ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS /
5 NORMAL ACADEMIC

Candidate Name	Class	Register Number

ADDITIONAL MATHEMATICS 4049/01

Paper 1

28 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, or correction fluid.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 9**0**.

90

Setter: Ms Lee Sock Kee
Vetter: Mr Francis Tan and Mr Lionel Ang

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

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1. **(a)** Solve the simultaneous equations

$$x - 2y + 4 = 0$$
$$x^2 + y^2 = 2x + 4$$

[3]

(b) Explain the geometrical meaning of your answer in **(a)**. [1]

[Turn over

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2. **(a)** Express $y = 3 - 8x - 2x^2$ in the form $y = a(x+b)^2 + c$ and hence state the maximum value of y. [3]

(b) Show that there are no values of p for which the curve $y = (p-3)x^2 + 2px + (p+1)$ is always positive. [3]

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(c) A quadratic equation is given by $hx^2 - 2kx + 6k - 9h = 0$, where h and k are constants and $h \neq 0$.

(i) Show that the equation has real roots for all values of h and k. [3]

(ii) In the case where the equation has two real and equal roots, express h in terms of k. [2]

[Turn over

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3. Given that $\cos A = \sqrt{\frac{2}{11}}$ where $180^\circ < A < 360^\circ$, find, without the use of a calculator, the value of

(a) $\tan A$, [2]

(b) $\sin(A - 90°)$, [2]

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(c) $\dfrac{1}{\sec 2A}$. [2]

[Turn over

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4. **(a)** Factorise $27x^3 - \frac{y^3}{8}$ completely. [2]

(b) Express $\frac{8x^3 - 7x^2 + 4x - 3}{(2x^2 - x)(2x - 1)}$ in partial fractions. [6]

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5. A curve is such that $\frac{dy}{dx} = \frac{2x - ax^2}{3}$, where a is a constant.

(a) Given that the curve has a turning point at $(3, 7)$, show that the value of a is $\frac{2}{3}$. [1]

(b) Find the range of values of x for which y decreases as x increases. [3]

[Turn over

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(c) Find the equation of the curve. [4]

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6. **(a)** Prove the identity $(\cot x - \operatorname{cosec} x)^2 = \dfrac{1-\cos x}{1+\cos x}$. [4]

(b) Hence, solve the equation $2(\cot x - \operatorname{cosec} x)^2 = 3\cos x$ for $0 \le x \le 2\pi$. [3]

(c) State the number of solutions of the equation $2(\cot 2x - \operatorname{cosec} 2x)^2 = 3\cos 2x$ in the range $-2\pi \le 2x \le 2\pi$. [1]

[Turn over

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7 **(a)** The graph of $y = \log_a x$ passes through the points with coordinates (125, 3) and (1, b).

(i) Determine the values of a and b. [2]

(ii) Sketch the graph of $y = \log_{a^{-1}} x$ indicating clearly any intercept on the axes. [2]

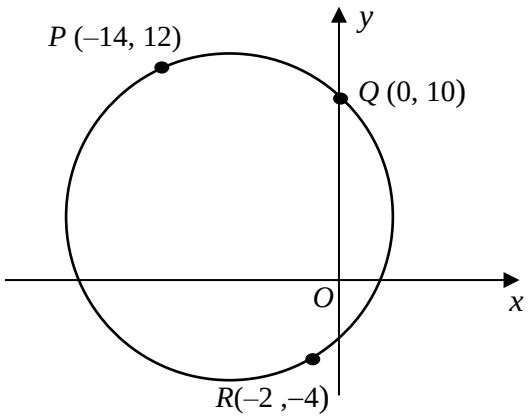
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(b) Find the values of a and b such that $\lg\left(\frac{8}{y}\right)+4\lg y = a\lg(by)$. [4]

[Turn over

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8 Solutions to this question by accurate drawing will not be accepted.



In the diagram which is not drawn to scale, P, Q and R are points on the circle.

(a) Show that PR is the diameter of the circle and hence find the centre of the circle. [5]

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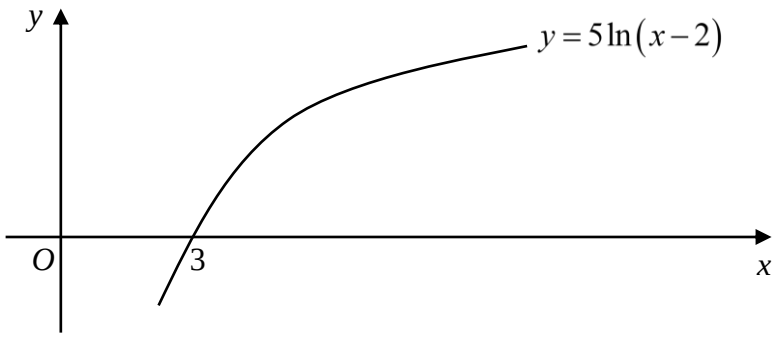
(b) Find the equation of the circle that passes through the points P, Q and R. [2]

(c) Determine whether the point $S(-14, -2)$ lies inside or outside the circle. [2]

[Turn over

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9 The diagram shows part of the curve $y = 5\ln(x-2)$.



(a) Find the exact value of $\int_0^5 x \, dy$. [3]

(b) On the diagram above, shade the region whose area is $\int_0^5 x \, dy$, showing your upper limit clearly. [1]

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(c) Hence find $\int_3^{e+2} 5\ln(x-2)\ \mathrm{d}x$. [3]

[Turn over

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10 Water is being added at a constant rate of 4 cm^3/s to an inverted right cone. The height of the cone is twice the radius of the cone.

[The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

(a) Show that the height of the water level in the cone is 6 cm when the volume of water in the cone is $18\pi\ cm^3$. [2]

(b) Calculate the rate of change of height of the water level when the volume of water is $18\pi\ cm^3$. Leave your answer in its exact form. [3]

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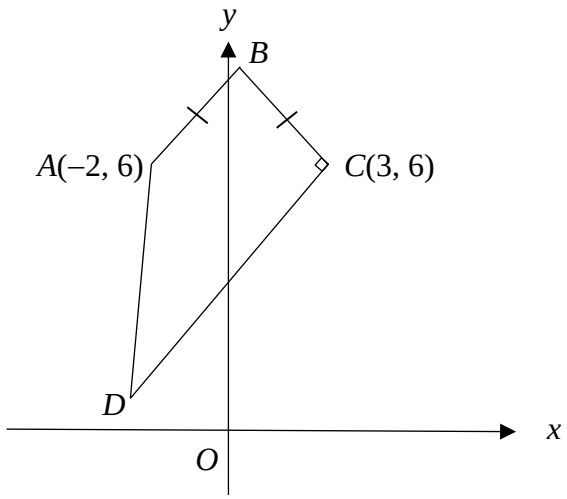
11 **(a)** Given that $y = (x+5)\sqrt{2x-5}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{kx}{\sqrt{2x-5}}$, where k is a constant. [2]

(b) Hence, find $\int \frac{x-4}{\sqrt{2x-5}}\, dx$. [4]

[Turn over

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12 Solutions to this question by accurate drawing will not be accepted.



The diagram (not drawn to scale) shows a quadrilateral *ABCD* such that *AB* = *BC* and angle $BCD = 90°$. Point *A* is $(-2, 6)$ and point *C* is $(3, 6)$. Given that the area of triangle *ABC* is 7.5 square units and point *D* lies on the line $y + x + 2 = 0$,

(a) show that the coordinates of *B* is $\left(\frac{1}{2}, 9\right)$. [2]

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(b) Find the coordinates of D. [4]

[Turn over

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(c) Find the area of *ABCD*. [2]

(d) If *ABCT* is a parallelogram, find the coordinates of *T*. [2]

-End of paper-

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