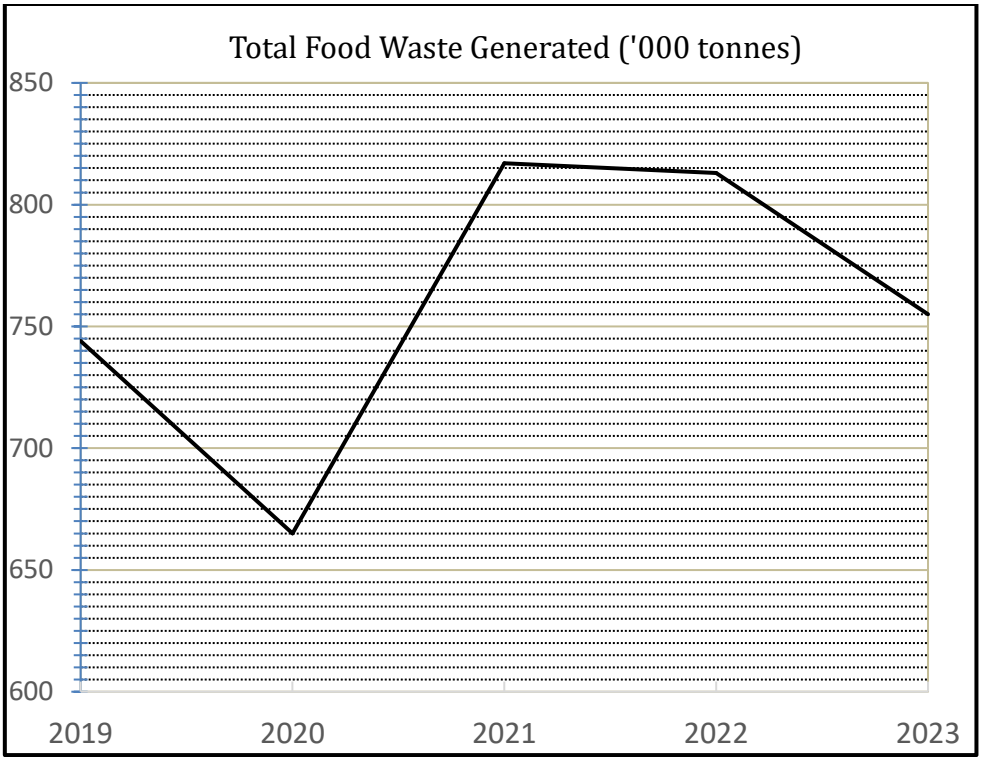
= 0.354 cm² ------- A1

Answer …… cm² [3]

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9. To achieve **Singapore's Zero Waste Masterplan goal**, the government aims to reduce food waste by 30% by 2030.

The annual waste and recycling statistics from the National Environment Agency shows the amount of food waste generated (in 1000 tonnes) in Singapore from 2019 to 2023.



[Source: NEA Website on Waste Statistics]

Year	Estimated Singapore Population
2022	5.64 million
2023	5.9 million

The National Environment Agency (NEA) reported that in **2022**, Singapore generated 813 000 tonnes of food waste annually.

[1 Tonne = 1000 kg]

(a) Express 813,000 tonnes in kg, in standard form.

8.13×10^{8} ------- B1

Answer : …… kg [1]

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(b) Calculate the average mass, in kg, of food waste per person, per day, in **2022**.

$\frac{8.13\times10^{8}}{5.64\times10^{6}} \div 365$ ------- M1 (using 'their' (a))

= **0.395 kg** (3 sf) ------- A1

Answer : …… kg [2]

Mrs Kim's household is made up of **5 persons** altogether.

The table below shows the estimated average mass of food wastage (in kg) for the three meals in her household in 2022.

Breakfast	Lunch	Dinner
0.25	0.9	1.25

(c) Calculate the average mass, in kg, of food waste per person in Mrs Kim's household, per day, in 2022.

(0.25+0.9+1.25) ÷5 = **0.48 kg** ------- B1

Answer : …… kg [1]

According to NEA, a household of 5 persons can reduce food waste in these 4 ways.

Ways to reduce food waste	**Estimated food waste reduction**
Composting	4.9 kg per week
Utilising leftovers	175 g per day
Keeping track of expiry dates	0.8 kg per week
Proper storage	2.4 kg per month

In 2023, Mrs. Kim encouraged her family to reduce their food waste by implementing the methods listed above. However, Mrs. Kim's household does not have a food composting

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(d) Determine if Mrs. Kim's household managed to reduce their food waste more effectively than the national average mass. Provide your reasoning and calculations.

Compare Yearly

National food waste per person per **year** in 2023

$= \frac{755000000}{5900000}$ ------ M1

= 127.9661 kg

National food waste reduction per person from 2022 to 2023

$= \left(\frac{8.13\times10^{8}}{5.64\times10^{6}}\right) - 126.9661$ ------ M1

$= 144.1489 - 126.9661$

= **16.1828 kg**

Per day calculations (in kg) --- M1 (at least **2** calculations)

Leftovers = 0.175 × 365 = 63.875 kg
Expiry = 0.8 × 52 = 41.6 kg (0.8×4×12=7.86kg)
Storage = 2.4 × 12 = 28.8 kg

Each person waste (household has 5 person)

$= \frac{63.875 + 41.6 + 28.8}{5}$ ---- M1 (add all 3 parts,÷ 5 people)

= **26.855 kg / 26.215 kg**

> **16.1828 kg** ------ M1 (comparison*)

Mrs Kim managed to reduce food waste <u>more effectively</u> than the national average mass.
--- A1 (follow through from their comparisions)

*Percentage comparison will be
15.625% > 11.24% ------ M1 (comparison)

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Compare % reduction

Food waste per person per day in 2023

$= \frac{755000000}{5900000} \div 365$ ------ M1

= 0.350592 kg

Percentage of food reduction from 2022 to 2023

$= \frac{0.395 - 0.350592}{0.395} \times 100$ ------ M1

$= \frac{0.044408}{0.395} \times 100$

= **11.24 %**

Per day calculations (in kg) --- M1 (at least **2** calculations)

Leftovers = 175 g = 0.175 kg

Expiry $= \frac{0.8}{7}$ = 0.114285 kg

Storage $= \frac{2.4}{4} \div 7 = \frac{3}{35}$ kg

Each person waste reduced (household has 5 person)

$= \frac{0.175 + 0.114285 + \frac{3}{35}}{5}$ ---- M1 (add all 3 parts,÷ 5 people)

= 0.075 kg

% savings $= \frac{0.075}{0.48} \times 100$

= **15.625%** ------ M1

> 11.24%

Mrs Kim managed to reduce food waste <u>more effectively</u> than the national average mass.

--- A1 (follow through from their comparisions)

Compare absolute value

Food waste per person per day in 2023

$= \frac{755000000}{5900000} \div 365$ ------ M1

= 0.350592 kg

Percentage of food reduction from 2022 to 2023

= 0.395 – 0.350592 x 5 ------ M1

= **0.044408 kg**

← same M1

Each person waste reduced (household has 5 person)

$= \frac{0.175 + 0.114285 + \frac{3}{35}}{5}$ ------- M1

= 0.075 kg ------- M1

> 0.044408 kg

Mrs Kim managed to reduce food waste <u>more effectively</u> than the national average mass.

--- A1 (follow through from their comparisions)

[6]

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Per person

	Per day	Per Week	Per Month	Per Year
Kim	0.075	0.51644	2.2379	26.855
National	0.055	0.3112	1.348	16.1828

Per 5 persons

	Per day	Per Week	Per Month	Per Year
Kim	0.375	2.582	11.19	134.275
National	0.22	1.561	6.742	80.914

End of Paper

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