XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

ADDITIONAL MATHEMATICS

4049/02

Paper 2

Secondary 4 Express

26 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		*Marks Awarded*	
Presentation		*Marks Penalised*	

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90

Parent's/Guardian's Signature:

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1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2}bc\sin A$$

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1 A company purchased a colour copier machine at a cost of \$8500. The value of this machine decreases with time such that its value, \$$V$, after t months of usage is given by $V = 8500e^{-kt}$, where k is a constant.

(a) The value of the copier machine is expected to fall to \$6400 after 8 months of usage. Estimate the value, to the nearest dollar, of the machine after 2 years of usage. [4]

(b) Copier machines are to be replaced when its value reaches $\frac{1}{7}$ of its initial value.

The company's manager, Mrs Lee, claims that the machine will last for at least 5 years before a replacement is due. Showing all necessary working, explain whether you agree with Mrs Lee. [2]

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2 **(a)** Differentiate $2x\sin\frac{x}{2}$. [2]

(b) Use the result in part **(a)** to evaluate $\int_0^{\pi} 3x\cos\frac{x}{2}dx$, leaving your answer as an exact value in the form $a\pi - b$, where a and b are constants. [4]

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3 It is given that $f(x) = 2x^3 + px^2 + qx + 3$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -75 when divided by $x + 2$.

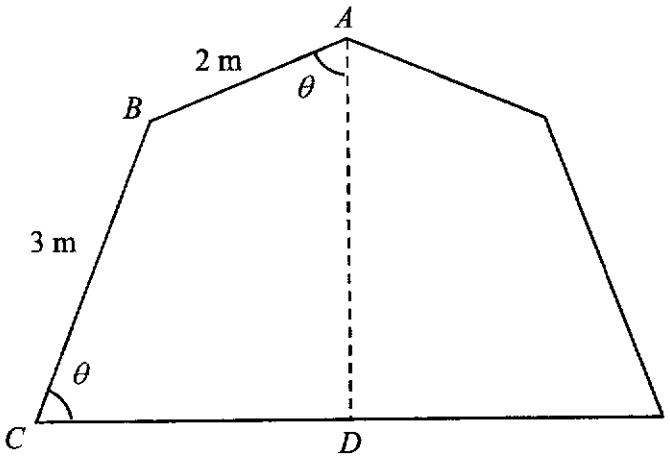
(a) Show that $p = -15$ and $q = 1$. [4]

(b) Solve the equation $f(x) = 0$. [4]

(c) Hence, solve the equation $2k\sqrt{k} + pk + q\sqrt{k} + 3 = 0$. [2]

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4 The diagram shows a vertical cross section of a tent in which AB = 2 m, BC = 3 m and angle BAD = angle $BCD = \theta$. The tent is symmetrical about its vertical height AD and it is set up on horizontal ground.



(a) Show that $AD = 3\sin\theta + 2\cos\theta$. [2]

(b) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

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(c) Given that the vertical height of the tent is 3.45 m, calculate the value of θ. [2]

(d) Find the value of θ for which AD is a maximum. [2]

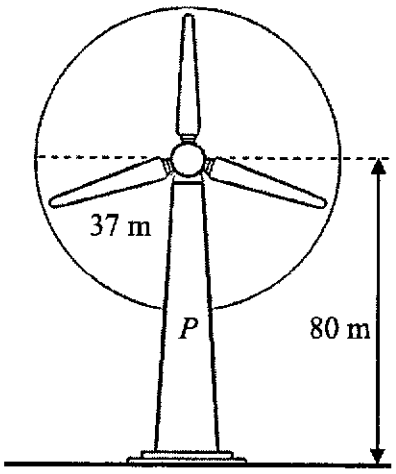
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5 **(a)** Given that $\log_p A^2 = 10$ and $\log_p B = 2$, find the value of $\log_A pB$. [3]

(b) Solve $3^x = 6 - 5\left(3^{-x}\right)$. [4]

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6 The diagram shows a wind turbine with propeller-like blades that have a length of 37 m each. Wind turns the blades that spin around a rotor in the centre to generate electricity. The height, h m, of the tip of each blade above the ground, t seconds after leaving a particular point P, can be modelled by $h = a - 37\cos bt$, where a and b are constants.



The centre of the wind turbine's rotor is 80 m from the ground and on average, the blades rotate in an anti-clockwise direction at a rate of 1 revolution every 8π seconds.

(a) Show that $a = 80$ and $b = \frac{1}{4}$. [2]

(b) Find the time taken, in seconds, for the blade to first reach a height of 89 m above ground after leaving P. [3]

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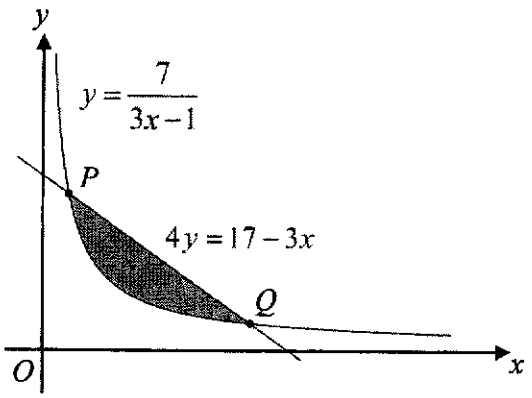
7 The equation of a curve is $y = 5\ln x$. The tangent to the curve at $x = e^2$ intersects the x-axis at A.

(a) Show that the coordinates of A are $\left(-e^2,\ 0\right)$. [5]

(b) Find the area bounded by the tangent, the line $x = e^2$ and the x-axis. [2]

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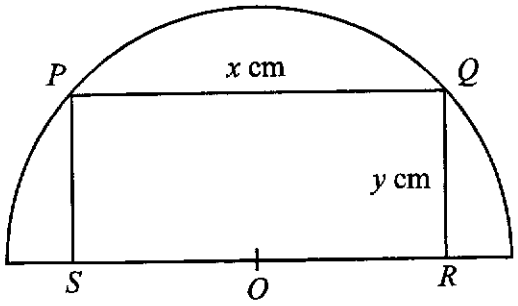


The diagram shows part of the curve $y=\frac{7}{3x-1}$ and the line $4y=17-3x$, where the curve intersects the line at points P and Q.

Find, showing all necessary working, the area of the shaded region that can be expressed in the form $a-b\ln 7$, where a and b are constants. [6]

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9 $PQRS$ is a rectangle with $PQ = x$ cm and $QR = y$ cm. It is inscribed in a semicircle with centre O and radius 10 cm.



(a) Show that the area of the rectangle, A cm^2, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [2]

(b) Given that x can vary, find the value of x for which the area of the rectangle is stationary. Leave your answer in the form $a\sqrt{b}$, where a and b are constants. [4]

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(c) Explain why the value of x in **part (b)** gives the largest possible value of A and hence, find the maximum area of rectangle $PQRS$. [3]

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10 AB is a chord of the circle C_1, where the coordinates of A and B are $(2, 5)$ and $(6, 3)$ respectively. The line $y = 5 - x$ passes through the centre of the circle.

(a) Find the coordinates of the centre of C_1. [4]

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(b) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [3]

(c) Another circle C_2 with centre $(2, 3)$ passes through the centre of C_1.

Explain if the C_2 lies entirely within C_1. [2]

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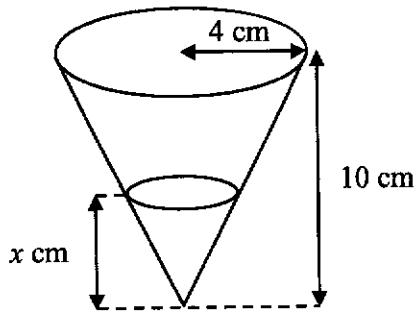
11 **(a)** Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$. [4]

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(b) Hence, solve the equation $\frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0° < x < 360°$. [4]

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12 Water is dispensed at a constant rate into an empty paper cup in the form of an inverted cone of height 10 cm and radius 4 cm. After t seconds, the depth of the water in the conical cup is x cm.



(a) Show that the volume, V cm^3, of water in the cup is given by $\frac{4\pi x^3}{75}$. [2]

(b) The water dispenser is a cylindrical container with radius 12 cm.
Given that the depth of water in the cylinder dispenser decreases at a constant rate of 0.0035 cm/s, find the rate of increase in the volume of water dispensed into the conical cup, leaving your answer in terms of π. [2]

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(c) Hence, find the rate of increase in the depth of water in the conical cup when the volume of water dispensed is $\frac{5\pi}{6}$ cm^3. [4]

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Preliminary Examination 2024

CANDIDATE NAME Mark Scheme

CLASS ___ INDEX NUMBER ___

ADDITIONAL MATHEMATICS 4049/02

Secondary 4 Express

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If working is needed for any question it must be shown in the space below the question.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Errors	Qn No.	Errors	Qn No.
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Brackets		Units	
Geometry		[redacted]	
Presentation		[redacted]	

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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1 A company purchased a colour copier machine at a cost of \$8500. The value of this machine decreases with time such that its value, \$$V$, after t months of usage is given by $V = 8500e^{-kt}$, where k is a constant.

(a) The value of the copier machine is expected to fall to \$6400 after 8 months of usage. Estimate the value, to the nearest dollar, of the machine after 2 years of usage. [4]

At $t=8$,

$6400 = 8500e^{-8k}$ [M1] sub in values

$e^{-8k} = \frac{64}{85}$

$-8k = \ln\frac{64}{85}$

$k = -\frac{1}{8}\ln\frac{64}{85}$

$= 0.035471$

either [M1] value of k

At $t=24$, allow ecf

$V = 8500e^{-0.035471\times24}$ [M1] sub in value

$= \$3628.29$

$= \$3628$ [A1] nearest dollar

(b) Copier machines are to be replaced when its value reaches $\frac{1}{7}$ of its initial value.

The company's manager, Mrs Lee, claims that the machine will last for at least 5 years before a replacement is due. Showing all necessary working, explain whether you agree with Mrs Lee. [2]

$\frac{1}{7}(8500) = 8500e^{-0.035471t}$

$\ln\frac{1}{7} = -0.035471t$

$t = \frac{\ln\frac{1}{7}}{-0.035471}$

$= 54.859$ months

either (allow ecf) [M1] value of t in months

≈ 4.57 years < 5 years

* must show comparison w respect to context

[A1] elaboration/reference to 5 years explained.

Disagree, it will require a replacement in less than 5 years.

[Alternative]

$\text{Value}_0 = \frac{1}{7}\times8500$

$= \$1214.29$

$\text{Value}_n = 8500e^{(-0.035471)(60)}$ [M1] allow ecf

$= \$1011.88$

Disagree,
since \$1011.88 < \$1214.29,
it will require replacement in
less than 5 years.

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2 **(a)** Differentiate $2x\sin\frac{x}{2}$. [2]

Penalise under [Presentation] if '$\frac{dy}{dx}$' is written without defining $y = 2x\sin\frac{x}{2}$.

$\frac{d}{dx}\left(2x\sin\frac{x}{2}\right) = 2x\cdot\frac{1}{2}\cos\frac{x}{2} + 2\sin\frac{x}{2}$ [M1] product rule

$= x\cos\frac{x}{2} + 2\sin\frac{x}{2}$ [A1]

(b) Use the result in part **(a)** to evaluate $\int_0^{\pi} 3x\cos\frac{x}{2}\,dx$, leaving your answer as an exact value in the form $a\pi - b$, where a and b are constants. [4]

From (a),

$\int_0^{\pi} x\cos\frac{x}{2} + 2\sin\frac{x}{2}\,dx = \left[2x\sin\frac{x}{2}\right]_0^{\pi}$

$\int_0^{\pi} x\cos\frac{x}{2}\,dx = \left[2x\sin\frac{x}{2}\right]_0^{\pi} - \int_0^{\pi} 2\sin\frac{x}{2}\,dx$ — [M1] —

$\therefore \int_0^{\pi} 3x\cos\frac{x}{2}\,dx = 6\left[x\sin\frac{x}{2}\right]_0^{\pi} - 6\left[-2\cos\frac{x}{2}\right]_0^{\pi}$ — [M1] for $\int_0^{\pi}\sin\frac{x}{2}\,dx$ or $\int_0^{\pi} 2\sin\frac{x}{2}\,dx$

$= 6\left[x\sin\frac{x}{2} + 2\cos\frac{x}{2}\right]_0^{\pi}$

$= 6\left(\pi\sin\frac{\pi}{2} + 2\cos\frac{\pi}{2} - 2\cos 0\right)$ — [M1] definite integral value

$= 6(\pi - 2)$

$= 6\pi - 12$ [A1]

[Alternative]

$\int_0^{\pi} x\cos\frac{x}{2}\,dx = \left[2x\sin\frac{x}{2}\right]_0^{\pi} - \int_0^{\pi} 2\sin\frac{x}{2}\,dx$

$= (2\pi\sin\frac{\pi}{2} - 0) - \left[-4\cos\frac{x}{2}\right]_0^{\pi}$

$= 2\pi\sin\frac{\pi}{2} + (4\cos\frac{\pi}{2} - 4\cos 0)$

$= 2\pi - 4$

$\therefore \int_0^{\pi} 3x\cos\frac{x}{2}\,dx = 3(2\pi - 4)$

$= 6\pi - 12$

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3 It is given that $f(x) = 2x^3 + px^2 + qx + 3$, where p and q are constants, has a factor of $2x-1$ and leaves a remainder of -75 when divided by $x+2$.

(a) Show that $p = -15$ and $q = 1$. [4]

(-1m) from qa if students do not show "= 0" at all.

$f(\frac{1}{2}) = 0$

$2(\frac{1}{2})^3 + p(\frac{1}{2})^2 + q(\frac{1}{2}) + 3 = 0$ [M1] factor theorem

$\frac{1}{4}p + \frac{1}{2}q + \frac{13}{4} = 0$

$p + 2q + 13 = 0$

$p = -2q - 13$ —①

$f(-2) = -75$

$2(-2)^3 + p(-2)^2 + q(-2) + 3 = -75$ [M1] Remainder theorem

$4p - 2q - 13 = -75$

$2p = q - 31$ —②

Sub ① into ②:

$2(-2q-13) = q - 31$

$-4q - 26 = q - 31$

$5q = 5$

$q = 1$

[A1] for 1st unknown shown through substitution or elimination method.

$\therefore p = -2(1) - 13$

$= -15$

[A1]

(b) Solve the equation $f(x) = 0$. [4]

(-1m) from qa if no "= 0" at all.

Note: If both [M1] not awarded, no [A2]

$f(x) = 2x^3 - 15x^2 + x + 3$

Method #1: ⟷ either [M1] ⟷ Method #2

$$\begin{array}{r} x^2 - 7x - 3 \\ 2x-1 \overline{)\,2x^3 - 15x^2 + x + 3} \\ -(2x^3 - x^2) \\ \hline -14x^2 + x \\ -(-14x^2 + 7x) \\ \hline -6x + 3 \\ -(-6x + 3) \\ \hline 0 \end{array}$$

Let $f(x) = (2x-1)(Ax^2 + Bx + C)$

Comparing coeff. of x^3 and constant:

$\Rightarrow A = 1,\ C = -3$

Comparing coeff. of x:

$1 = 2(-3) - B$

$B = -6 - 1$

$= -7$

$\Rightarrow f(x) = (2x-1)(x^2 - 7x - 3)$

$\Rightarrow (2x-1)(x^2 - 7x - 3) = 0$ [M1]

$2x - 1 = 0$ or $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-3)}}{2(1)}$

$x = \frac{1}{2}$ [A1]

$= 7.4051$ or -0.40512

$= 7.41$ or -0.405 [A1] only awarded when quadratic formula shown.

OR $x = \frac{7 \pm \sqrt{61}}{2}$

(c) Hence, solve the equation $2k\sqrt{k} + pk + q\sqrt{k} + 3 = 0$. [2]

[Alternative]

Let $k = x^2$

⋮

$k = \frac{1}{4}, 54.8, 0.164$

Let $x = \sqrt{k}$,

$\sqrt{k} = \frac{1}{2}$ OR $\sqrt{k} = 7.4051$ OR $\sqrt{k} = -0.40512$ [rej.]

$k = \frac{1}{4}$ [B1]

$k = 7.4051^2$

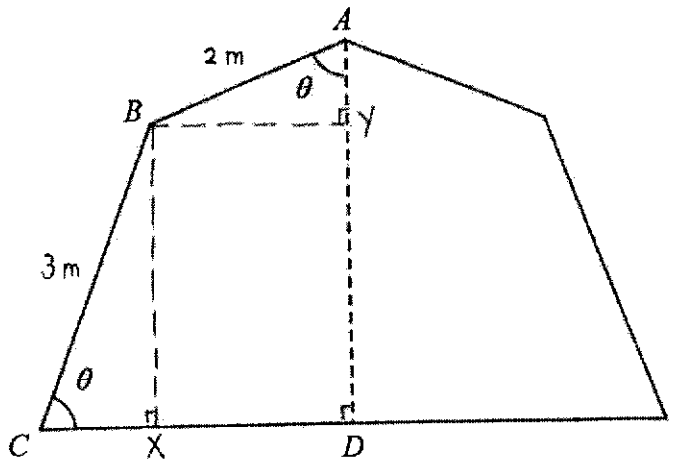
$= 54.8$

[A1] both must show rejected value. (since $\sqrt{k} > 0$)

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angle BAD = angle BCD = θ.

4 The diagram shows a vertical cross section of a tent in which $AB = 2$ m, BC = 3m and It is symmetrical about its vertical height AD and it is set up on horizontal ground.



(a) Show that $AD = 3\sin\theta + 2\cos\theta$. [2]

must be shown.

$*\cos\theta = \frac{AY}{2}$

$AY = 2\cos\theta$

$*\sin\theta = \frac{BX}{3}$

$BX = 3\sin\theta$

either [M1]

$AD = AY + YD$

$= AY + BX$

$= 2\cos\theta + 3\sin\theta$ [A1] shown

(b) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

Let $R\cos(\theta - \alpha) = 2\cos\theta + 3\sin\theta$

$R = \sqrt{2^2 + 3^2}$ [M1]

$= \sqrt{13}$

$\tan\alpha = \frac{3}{2}$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$ [M1]

$= 56.309°$

$= 56.3°$

$\Rightarrow AD = \underline{\sqrt{13}\cos(\theta - 56.3°)}$ [A1]

OR $3.61\cos(\theta - 56.3°)$

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(c) Given that the vertical height of the tent is 3.45 m, calculate the value of θ. [2]

$\sqrt{13}\cos(\theta - 56.309°) = 3.45$

$\cos(\theta - 56.309°) = \frac{3.45}{\sqrt{13}}$ or 0.95685 [M1] allow ecf (max 1m) for error from (b)

ref. $\angle = \cos^{-1}\left(\frac{3.45}{\sqrt{13}}\right)$

$= 16.891°$

since θ is acute,

$\theta - 56.309° = 16.891°$

$\theta = 16.891° + 56.309°$

$= \underline{73.2°}$ [A1]

(d) Find the value of θ for which AD is a maximum. [2]

Maximum value of AD occurs at:

$\cos(\theta - 56.309°) = 1$ [M1] allow ecf from (b)

$\theta - 56.309° = 0°$

$\therefore \theta = \underline{56.3°}$ [A1]

[Alternative]

$\frac{d}{d\theta}(3\sin\theta + 2\cos\theta)$

$= 3\cos\theta - 2\sin\theta$

$3\cos\theta - 2\sin\theta = 0$

$2\tan\theta = 3$

$\theta = 56.3°$ (acute) [A1]

[M1] allow ecf need to show $\frac{d}{d\theta}\ldots = 0$ for maximum.

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5 **(a)** Given that $\log_p A^2 = 10$ and $\log_p B = 2$, find the value of $\log_A pB$. [3]

$2\log_p A = 10$

$\log_p A = 5$ ↖

$\log_p B = 2$ ↙ either [M1]

$p^2 = B$

$\log_A pB = \frac{\log_p pB}{\log_p A}$ [M1] change base

$= \frac{\log_p (p \times p^2)}{5}$

$= \frac{\log_p (p^3)}{5}$

$= \frac{3}{5}$ [A1]

[Alternative ①]

$2\log_p A = 10$

$\log_p A = 5$ [M1]

$\log_A pB = \frac{\log_p pB}{\log_p A}$ [M1]

$= \frac{\log_p p + \log_p B}{5}$

$= \frac{1+2}{5}$

$= \frac{3}{5}$ [A1]

[Alternative ②]

$2\log_p A = 10$

$\log_p A = 5$

$\log_A p = \frac{\log_p p}{\log_p A} = \frac{1}{5}$ } either [M1]

$\log_A B = \frac{\log_p B}{\log_p A} = \frac{2}{5}$ [M1]

$\therefore \log_A pB = \log_A p + \log_A B$

$= \frac{1}{5} + \frac{2}{5}$

$= \frac{3}{5}$ [A1]

[Alternative ③]

change to base A

$\log_p A = 5$

$\log_A p = \frac{1}{5}$ } either [M1]

$\frac{\log_A B}{\log_A p} = 2$ [M1]

$\log_A B = 2 \times \frac{1}{5} = \frac{2}{5}$

$\log_A p + \log_A B = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ [A1]

(b) Solve $3^x = 6 - 5\left(3^{-x}\right)$. [4]

$3^x = 6 - \frac{5}{3^x}$

$3^{2x} = 6(3^x) - 5$ [M1] multiply by 3^x throughout eqn

$3^{2x} - 6(3^x) + 5 = 0$

Let $u = 3^x$,

$u^2 - 6u + 5 = 0$

$(u-1)(u-5) = 0$ } [M1] both (substitution & factorise)

$u = 1$ or $u = 5$

$\Rightarrow 3^x = 3^0$ $\Rightarrow 3^x = 5$

$\therefore x = 0$ $\lg 3^x = \lg 5$

[A1] $x = \frac{\lg 5}{\lg 3}$

$= 1.46$

[A1]

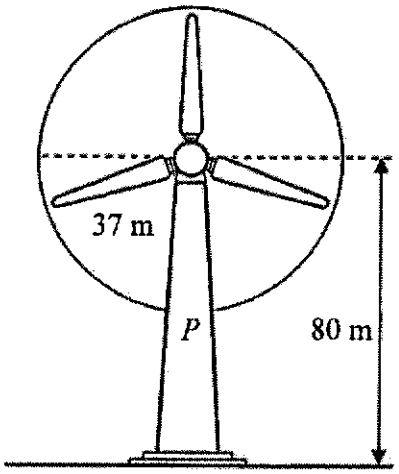
(accept $x = \log_3 5$)

-1 deduct [A1] if any x values rejected.

-1 lg written as log / $\log_{10}$

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6 The diagram shows a wind turbine with propeller-like blades that have a length of 37 m each. Wind turns the blades that spin around a rotor in the centre to generate electricity. The height, h m, of the tip of each blade above the ground, t seconds after leaving a particular point P, can be modelled by $h = a - 37\cos bt$, where a and b are constants.



The centre of the wind turbine's rotor is 80 m from the ground and the blades rotate in an anti-clockwise direction at a rate of 1 revolution every 8π seconds.

(a) Explain why $a = 80$ and show that $b = \frac{1}{4}$. [2]

For $a = 80$
$a \neq$ amplitude

at point P, $t = 0$

$h = 80 - 37$

$= 43$ m

$\Rightarrow 43 = a - 37\cos(0)$

$\therefore a = 80$ [A1]

[Alternatives]
Accept all other equivalent answers:
- when $t = 2\pi$, $h = 80$
- when $t = 4\pi$, $h = 117$
- when $t = 6\pi$, $h = 80$

OR
max. $h = 117$
min. $h = 43$
$a = \frac{117 + 43}{2} = 80$ [A1]

For $b = \frac{1}{4}$

Period $= 8\pi$

$\frac{2\pi}{b} = 8\pi$

$b = \frac{2\pi}{8\pi}$

$= \frac{1}{4}$ [A1]

OR $\cos(2\pi b) = 0$

$b = \frac{\pi}{2\pi} = \frac{1}{4}$

(b) Find the time taken, in seconds, for the blade to first reach a height of 89 m above ground after leaving P. [3]

$h = 80 - 37\cos\frac{t}{4}$

$80 - 37\cos\frac{t}{4} = 89$

$\cos\frac{t}{4} = \frac{89 - 80}{-37}$

$\cos\frac{t}{4} = -\frac{9}{37}$ [M1]

$\alpha_{(ref.\angle)} = \cos^{-1}\left(\frac{9}{37}\right)$

$= 1.3250$ rad.

$\therefore \frac{t}{4} = \pi - 1.3250, \pi + 1.3250$

$= 1.8165, 4.4665$ [M1]

$t = 7.27$, 17.9 (rej.)

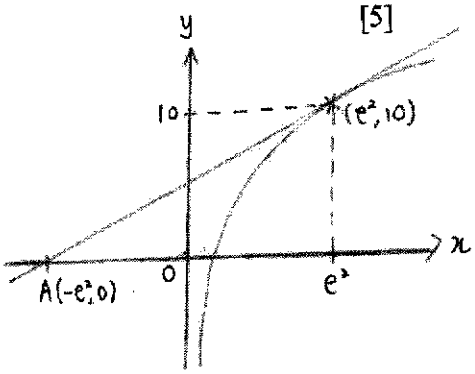
[A1]

[* since first t value,
benefit of doubt:
accept $\frac{t}{4} = \cos^{-1}\left(-\frac{9}{37}\right)$
To take note of α (ref. $\angle$) misconception.]

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7 The equation of a curve is $y = 5\ln x$. The tangent to the curve at $x = e^2$ intersects the x-axis at A.

(a) Show that the coordinates of A are $(-e^2, 0)$. [5]



at $x = e^2$, $y = 5\ln e^2$

$= 5(2\ln e)$

$= 10$ [M1]

$\frac{dy}{dx} = 5\left(\frac{1}{x}\right)$

$= \frac{5}{x}$ [M1]

$m_{(at\ x=e^2)} = \frac{5}{e^2}$

∴ Eqn of tangent: $y - 10 = \frac{5}{e^2}(x - e^2)$

$y = \frac{5}{e^2}x + 5$ [M1] allow ecf

⇒ intersects x-axis, $y = 0$.

∴ $\frac{5}{e^2}x + 5 = 0$ [M1] allow ecf.

$\frac{5}{e^2}x = -5$

$\frac{x}{e^2} = -1$

$\therefore x = -e^2$ } Solve for x [A1]

⇒ coordinates of $A(-e^2, 0)$. (shown)

OR Let $A(x, 0)$,

$\frac{10 - 0}{e^2 - x} = \frac{5}{e^2}$ [M1]

$10e^2 = 5(e^2 - x)$ [M1]

$-5x = 5e^2$

$x = \frac{5e^2}{-5}$

$= -e^2$

$\therefore A(-e^2, 0)$ } [A1]

(b) Find the area bounded by the tangent, the line $x = e^2$ and the x-axis. [2]

Area of bounded region

$= \frac{1}{2} \times 2e^2 \times 10$ [M1]

$= 10e^2$ units^2, [A1]

OR 73.9 units^2

[Alternative]

Area $= \int_{-e^2}^{e^2} \frac{5}{e^2}x + 5\,dx$

$= \left[\frac{5}{2e^2}x^2 + 5x\right]_{-e^2}^{e^2}$ [M1]

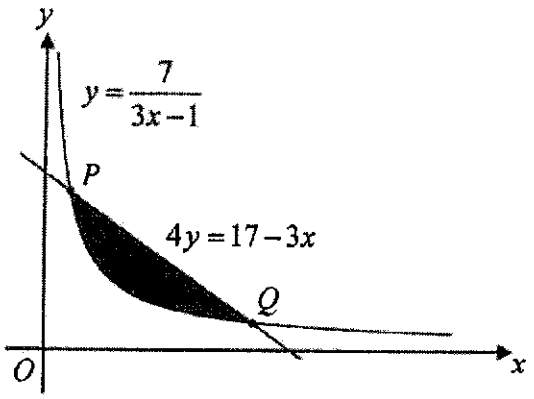
$= \left(\frac{5}{2}e^2 + 5e^2\right) - \left(\frac{5}{2}e^2 - 5e^2\right)$

$= 5e^2 + 5e^2$

$= 10e^2$ units^2 [A1]

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8



The diagram shows part of the curve $y=\frac{7}{3x-1}$ and the line $4y=17-3x$, where the curve intersects the line at points P and Q.

Find, showing all necessary working, the area of the shaded region that can be expressed in the form $a-b\ln 7$, where a and b are constants. [6]

$y=\frac{7}{3x-1}$ —①

$4y=17-3x$

$y=\frac{17-3x}{4}$ —②

sub ② into ①:

$\frac{17-3x}{4}=\frac{7}{3x-1}$ [M1]

$(17-3x)(3x-1)=28$

$51x-17-9x^2+3x=28$

$9x^2-54x+45=0$
$x^2-6x+5=0$
$(x-1)(x-5)=0$
overall -1m for 68 for lack of equivalence or not factorised (eg: skip steps for ÷9)

$x=1$ or 5 [M1]

Area of shaded region

$=\int_1^5 \frac{17-3x}{4}-\frac{7}{3x-1}\,dx$

$=\underbrace{\left[\frac{17}{4}x-\frac{3}{8}x^2\right]_1^5}_{\text{[M1] area under line}}-\underbrace{\frac{7}{3}\left[\ln(3x-1)\right]_1^5}_{\text{[M1] area under curve}}$

$=\underbrace{\left[\frac{17}{4}(5)-\frac{3}{8}(5)^2\right]-\left(\frac{17}{4}-\frac{3}{8}\right)}-\underbrace{\frac{7}{3}\left[\ln 14-\ln 2\right]}_{\text{OR}}$ [M1] either definite integral value

$=\frac{95}{8}-\frac{31}{8}-\frac{7}{3}\ln\left(\frac{14}{2}\right)$

$=\underline{8-\frac{7}{3}\ln 7}$ [A1]

[Alternative]

Area $=\underbrace{\frac{1}{2}(3.5+0.5)(5-1)}_{\text{area of trapezium [M1]}}-\int_1^5 \frac{7}{3x-1}\,dx$

$=\frac{1}{2}(4)(14)-\underbrace{\frac{7}{3}\left[\ln(3x-1)\right]_1^5}_{\text{[M1] area under curve}}$

$=8-\underline{\frac{7}{3}(\ln 14-\ln 2)}$ [M1]

$=8-\frac{7}{3}\ln\left(\frac{14}{2}\right)$

$=8-\frac{7}{3}\ln 7$

[Alternative] against the y-axis

Area $=\int_{0.5}^{3.5}\frac{17-4y}{3}-\frac{7+y}{3y}\,dy$

$=\underbrace{\left[\frac{17}{3}-\frac{4}{3}y\right]_{0.5}^{3.5}}_{\text{[M1]}}-\underbrace{\left[\frac{7}{3}\ln y+\frac{1}{3}y\right]_{0.5}^{3.5}}_{\text{[M1]}}$

$=\underbrace{\left(\frac{119}{6}-\frac{49}{6}\right)-\left(\frac{17}{6}-\frac{1}{6}\right)}-\underbrace{\left[\frac{7}{3}\ln 3.5+\frac{7}{6}-\frac{7}{3}\ln 0.5-\frac{1}{6}\right]}$

[M1] either

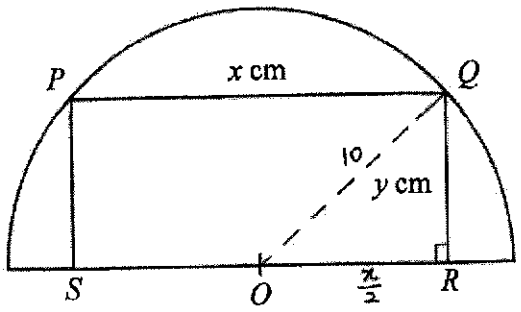
$=9-\left(1+\frac{7}{3}\ln 7\right)$

$=\underline{8-\frac{7}{3}\ln 7}$ [A1]

[Turn over

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9 $PQRS$ is a rectangle with $PQ = x$ cm and $QR = y$ cm. It is inscribed in a semicircle with centre O and radius 10 cm.



(a) Show that the area of the rectangle, A cm^2, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [2]

$y^2 + \left(\frac{x}{2}\right)^2 = 10^2$

$y = \sqrt{100 - \frac{x^2}{4}}$ [M1]

(since $y > 0$)

$\therefore$ Area $= xy$

$= x\sqrt{100 - \frac{x^2}{4}}$

$= x\sqrt{\frac{400 - x^2}{4}}$ manipulation must be shown

$= \frac{x}{2}\sqrt{400 - x^2}$ [A1] shown

OR

Area $= xy$

$= x\sqrt{100 - \frac{x^2}{4}}$

must be shown $= \frac{x}{2}(2)\sqrt{100 - \frac{x^2}{4}}$

$= \frac{x}{2}\sqrt{4}\sqrt{100 - \frac{x^2}{4}}$

$= \frac{x}{2}\sqrt{400 - x^2}$ } [A1]

OR

$y^2 = 100 - \frac{x^2}{4}$

$4y^2 = 400 - x^2$

$2y = \sqrt{400 - x^2}$ } [M1]

$y = \frac{1}{2}\sqrt{400 - x^2}$ sub

Area $= xy$

$= x\left(\frac{1}{2}\sqrt{400 - x^2}\right)$

$= \frac{x}{2}\sqrt{400 - x^2}$ } [A1]

(b) Given that x can vary, find the value of x for which the area of the rectangle is stationary. Leave your answer in the form $a\sqrt{b}$, where a and b are constants. [4]

$\frac{dA}{dx} = \frac{1}{2}x \cdot \frac{1}{2}(400 - x^2)^{-\frac{1}{2}}(-2x) + \frac{1}{2}(400 - x^2)^{\frac{1}{2}}$ [M1] OR quotient rule

$= -\frac{x^2}{2\sqrt{400 - x^2}} + \frac{\sqrt{400 - x^2}}{2}$

$= \frac{-x^2 + 400 - x^2}{2\sqrt{400 - x^2}}$

$= \frac{200 - x^2}{\sqrt{400 - x^2}}$ } any acceptable [M1]

When $\frac{dA}{dx} = 0$,

$\frac{200 - x^2}{\sqrt{400 - x^2}} = 0$ [M1] e.e.f. allowed. for equating $\frac{dA}{dx} = 0$, using their $\frac{dA}{dx}$ above.

$200 = x^2$

$x = \sqrt{200}$ (since $x > 0$)

$= 10\sqrt{2}$ [A1]

(accept $5\sqrt{8}$ as per $a\sqrt{b}$ form)

(P)^1 for any errors in expressions for $\frac{dA}{dx}$

Not accepted:

$\frac{dA}{dx}\left(\frac{x}{2}\sqrt{400 - x^2}\right)$

or $\frac{dy}{dx}$ etc ...

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(c) Explain why the value of x in (b) gives the largest possible value of A and hence, find the maximum area of the rectangle. [3]

⇒ Proving/finding maximum area

Ⓟ$^{-1}$ $\frac{dy}{dx}$ stated instead of $\frac{dA}{dx}$

Method #1 (2nd derivative test)

$$\frac{d^2A}{dx^2} = (200-x^2)\cdot -\tfrac{1}{2}(400-x^2)^{-\frac{3}{2}}(-2x) + (-2x)(400-x^2)^{-\frac{1}{2}}$$

$$= \frac{x(200-x^2)}{(400-x^2)^{\frac{3}{2}}} - \frac{2x}{(400-x^2)^{\frac{1}{2}}}$$

[M1] any equivalent $\frac{d^2A}{dx^2}$ expression c.f.a. from (b)

$$= \frac{200x - x^3 - 800x + 2x^3}{(400-x^2)^{\frac{3}{2}}}$$

$$= \frac{x^3 - 600x}{(400-x^2)^{\frac{3}{2}}}$$

at $x = 10\sqrt{2}$,

$$\frac{d^2A}{dx^2} = \frac{(10\sqrt{2})^3 - 600(10\sqrt{2})}{(400-200)^{\frac{3}{2}}}$$

$$= -2 < 0$$

Since $\frac{d^2A}{dx^2} < 0$, maximum area occurs at $x = 10\sqrt{2}$.

[A1] $\frac{d^2A}{dx^2}$ value must be shown

$$\therefore \text{Max. area} = \frac{10\sqrt{2}}{2}\sqrt{400-(10\sqrt{2})^2}$$

$$= 5\sqrt{2}\cdot\sqrt{200}$$

$$= 5\sqrt{400}$$

$$= 100\ \text{cm}^2, \quad \text{[A1]}$$

Method #2 (1st derivative test)

x	$(10\sqrt{2})^-$	$10\sqrt{2}$	$(10\sqrt{2})^+$
$\frac{dA}{dx}$	>0	$=0$	<0
sketch of tangent	/	—	\

[M1]

By the 1st derivative test,
⇒ Maximum area occurs at $x = 10\sqrt{2}$ [A1]

$$\therefore \text{Max. area} = \frac{10\sqrt{2}}{2}\sqrt{400-(10\sqrt{2})^2}$$

$$= 100\ \text{cm}^2, \quad \text{[A1]}$$

Explanations NOT accepted:

'$x = 10\sqrt{2}$ is a maximum point'.

'$x = 10\sqrt{2}$ is maximum' ← any unclear/inaccurate indication of the meaning of x.

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10 AB is a chord of the circle C_1, where the coordinates of A and B are $(2, 5)$ and $(6, 3)$ respectively. The line $y = 5 - x$ passes through the centre of the circle.

(a) Find the coordinates of the centre of C_1. [4]

Method #1

Mid-point of AB $= \left(\frac{2+6}{2}, \frac{5+3}{2}\right)$

$= (4, 4)$ [M1]

$m_{AB} = \frac{5-3}{2-6}$

$= -\frac{1}{2}$

$\therefore m_{\perp} = 2$ [M1]

Eqn of ⊥ bisector:

$y - 4 = 2(x - 4)$

$y = 2x - 4$ —①

$y = 5 - x$ —②

Sub ① into ②:

$2x - 4 = 5 - x$ [M1] allow ecf intersection of ⊥ bisector and line

$3x = 9$

$x = 3$

$\therefore y = 5 - 3$

$= 2$

⇒ centre: $(3, 2)$ [A1]

Method #2

Let centre of C_1 be (a, b)

Length of AC_1 = Length of BC_1

$\sqrt{(5-b)^2 + (2-a)^2} = \sqrt{(3-b)^2 + (6-a)^2}$ [M1]

$25 - 10b + b^2 + 4 - 4a + a^2 = 9 - 6b + b^2 + 36 - 12a + a^2$

$29 - 10b - 4a = 45 - 6b - 12a$

$0 = 16 + 4b - 8a$

$b - 2a + 4 = 0$

$b = 2a - 4$ —①

For line $y = 5 - x$, at (a, b),

$b = 5 - a$ —② [M1]

Sub ① into ②:

$2a - 4 = 5 - a$ [M1]

$3a = 9$

$a = 3$

$\therefore b = 5 - 3$

$= 2$

⇒ centre $(3, 2)$ [A1]

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(b) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [3]

Method #1

radius $= \sqrt{3^2+1^2}$

$= \sqrt{10}$ [M1]

$(x-3)^2 + (y-2)^2 = (\sqrt{10})^2$ [M1] allowed

$x^2 - 6x + 9 + y^2 - 4y + 4 = 10$

$x^2 + y^2 - 6x - 4y + 3 = 0$ [A1]

Method #2

radius $= \sqrt{3^2+1^2}$

$= \sqrt{10}$ [M1]

using $x^2+y^2+2gx+2fy+c=0$,

centre of circle: $C(-g,-f)$

$\Rightarrow -g = 3 \quad \Rightarrow -f = 2$

$g = -3 \quad f = -2$

$2g = -6 \quad 2f = -4$

radius $= \sqrt{f^2+g^2-c}$

$\sqrt{10} = \sqrt{(-2)^2+(-3)^2-c}$ either values of both p and q [M1] or value of r. allowed

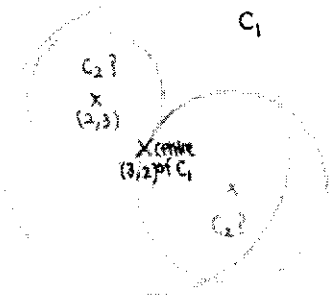
$10 = 13 - c$

$c = 3$

$\therefore x^2+y^2-6x-4y+3=0$ [A1]

(c) Another circle C_2 with centre $(2, 3)$ passes through the centre of C_1.

Explain if the C_2 lies entirely within C_1. [2]



for C_2 to lie entirely within C_1,

diameter of C_2 < radius of C_1

radius of $C_2 = \sqrt{1^2+1^2} = \sqrt{2}$ [M1] radius of C_2

$\therefore$ diameter of $C_2 = 2\sqrt{2} < \sqrt{10}$ [A1] explained with comparison.

Since the diameter of circle C_2 is shorter than the radius of circle C_1, it lies entirely within C_1.

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11 (a) Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$ [4]

$$\text{LHS} = \frac{2\sin x\cos x - (1 - 2\sin^2 x) + 1}{2\sin x\cos x + (2\cos^2 x - 1) + 1}$$ [M1] double angle formula for $\sin 2x$

[M1] either $(2\cos^2 x - 1)$ selected in numerator or $(1 - 2\sin^2 x)$ selected in denominator

$$= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x}$$

$$= \frac{2\sin x(\cos x + \sin x)}{2\cos x(\sin x + \cos x)}$$ [M1] factorisation

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$ [A1] shown.

Alternative

$$\frac{2\sin x\cos x - (\cos^2 x - \sin^2 x) + 1}{2\sin x\cos x + (\cos^2 x - \sin^2 x) + 1}$$ [M1]

$$= \frac{2\sin x\cos x - \cos^2 x + \sin^2 x + (\sin^2 x + \cos^2 x)}{2\sin x\cos x + \cos^2 x - \sin^2 x + (\sin^2 x + \cos^2 x)}$$

$$= \frac{2\sin x\cos x + 2\sin^2 x}{2\sin x\cos x + 2\cos^2 x} = \frac{2\sin x(\cos x + \sin x)}{2\cos x(\sin x + \cos x)}$$ [M1]

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$ [A1]

Alternative:

$$\text{LHS} = \frac{2\sin x\cos x - (2\cos^2 x - 1) + 1}{2\sin x\cos x + (1 - 2\sin^2 x) + 1}$$ [M1] double angle formula for $\sin 2x$

$$= \frac{2\sin x\cos x - 2\cos^2 x + 2}{2\sin x\cos x - 2\sin^2 x + 2}$$

$$= \frac{2(\sin x\cos x - \cos^2 x + 1)}{2(\sin x\cos x - \sin^2 x + 1)}$$

$$= \frac{\sin x\cos x - (1 - \sin^2 x) + 1}{\sin x\cos x - (1 - \cos^2 x) + 1}$$ [M1] either numerator: $\cos 2x = 2\cos^2 x - 1$ selected first, then $\cos^2 x = 1 - \sin^2 x$

or denominator: $\cos 2x = 1 - 2\sin^2 x$ selected first, then $\sin^2 x = 1 - \cos^2 x$.

$$= \frac{\sin x\cos x + \sin^2 x}{\sin x\cos x + \cos^2 x}$$

$$= \frac{\sin x(\cos x + \sin x)}{\cos x(\sin x + \cos x)}$$ [M1] factorisation

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$ [A1] shown

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(b) Hence, solve the equation $\frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0° < x < 360°$. [4]

Hence, $\tan x = 5 - 2\sec^2 x$

$\tan x = 5 - 2(1 + \tan^2 x)$ [M1] $\sec^2 x = 1 + \tan^2 x$

award M1 even if solution from (a) is applied incorrectly

$2\tan^2 x + \tan x - 3 = 0$

$(2\tan x + 3)(\tan x - 1) = 0$ [M1]

$\tan x = -\frac{3}{2}$ or $\tan x = 1$

$\alpha = \tan^{-1}\left(\frac{3}{2}\right)$ $\alpha = 45°$

$= 56.309°$ $\therefore x = 45°, 225°$ [A1]

$\therefore x = 180° - \alpha, 360° - \alpha$

$= 180° - 56.309°, 360° - 56.309°$

$= 123.7°, 303.7°$ [A1]

$\Rightarrow x = 45°, 123.7°, 225°, 303.7°$.

(ii) $^{-1}$ degrees must be stated.

[A1] deducted for <1dp answers.
(eg. 123°/124° or 303°/304°)

[Alternative]

$\tan x = 5 - 2\sec^2 x$

$\frac{\sin x}{\cos x} = 5 - \frac{2}{\cos^2 x}$

$\sin x \cos x = 5\cos^2 x - 2$

$5\cos^2 x - \sin x \cos x - 2 = 0$

$5\cos^2 x - \sin x \cos x - 2(\sin^2 x + \cos^2 x) = 0$ [M1]

$3\cos^2 x - \sin x \cos x - 2\sin^2 x = 0$

$(3\cos x + 2\sin x)(\cos x - \sin x) = 0$ [M1]

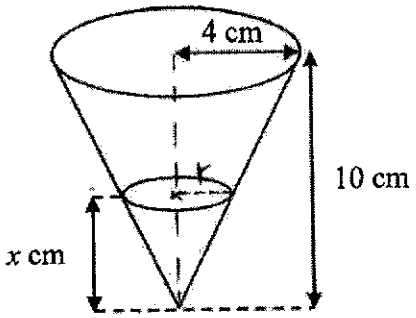
$3\cos x = -2\sin x$ or $\cos x = \sin x$

$\tan x = -\frac{3}{2}$ $\tan x = 1$

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12 Water is dispensed at a constant rate into an empty paper cup in the form of an inverted cone of height 10 cm and radius 4 cm. The water dispenser is a cylindrical container with radius 12 cm. After t seconds, the depth of the water in the conical cup is x cm.

* Penalise 1m overall for 811 for absence of units in (b), (c)



(a) Show that the volume of water in the cup is $\frac{4\pi x^3}{75}$ cm^3. [2]

$\frac{r}{4} = \frac{x}{10}$

$r = \frac{2}{5}x$ [M1]

Volume $= \frac{1}{3}\pi\left(\frac{2}{5}x\right)^2(x)$

$= \frac{2^2\pi x^3}{3\times 5^2}$

$= \frac{4\pi x^3}{75}$

[A1] shown with substitution of r into vol. of cone formula.

OR

$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^3$

$\frac{V_1}{\frac{1}{3}\pi(4)^2(10)} = \left(\frac{x}{10}\right)^3$ [M1]

$V_1 = \frac{x^3}{1000} \times \frac{160\pi}{3}$

$= \frac{160\pi x^3}{3000}$

$= \frac{4\pi x^3}{75}$ [A1]

(b) Given that the depth of water in the cylinder dispenser decreases by 0.0035 cm/s, find the rate of increase in the volume of water dispensed in the conical cup, in terms of π. [2]

Common misconception: misinterpretation of variables in 2nd, values happen to give same final ans but NO marks awarded.

$\frac{dx}{dt} \neq 0.0035$

rate of increase in the height of water in cone is not a constant.

Let h be the depth of water in cylinder.

$\frac{dh}{dt} = -0.0035$ cm/s

rate of increase in cup (in vol.) = rate of decrease in cylinder (in vol.)

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

Volume of cylinder $= \pi(12^2)h$

$= 144\pi h$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$= 144\pi \times -0.0035$ [M1]

$= -0.504\pi$ cm^3/s

$\Rightarrow$ rate of increase in cup

$= \underline{0.504\pi \text{ cm}^3/\text{s}}$ (or $\frac{63\pi}{125}$ cm^3/s) [A1] accept positive $\frac{dV}{dt}$ directly.

[Alternative]

(cylinder) $\frac{dV}{dt} = \pi r^2(\Delta h)$

$= \pi(12)^2(-0.0035)$ [M1]

$= -0.504\pi$ cm^3/s

$\therefore \frac{dV}{dt}$ (cone) $= 0.504\pi$ cm^3/s [A1]

① $^{-1}$ cm^3/s

$\frac{dV}{dt} = 144\pi \times 0.0035$

$= 0.504\pi$ cm^3/s

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(c) Hence, find the rate of increase in the depth of water in the conical cup when the volume of water dispensed is $\frac{5\pi}{6}$ cm^3. [4]

$$\frac{dV}{dt}_{(cup)} = \frac{dV}{dx} \times \frac{dx}{dt}$$

When $vol_{cup} = \frac{5\pi}{6}$,

$$\frac{4\pi x^3}{75} = \frac{5\pi}{6}$$

$$x^3 = \frac{5}{6}\left(\frac{75}{4}\right)$$

$$x = \sqrt[3]{\frac{125}{8}}$$

$$= 2.5 \text{ cm} \quad [M1]$$

$$\frac{dV}{dx} = \frac{3(4\pi)x^2}{75}$$

$$= \frac{4\pi x^2}{25} \quad [M1]$$

$$\therefore \underline{0.504\pi} = \frac{4\pi(2.5)^2}{25} \times \frac{dx}{dt} \quad [M1] \quad \text{(allow ecf from part (b) or incorrect x-value above.)} \quad \underline{\text{OR}} \quad \frac{dx}{dt} = \frac{63}{20x^2} \quad [M1]$$

$$\therefore \frac{dx}{dt} = \frac{0.504\pi(25)}{4\pi(2.5)^2}$$

$$= 0.504 \text{ cm/s} \quad [A1] \quad \left(\text{or } \frac{63}{125} \text{ cm/s}\right)$$

(ii) -1 cm/s

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Answer Key

Qn No.		Answers
1	(a)	\$3628
	(b)	Disagree.
2	(a)	$x\cos\frac{x}{2}+2\sin\frac{x}{2}$
	(b)	$6\pi-12$
3	(b)	$x=7.41$ or -0.405
	(c)	$k=0.25$ or 54.8
4	(b)	$\sqrt{13}\cos(\theta-56.3°)$
	(c)	$73.2°$
	(d)	$56.3°$
5	(a)	$\frac{3}{5}$
	(b)	$x=0$ or 1.46
6	(b)	$t=7.27$ s
7	(b)	$10e^2$ or 73.9 units^2
8		$8-\frac{7}{3}\ln 7$
9	(b)	$10\sqrt{2}$
	(c)	Maximum area 100 cm^2
10	(a)	$(3,2)$
	(b)	$x^2+y^2-6x-4y+3=0$
	(c)	C_2 lies entirely within C_1
11	(b)	$x=45°,123.7°,225°,303.7°$
12	(b)	0.504π cm^3/s or $\frac{63\pi}{125}$ cm^3/s
	(c)	0.504 cm/s

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