SWISS COTTAGE SECONDARY SCHOOL
SECONDARY FOUR AND FIVE
PRELIMINARY EXAMINATION

O

Name: _____ ( ) Class: _____

ADDITIONAL MATHEMATICS

Paper 2

4049/02
Tuesday 10 September 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Questions	1	2	3	4	5	6
Marks						

This document consists of **18** printed pages and **2** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+.....+\binom{n}{r}a^{n-r}b^r+....+b^n$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1).....(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}bc\sin A$$

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Answer **all** the questions.

Section A (41 marks)

1 A man bought a new car. The value of the car depreciated with time so that its value, $\$P$, after t months' use is given by $P = 175\ 000e^{-kt}$, where k is a constant.

(a) Find the value of the car, $\$P$, when the man bought it. [1]

The value of the car is expected to be \$162 000 after eight months' use.

(b) Show that $k = 0.01$. [2]

(c) Use the result from **part (b)** to determine the age of the car correct to the nearest month, when its value reached half of the original value when the man bought it. [2]

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2 A calculator must not be used in this question.

(a) Show that $\cot 15° = \sqrt{3} + 2$. [4]

(b) Use the result from **part (a)** to find an expression for $\text{cosec}^2 15°$, in the form $p + q\sqrt{3}$ where p and q are integers. [2]

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3 Given that $\int_0^m \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) \mathrm{d}x = \frac{3}{4}$, where m is a positive constant,

(a) show that $4e^{4m} - 12e^{2m} + 5 = 0$. [3]

(b) Use the result from **part (a)** and a suitable substitution to find the value of m. [4]

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4 The point A lies on the curve $y = x \ln x$. The tangent to the curve at A is parallel to the line $y = 2x + 3$.

(a) Find the exact coordinates of A. [4]

The normal to the curve $y = x \ln x$ at A meets the line $y = 2x + 3$ at the point B.

(b) Show that the x-coordinate of B is $k(e-2)$, where k is a constant to be found. [3]

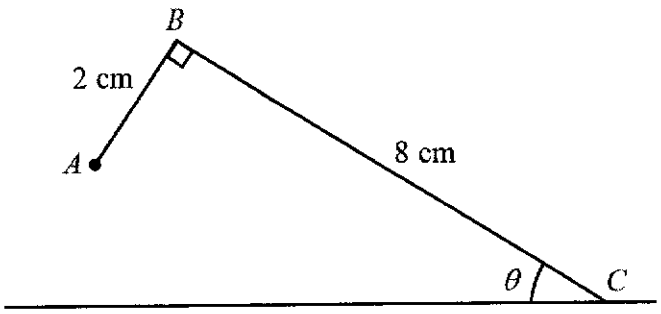
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5 **(a)** Prove the identity $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=\tan x$. [3]

(b) Hence solve the equation $\dfrac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}=3\cot 2x$ for $0°<x<180°$. [4]

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6



The diagram shows two rods AB and BC rigidly hinged at B so that angle $ABC = 90°$. The lengths of AB and BC are 2 cm and 8 cm respectively. The point C is fixed on horizontal ground and the rod BC rotates in a vertical plane with the rod BC inclined at an angle θ to the ground.

(a) Show that the height, h cm, of A above the ground is given by $h = a\sin\theta - b\cos\theta$, where a and b are integers to be found. [2]

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(b) Using the values of a and b found in **part (a)**, express h in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0\leq\alpha\leq\frac{\pi}{2}$. [4]

(c) Hence, state the maximum value of h and find the corresponding value of θ. [3]

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10

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Name: _____ ( )

Class: _____

Questions	7	8	9	10	11
Marks					

Section B (49 marks)

7 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x)=0$ are m, $2m$ and $(1-m)$, where $m>0$. It is given that $f(x)$ has a remainder of 30 when divided by $1-x$.

(a) Show that $2m^3-3m^2+m-30=0$. [3]

(b) Hence, find a value for m and show that there are no other real values of m which satisfy this equation. [4]

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8 It is given that $y = \dfrac{x-1}{\sqrt{1+x}}$.

(a) Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{x+3}{p\sqrt{(1+x)^3}}$, where p is a constant. [4]

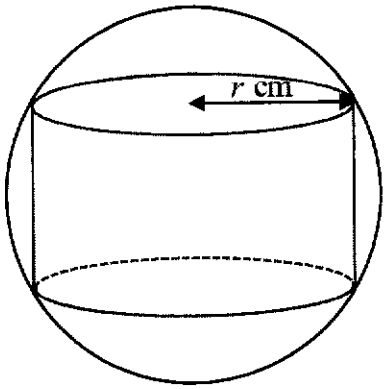
(b) Given that y is changing at a consant rate of 0.6 units per second, find the rate of change of x when $x = 3$. [2]

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(c) Use the result from **part (a)** to evaluate $\int_0^3 \frac{x+3}{3\sqrt{(1+x)^3}}\,dx$. [4]

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9



The diagram shows a cylinder of radius r cm inscribed in a sphere with a fixed internal radius of 8 cm.

(a) Given that the curved surface area of the cylinder is S cm^2, show that

$$\frac{dS}{dr} = \frac{8\pi\left(32 - r^2\right)}{\sqrt{64 - r^2}}.$$ [5]

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(b) Given that r varies, show that S has a stationary value when the height of the cylinder is equal to twice the radius of the cylinder. [3]

(c) Determine whether this value of S is a maximum or a minimum. [2]

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10 The equation of a circle is $x^2 + y^2 + 4x - 6y - 12 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

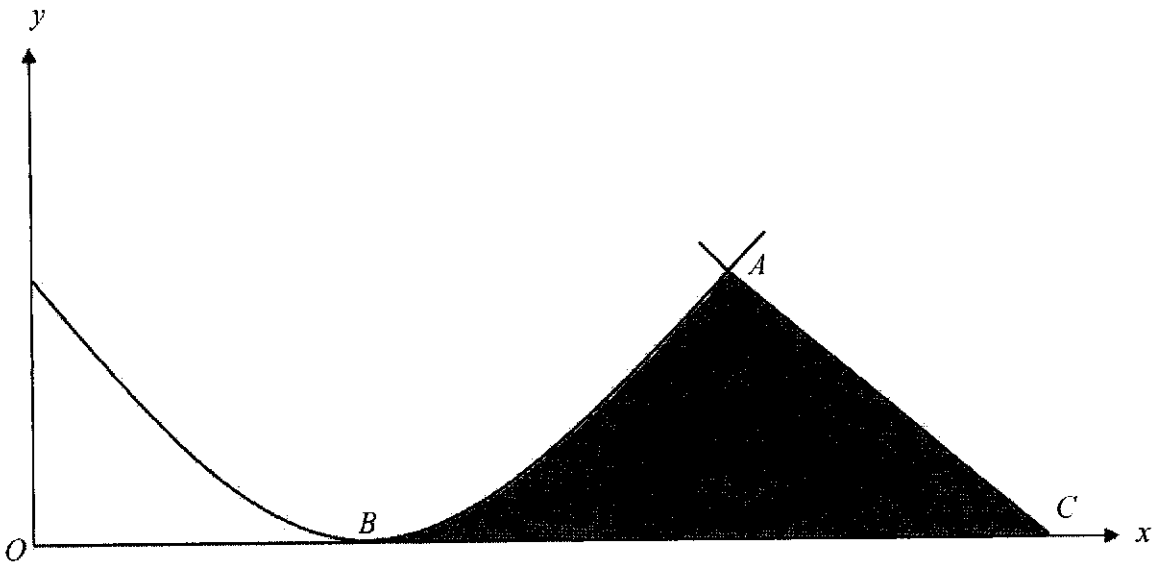
(b) Find the shortest distance of the centre of the circle to the line $y = 2x - 3$ and hence explain whether the circle intersects the line $y = 2x - 3$. [6]

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Continuation of working space for Question **10(b).**

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11



The diagram shows part of the curve $y = 1 - \sin x$ passing through the point A. The curve touches the x-axis at the point B.

The gradient of the tangent to the curve at A is 1 and the normal to the curve at A meet the x-axis at C.

Show that the area of the shaded region is $\frac{1}{2}(\pi - 1)$ units^2. [12]

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Continuation of working space for question **11**.

END OF PAPER

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2024 4G3 Additional Mathematics Preliminary Examinations Marking Scheme

Section A (41 marks)

1 A man bought a new car. The value of the car depreciated with time so that its value, $\$P$, after t months' use is given by $P = 175\ 000e^{-kt}$, where k is a constant.

(a) Find the value of the car, $\$P$, when the man bought it. [1]

When $t = 0$,
$P = 175000e^{-k(0)}$,
$P = 175000$ [B1]

The value of the car is expected to be \$162 000 after eight months' use.

(b) Show that $k = 0.01$. [2]

When $t = 8$,
$175000e^{-8t} = 162000$ [M1]
$e^{-8k} = \frac{162000}{175000}$
$k = -\frac{1}{8}\ln\frac{162000}{175000}$
$k = 0.0096487$
$k = 0.01$ [A1]

(c) Use the result from **part (b)** to determine the age of the car correct to the nearest month, when its value reached half of the original value when the man bought it. [2]

$175000e^{-0.01t} = \frac{175000}{2}$ [M1]
$e^{-0.01t} = \frac{1}{2}$
$-0.01t = \ln\frac{1}{2}$
$t = \frac{1}{-0.01}\ln\frac{1}{2}$
$t = 69.31$
$t = 70$ months [A1]

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2 **A calculator must not be used in this question.**

(a) Show that $\cot 15° = \sqrt{3} + 2$. [4]

LHS $= \cot 15°$

$$= \frac{1}{\tan(60° - 45°)}$$

$$= 1 \div \left[\frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°} \right]$$ [M1 – application of additional formula]

$$= 1 \div \left[\frac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)} \right]$$ [M1 – exact values of trigonometric functions for special angles]

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1}$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$ [M1 – multiplication by conjugate]

$$= \frac{1 + 2\sqrt{3} + 3}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{2\sqrt{3} + 4}{2}$$ [A1]

$$= \sqrt{3} + 2$$

= RHS (shown)

OR

$\cot 15$

$$= \frac{\cos 15}{\sin 15}$$

$$= \frac{\cos 60 \cos 45 + \sin 60 \sin 45}{\sin 60 \cos 45 - \cos 60 \sin 45} [M1]$$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}} [M1]$$

$$= \frac{\frac{1 + \sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3} - 1}{2\sqrt{2}}}$$

$$= \frac{1 + \sqrt{3}}{\sqrt{3} - 1} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} [M1]$$

$$= \frac{3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= 2 + \sqrt{3} [A1]$$

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OR

$\cot 15$

$= \dfrac{1}{\tan(45-30)}$

$= \dfrac{1+\tan 45\tan 30}{\tan 45-\tan 30}[M1]$

$= \dfrac{1+1\times\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}[M1]$

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{1+\sqrt{3}}{1+\sqrt{3}}[M1]$

$= \dfrac{3+2\sqrt{3}+1}{3-1}$

$= 2+\sqrt{3}[A1]$

OR

$\cot 15$

$= \dfrac{\cos 15}{\sin 15}$

$= \dfrac{\cos 45\cos 30+\sin 45\sin 30}{\sin 45\cos 30-\cos 45\sin 30}[M1]$

$= \dfrac{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}}{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}}[M1]$

$= \dfrac{\frac{1+\sqrt{3}}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\times\dfrac{1+\sqrt{3}}{1+\sqrt{3}}[M1]$

$= \dfrac{3+2\sqrt{3}+1}{3-1}$

$= 2+\sqrt{3}[A1]$

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(b) Use the result from **part (a)** to find an expression for $\text{cosec}^2 15°$, in the form $p+q\sqrt{3}$ where p and q are integers. [2]

$\text{cosec}^2 15° = 1+\cot^2 15°$ [M1 – application of special identities]

$= 1+\left(\sqrt{3}+2\right)^2$

$= 1+3+4\sqrt{3}+4$

$= 8+4\sqrt{3}$ [A1]

$= 4\left(2+\sqrt{3}\right)$

OR

$\text{cosec}^2 15°$

$= \dfrac{1}{\sin^2 15°}$

$= \cot 15° \times \dfrac{1}{\sin 15°} \div \cos 15°$

$= \left(\sqrt{3}+2\right) \times \dfrac{1}{\sin 15° \cos 15°}$

$= \left(\sqrt{3}+2\right) \times \dfrac{1}{\sin(45\text{-}30)° \cos(45-30)°}$

$= \left(\sqrt{3}+2\right) \times \dfrac{1}{\frac{\sqrt{2}\sqrt{3}-\sqrt{2}}{4} \times \frac{\sqrt{2}\sqrt{3}+\sqrt{2}}{4}}$ [$M1$ – app of correctly formed special identities]

$= \dfrac{\sqrt{3}+2}{\frac{6-2}{10}}$

$= \left(\sqrt{3}+2\right) \times \dfrac{16}{4}$

$= 4\sqrt{3}+8$ [$A1$]

3 Given that $\int_0^m \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) dx = \frac{3}{4}$, where m is a positive constant,

(a) show that $4e^{4m} - 12e^{2m} + 5 = 0$. [3]

$\int_0^m \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) dx = \frac{3}{4}$

$\left[e^{2x} + \dfrac{5e^{-2x}}{4}\right]_0^m = \dfrac{3}{4}$ [M1 – integration]

$\left[e^{2m} + \dfrac{5e^{-2m}}{4}\right] - \left[1+\dfrac{5}{4}\right] = \dfrac{3}{4}$ [M1 – substitution of limits]

$e^{2m} + \dfrac{5}{4e^{2m}} - 3 = 0$ [A1]

$4e^{4m} - 12e^{2m} + 5 = 0$ (shown)

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(b) Use the result from **part (a)** and a suitable substitution to find the value of m. [4]

$4e^{4m}-12e^{2m}+5=0$

Let $u=e^{2m}$,

$4u^2-12u+5=0$ [M1 – substitution]

$(2u-5)(2u-1)=0$ [M1 – factorisation of quadratic equation]

$u=\frac{5}{2}$ or $u=\frac{1}{2}$

$e^{2m}=\frac{5}{2}$ or $e^{2m}=\frac{1}{2}$

$2m=\ln\frac{5}{2}$ or $2m=\ln\frac{1}{2}$ [M1 – application of ln]

$m=\frac{1}{2}\ln\frac{5}{2}$ or $m=\frac{1}{2}\ln\frac{1}{2}$

$m=0.458$ or $m=-0.347$ (reject)

Therefore, $m=0.458$. [A1 – includes rejecting $m=-0.347$]

4 The point A lies on the curve $y=x\ln x$. The tangent to the curve at A is parallel to the line $y=2x+3$.

(a) Find the exact coordinates of A. [4]

$m_A=2$

$\frac{dy}{dx}=2$ [M1]

Given $y=2x\ln x$,

$\frac{dy}{dx}=x\left(\frac{1}{x}\right)+\ln x$ [M1 – differentiation using product rule]

$\frac{dy}{dx}=1+\ln x$

$1+\ln x=2$

$\ln x=1$

$x=e$ [M1]

When $x=e$,

$y=e\ln e$

$y=e$

$A(e,e)$ [A1]

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	The normal to the curve $y = x\ln x$ at A meets the line $y = 2x+3$ at the point B.	
	(b)	Show that the x-coordinate of B is $k(e-2)$, where k is a constant to be found. [3]

Let equation of normal at A be $y = mx + c$.

$m = -\frac{1}{2}$, $\quad A(e,e)$

$y - e = -\frac{1}{2}(x-e)$ [M1]

$y = -\frac{1}{2}x + \frac{1}{2}e + e$

$y = -\frac{1}{2}x + \frac{3}{2}e$ - eq (1)

$y = 2x + 3$ - eq (2)

$(1) = (2)$

$-\frac{1}{2}x + \frac{3}{2}e = 2x + 3$ [M1]

$\frac{5}{2}x = \frac{3}{2}e - 3$

$x = \frac{3}{5}e - \frac{6}{5}$

$x = \frac{3}{5}(e-2)$ (shown) [A1]

5	**(a)**	Prove the identity $\frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x} = \tan x$. [3]

LHS $= \frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}$

$= \frac{1-(1-2\sin^2 x)+2\sin x\cos x}{1+(2\cos^2 x-1)+2\sin x\cos x}$ [M1 – application of trigonometric identity where $2x$ is removed and this step allows for factorization next]

$= \frac{1-1+2\sin^2 x+2\sin x\cos x}{1+2\cos^2 x-1+2\sin x\cos x}$

$= \frac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$

$= \frac{2\sin x(\sin x+\cos x)}{2\cos x(\sin x+\cos x)}$ [M1 – factorisation and simplification]

$= \frac{\sin x}{\cos x}$ [A1]

$= \tan x$

OR

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LHS

$$= \frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x}$$

$$= \frac{1-\left(\cos^2 x-\sin^2 x\right)+2\sin x\cos x}{1+\left(\cos^2 x-\sin^2 x\right)+2\sin x\cos x}$$

$$= \frac{\cos^2 x+\sin^2 x-\cos^2 x-\sin^2 x+2\sin x\cos x}{\cos^2 x+\sin^2 x+\cos^2 x-\sin^2 x+2\sin x\cos x}$$ [M1 – application of trigonometric identity where $2x$ is removed and this step allows for factorization next]

$$= \frac{2\sin^2 x+2\sin x\cos x}{2\cos^2 x+2\sin x\cos x}$$

$$= \frac{2\sin x\left(\sin x+\cos x\right)}{2\cos x\left(\sin x+\cos x\right)}$$ [M1 – factorisation and simplification]

$$= \frac{\sin x}{\cos x}$$ [A1]

$$= \tan x$$

(b) Hence solve the equation $\frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x} = 3\cot 2x$ for $0° < x < 180°$. [4]

$$\frac{1-\cos 2x+\sin 2x}{1+\cos 2x+\sin 2x} = 3\cot 2x$$

$$\tan x = 3\cot 2x$$

$$\tan x = \frac{3}{\tan 2x}$$

$$\tan x = \frac{3}{\frac{2\tan x}{1-\tan^2 x}}$$ [M1 – application of double angle formula]

$$\tan x = \frac{3\left(1-\tan^2 x\right)}{2\tan x}$$

$$2\tan^2 x = 3-3\tan^2 x$$

$$5\tan^2 x = 3$$

$$\tan^2 x = \frac{3}{5}$$

$$\tan x = \pm\sqrt{\frac{3}{5}}$$ [M1 – forming trigonometric equation]

Range: $0° < x < 180°$.

x is in all quadrants 1 and 2.

Reference angle $= \tan^{-1}\sqrt{\frac{3}{5}}$ [M1 – reference angle]

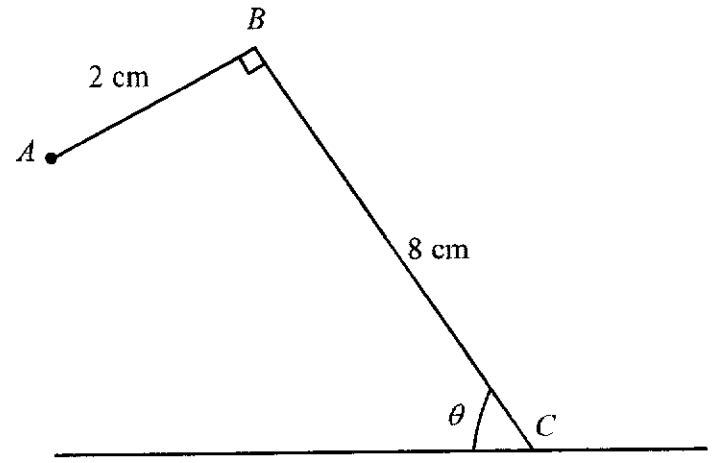
$= 37.76124°$

$x = 37.8°$ and $x = 180° - 37.76124°$

$x = 142.2°$ [A1 – for both answers]

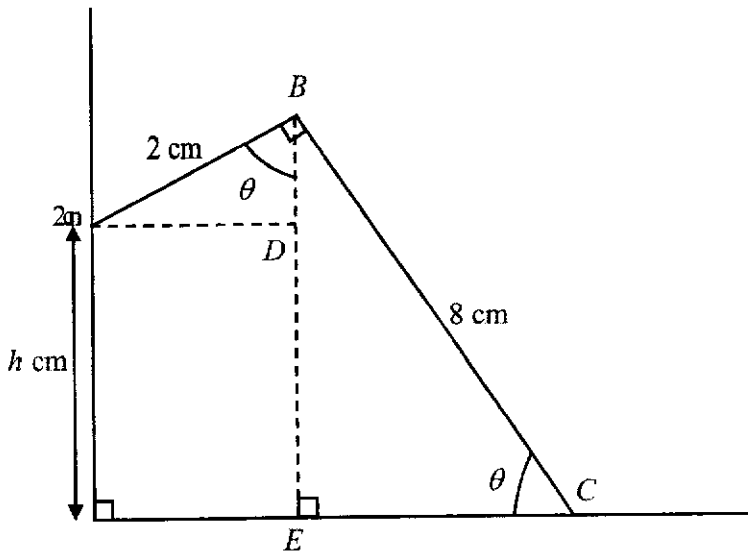
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The diagram shows two rods AB and BC rigidly hinged at B so that angle $ABC = 90°$. The lengths of AB and BC are 2 cm and 8 cm respectively. The point C is fixed on horizontal ground and the rod BC rotates in a vertical plane with the rod BC inclined at an angle θ to the ground.

(a) Show that the height, h cm, of A above the ground is given by $h = a\sin\theta - b\cos\theta$, where a and b are integers to be found. [2]



$BD = 2\cos\theta$

$BE = 8\sin\theta$ [M1 – both BD and BE]

$h = 8\sin\theta - 2\cos\theta$ [A1]

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(b) Using the values of a and b found in **part (a)**, express h in the form $R\sin(\theta-\alpha)$, where $R>0$ and $0\le\alpha\le\frac{\pi}{2}$. [4]

$8\sin\theta-2\cos\theta = R\sin(\theta-\alpha)$

$= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$R\cos\alpha = 8$ - eq (1)

$R\sin\alpha = 2$ - eq (2) [M1 – application of addition formula and forming 2 equations]

$R=\sqrt{8^2+2^2}$ [M1 – R]

$R=\sqrt{68}$

$R=2\sqrt{17}/8.25$

$\tan\alpha=\frac{2}{8}$ [M1 – $\tan\alpha$]

$\alpha=0.24498$ rad

Therefore $8\sin\theta-2\cos\theta = 2\sqrt{17}\sin(\theta-0.245)$ [A1]

(c) Hence, state the maximum value of h and find the corresponding value of θ. [3]

$h=2\sqrt{17}\sin(\theta-0.24498)$

$h_{max}=2\sqrt{17}/8.25$ cm [B1 – h_{max}]

At maximum,

$\sin(\theta-0.24498)=1$ [M1 – $\sin(\theta-0.24498)=1$]

$(\theta-0.24498)=\frac{\pi}{2}$

$\theta=1.82$ rad [A1]

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Name: _____ ( )

Class: _____

Questions	7	8	9	10	11	12
Marks						

Section B (49 marks)

7 The cubic polynomial $f(x)$ is such that the coefficient of x^3 is 1 and the roots of $f(x)=0$ are m, $2m$ and $(1-m)$, where $m>0$. It is given that $f(x)$ has a remainder of 30 when divided by $1-x$.

(a) Show that $2m^3-3m^2+m-30=0$. [3]

Given roots,

$f(x)=(x-m)(x-2m)(x-1+m)$ [M1]

$f(x)=(x^2-2mx-mx+2m^2)(x-1+m)$

$=(x^2-3mx+2m^2)(x-1+m)$

$=x^3-x^2+mx^2-3mx^2+3mx-3m^2x+2m^2x-2m^2+2m^3$ [M1 – expansion]

Given, $f(1)=30$,

$1-1+m-3m+3m-3m^2+2m^2-2m^2+2m^3=30$

$2m^3-3m^2+m-30=0$. (shown) [A1]

(b) Hence, find a value for m and show that there are no other real values of m which satisfy this equation. [4]

Let $g(m)=2m^3-3m^2+m-30$.

$g(3)=2(3)^3-3(3)^2+3-30=0$

Hence, $(m-3)$ is a factor of $g(m)$. [M1 – first factor]

$2m^3-3m^2+m-30=(m-3)(2m^2+km+10)$

Equating coefficient of m^2,

$-3=k-6$

$k=3$

Therefore,

$2m^3-3m^2+m-30=(m-3)(2m^2+3m+10)$

[M1 – quadratic factor $(2m^2+3m+10)$]

For $(2m^2+3m+10)$,

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Method 1

$b^2-4ac=3^2-4(2)(10)$ [M1 – discriminant]

$b^2-4ac=-71<0$

Since $b^2-4ac<0$, $2m^2+3m+10=0$ has no solution.

Method 2

$m=\dfrac{-3\pm\sqrt{3^2-4(2)(10)}}{2(2)}$ [OR M1 – quadratic formula]

$m=\dfrac{-3\pm\sqrt{-71}}{4}$

There is no solution.

Solving $2m^2+3m+10=0$,

$m=3$ is the only solution and hence $2m^2+3m+10=0$ has one real root, $m=3$. [A1]

8 It is given that $y=\dfrac{x-1}{\sqrt{1+x}}$.

(a) Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{x+3}{p\sqrt{(1+x)^3}}$, where p is a constant. [4]

$y = \dfrac{x-1}{\sqrt{1+x}}$

$\dfrac{dy}{dx} = \dfrac{\left(\sqrt{1+x}\right)(1)-(x-1)\left(\frac{1}{2}\right)(1+x)^{-\frac{1}{2}}}{\left(\sqrt{1+x}\right)^2}$ [M1 – application of quotient law for differentiation] [M1 – correct terms]

$= \dfrac{\frac{\sqrt{1+x}}{1}-\left(\frac{x-1}{2\sqrt{1+x}}\right)}{(1+x)}$

$= \dfrac{\frac{2(1+x)-(x-1)}{2\sqrt{1+x}}}{(1+x)}$ [M1 – Simplification]

$= \dfrac{2+2x-x+1}{2\sqrt{(1+x)^3}}$

$= \dfrac{x+3}{2\sqrt{(1+x)^3}}$ [A1]

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(b) Given that y is changing at a consant rate of 0.6 units per second, find the rate of change of x when $x=3$. [2]

$$\frac{dy}{dt} = \frac{dy}{dx}\times\frac{dx}{dt}$$

$$0.6 = \frac{x+3}{2\sqrt{(1+x)^3}}\times\frac{dx}{dt}$$ [M1 – connected rates of change]

$$\frac{dx}{dt} = 0.6\div\frac{x+3}{2\sqrt{(1+x)^3}}$$

$$= 0.6\times\frac{2\sqrt{(1+x)^3}}{x+3}$$

$$= \frac{6\sqrt{(1+x)^3}}{5(x+3)}$$

When $x=3$,

$$\frac{dx}{dt} = \frac{6\sqrt{(1+3)^3}}{5(3+3)}$$

$= 1.6$ units per second. [A1]

(c) Use the result from **part (a)** to evaluate $\int_0^3 \frac{x+3}{3\sqrt{(1+x)^3}}\,dx$. [4]

$$y = \frac{x-1}{\sqrt{1+x}}$$

$$\frac{dy}{dx} = \frac{x+3}{2\sqrt{(1+x)^3}}$$

$$\int_0^3 \frac{x+3}{2\sqrt{(1+x)^3}}\,dx = \left[\frac{x-1}{\sqrt{1+x}}\right]_0^3$$ [M1 – reverse of integration]

$$\int_0^3 \frac{x+3}{\sqrt{(1+x)^3}}\,dx = 2\left[\frac{x-1}{\sqrt{1+x}}\right]_0^3$$

$$3\int_0^3 \frac{x+3}{3\sqrt{(1+x)^3}}\,dx = 2\left[\frac{x-1}{\sqrt{1+x}}\right]_0^3$$

$$\int_0^3 \frac{x+3}{3\sqrt{(1+x)^3}}\,dx = \frac{2}{3}\left[\frac{x-1}{\sqrt{1+x}}\right]_0^3$$ [M1 – correct relationship]

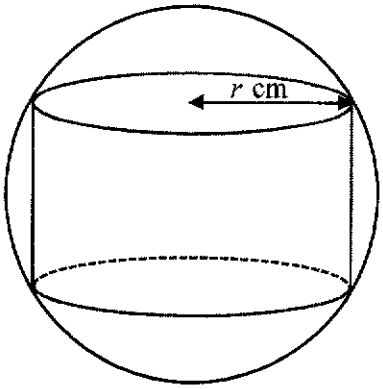
$$= \frac{2}{3}\left[\left(\frac{3-1}{\sqrt{1+3}}\right)-\frac{0-1}{\sqrt{1+0}}\right]$$ [M1 – application of limits]

$$= \frac{2}{3}[1+1]$$

$$= \frac{4}{3}$$ [A1]

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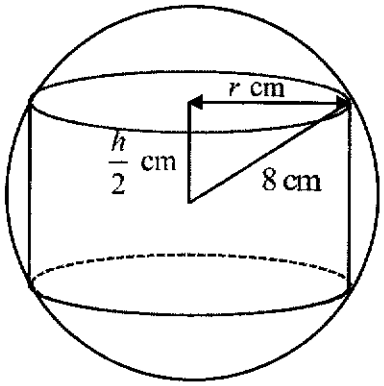


The diagram shows a cylinder of radius r cm inscribed in a sphere with a fixed internal radius of 8 cm.

(a) Given that the curved surface area of the cylinder is S cm^2, show that

$$\frac{dS}{dr} = \frac{8\pi(32 - r^2)}{\sqrt{64 - r^2}}.$$ [5]

Let the height of the cylinder be h.



By pythagoras theorem,

$\left(\frac{h}{2}\right)^2 + r^2 = 8^2$

$\frac{h^2}{4} + r^2 = 64$ [M1 – application of pythagoras theorem]

Because $h > 0$,

$h = \sqrt{4(64 - r^2)}$

$h = 2\sqrt{(64 - r^2)}$

$S = 2\pi rh$

$S = 4\pi r\sqrt{64 - r^2}$ [M1 – forming $S = 4\pi r\sqrt{64 - r^2}$]

$\frac{dS}{dr} = 4\pi\left[r\left(\frac{1}{2}\right)(64 - r^2)^{-\frac{1}{2}}(-2r) + (64 - r^2)^{\frac{1}{2}}\right]$ [M1 – differentiating using product rule]

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$$= 4\pi\left[\frac{-r^2}{\left(64-r^2\right)^{\frac{1}{2}}} + \frac{\left(64-r^2\right)^{\frac{1}{2}}}{1}\right]$$ [M1 – Simplification]

$$= 4\pi\left(\frac{-r^2+64-r^2}{\left(64-r^2\right)^{\frac{1}{2}}}\right)$$

$$= 4\pi\left(\frac{-2r^2+64}{\left(64-r^2\right)^{\frac{1}{2}}}\right)$$ [A1]

$$= \frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}}$$ (shown)

(b) Given that r varies, show that S has a stationary value when the height of the cylinder is equal to twice the radius of the cylinder. [3]

Method 1

For S to have a stationary value, $\frac{dS}{dr} = 0$.

$$\frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}} = 0$$ [M1 – $\frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}} = 0$]

$8\pi\left(32-r^2\right) = 0$

$r^2 = 32$

$r > 0$, hence $r = \sqrt{32} = 4\sqrt{2}$ [M1 – finding r]

When $r = \sqrt{32} = 4\sqrt{2}$.

$h = 2\sqrt{\left(64-\left(4\sqrt{2}\right)^2\right)}$

$h = 2\sqrt{32} = 8\sqrt{2}$

$h = 2r$ [A1]

Hence, the value of S has a stationary value when the height of the cylinder is equal to its diameter.

Method 2

Given $h = 2r$,

$2\sqrt{\left(64-r^2\right)} = 2r$

$\sqrt{\left(64-r^2\right)} = r$

$64-r^2 = r^2$

$2r^2 = 64$

$r^2 = 32$

$r = \sqrt{32}$ $(r > 0)$ [M1 – finding r]

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$$\frac{dS}{dr} = \frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}}$$

When $r=\sqrt{32}$,

$$\frac{dS}{dr} = \frac{8\pi\left(32-\sqrt{32}^2\right)}{\sqrt{64-\sqrt{32}^2}}$$ [M1 – subsitution of r]

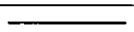
$$= 0$$ [A1]

Since $\frac{dS}{dr}=0$, S has a stationary value when the height of the cylinder is twice its radius.

(c) Determine whether this value of S is a maximum or a minimum. [2]

Method 1

$$\frac{dS}{dr} = \frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}}$$

x	$\sqrt{32}^-$	$\sqrt{32}$	$\sqrt{32}^+$
Sign of $\frac{dy}{dx}$	+	0	–
Sketch of tangent			
Sketch of curve			

[M1 – first derivative test]

Therefore, at $r=\sqrt{32}$, S is a maximum. [A1]

Method 2

$$\frac{dS}{dr} = \frac{8\pi\left(32-r^2\right)}{\sqrt{64-r^2}}$$

$$\frac{d^2S}{dr^2} = \frac{\left(64-r^2\right)^{\frac{1}{2}}(8\pi)(-2r)-8\pi\left(32-r^2\right)\left(\frac{1}{2}\right)\left(64-r^2\right)^{-\frac{1}{2}}(-2r)}{64-r^2}$$

[M1 – second derivative]

$$= \frac{-16\pi r\left(64-r^2\right)^{\frac{1}{2}}+8\pi r\left(32-r^2\right)\left(64-r^2\right)^{-\frac{1}{2}}}{64-r^2}$$

When $r=4\sqrt{2}$,

$$\frac{d^2S}{dr^2} = \frac{-16\pi\left(4\sqrt{2}\right)\left(64-\left(4\sqrt{2}\right)^2\right)^{\frac{1}{2}}+8\pi\left(4\sqrt{2}\right)\left(32-\left(4\sqrt{2}\right)^2\right)\left(64-\left(4\sqrt{2}\right)^2\right)^{-\frac{1}{2}}}{64-\left(4\sqrt{2}\right)^2}$$

$$\frac{d^2S}{dr^2} = \frac{-64\pi\sqrt{2}(32)^{\frac{1}{2}}-0}{32}$$

$$= \frac{-512\pi}{32}$$

$$= -50.3 < 0$$

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Since $\frac{d^2S}{dr^2} < 0$, S is maximum at $r = 4\sqrt{2}$. [A1]

10 The equation of a circle is $x^2 + y^2 + 4x - 6y - 12 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

Method 1

$(x-a)^2 + (y-b)^2 = r^2$

$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$

$-2a = 4 \qquad -2b = -6$

$a = -2 \qquad b = 3$ [B1 – a] [B1 – b]

Centre $(-2, 3)$

$a^2 + b^2 - r^2 = -12$

$(-2)^2 + (3)^2 - r^2 = -12$ [M1 – substitution]

$r = 5$ [A1 – r]

Method 2

$x^2 + y^2 + 2gx + 2fy + c = 0$

Centre is $(-g, -f)$

Given $x^2 + y^2 + 4x - 6y - 12 = 0$.

$2g = 4 \qquad 2f = -6$

$g = 2 \qquad f = -3$

Centre $(-2, 3)$ [B1 – a] [B1 – b]

$r = \sqrt{f^2 + g^2 - c}$

$r = \sqrt{(-3)^2 + (2)^2 - (-12)}$ [M1 – substitution]

$r = 5$ [A1 – r]

(b) Find the shortest distance of the centre of the circle to the line $y = 2x - 3$ and hence explain whether the circle intersects the line $y = 2x - 3$. [6]

The shortest distance is the line perpendicular to the line $y = 2x - 3$.

Let the equation of this line be $y = mx + c$.

$m = -\frac{1}{2}$ and Centre $(-2, 3)$ [M1 – gradient $m = -\frac{1}{2}$]

$y - 3 = -\frac{1}{2}(x + 2)$

$y = -\frac{1}{2}x + 2$ [M1 – equation of perpendicular to line]

To find the point of intersection D,

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$2x-3=-\frac{1}{2}x+2$ [M1 – simultaneous equataion]

$\frac{5}{2}x=5$

$x=2$

$y=2(2)-3$

$y=1$

$D(2,1)$ [B1 – coordinates of D]

Distance between Centre and D

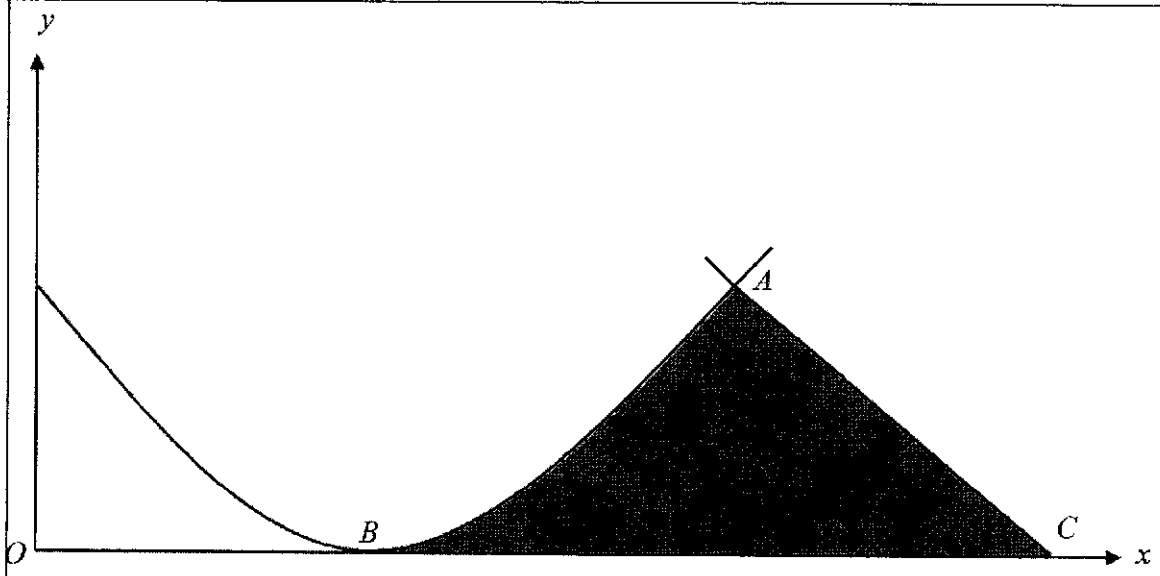
$=\sqrt{(2+2)^2+(1-3)^2}$ [M1 – line segment]

$=\sqrt{20}$

$=2\sqrt{5}/4.47$

Since $2\sqrt{5}/4.47<5$, the shortest distance is shorter than the radius of the circle, the circle intersects the line $y=2x-3$. [A1]

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The diagram shows part of the curve $y=1-\sin x$ passing through the point A. The curve touches the x-axis at the point B.

The gradient of the tangent to the curve at 2m is 1 and the normal to the curve at A meet the x-axis at C.

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Show that the area of the shaded region is $\frac{1}{2}(\pi-1)$ units^2. [12]

Let coordinates of B be $(b,0)$.

$1-\sin x=0$

$\sin x=1$

$b=x=\frac{\pi}{2}$

$B\left(\frac{\pi}{2},0\right)$ [B1 – coordinates of B]

Let coordinates of C be (x,y).

$\frac{dy}{dx}=-\cos x$ [M1 – $\frac{dy}{dx}$]

$-\cos x=1$

$\cos x=-1$

$x=\pi$

When $x=\pi$,

$y=1-\sin\pi$

$y=1$

$A(\pi,1)$. [B1 – coordinates of A]

Let equation of AC be $y=mx+c$

$m_{AC}=-1$ [M1 – gradient of normal, m_{AC}]

$A(\pi,1)$

$y-1=-(x-\pi)$

$y=-x+1+\pi$ [M1 – equation of line AC]

Let coordinates of C be $(c,0)$.

$-x+1+\pi=0$

$x=1+\pi$

$C(1+\pi,0)$ [B1 – coordinates of C]

Area of shaded region

$=\int_{\frac{\pi}{2}}^{\pi}(1-\sin x)\,dx+\frac{1}{2}\times[(1+\pi)-\pi]\times1$ [M1 – area under curve, $\int_{\frac{\pi}{2}}^{\pi}(1-\sin x)\,dx$]

$=\left[x-(-\cos x)\right]_{\frac{\pi}{2}}^{\pi}+\frac{1}{2}$ [M1 – area of triangle $\frac{1}{2}\times[(1+\pi)-\pi]\times1$]

$=\left[x+\cos x\right]_{\frac{\pi}{2}}^{\pi}+\frac{1}{2}$ [M1 – integration $\left[x+\cos x\right]_{\frac{\pi}{2}}^{\pi}$]

$=\left[(\pi+\cos\pi)-\left(\frac{\pi}{2}+\cos\frac{\pi}{2}\right)\right]+\frac{1}{2}$ [M1 – substitution of limits]

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	$=\left[\pi-1-\frac{\pi}{2}-0\right]+\frac{1}{2}$	[M1 – correct evaluation, including area of triangle]
	$=\frac{\pi}{2}-\frac{1}{2}$	
	$=\frac{1}{2}(\pi-1)$	[A1]

END OF PAPER

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