Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

19 August 2024 Monday 2 hours 15 min

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2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	*Marks Deducted*
Marks													

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

Setter: Ms Sabrina Tan
Vetter: Mr Tan Lip Sing

This question paper consists of **19** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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1 Express $\dfrac{3x^2+14x+6}{x^2(x+3)}$ as the sum of 3 partial fractions. [5]

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2 **(a)** State, in terms of π,

(i) the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$, [1]

(ii) the values between which the principal value of $\sin^{-1}x$ must lie. [1]

(b) Given that A is a reflex angle and $\cos A = -\frac{4}{5}$, find the exact value of $\cos(A + 30°)$ without the use of a calculator. [3]

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3 **(a)** Find the set of values of the constant k for which the curve $y = -kx^2 - 2x + 2k - 3$ does not intersect the x-axis. [3]

(b) Using the answer in **part (a)**, explain whether it is possible for $-kx^2 - 2x + 2k - 3$ to be positive for all x. [2]

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4 A roller coaster is being designed such that the height, h m, of a rider above the ground in a section of the roller coaster ride is given by $h = 10x - 2x^2 - 4$, where x is the horizontal distance of the rider from the starting point and $1 \leq x \leq 4$.

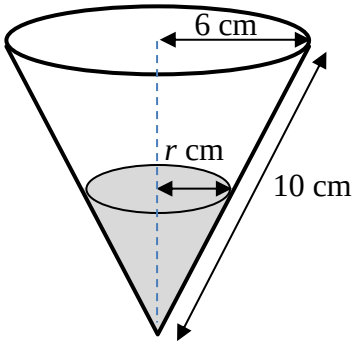
(a) Express h in the form $a + b(x + c)^2$ where a, b and c are constants to be determined. [3]

(b) Hence, explain why the rider cannot reach a height of 10 m. [1]

(c) After testing the prototype, the roller coaster designer wants to make the ride more exciting by moving the highest point of this section of the roller coaster ride up by 0.2 m and left by 0.1 m. Write down a possible new expression for h. [1]

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5 Water is poured into an empty inverted conical container with radius 6 cm and slant height 10 cm. After t seconds, the radius of the top surface of the water is r cm.



(a) Show that the surface area, S, of water in contact with the container at any time is given by $S = \frac{5}{3}\pi r^2$.

[The curved surface area of a cone of base radius r and slant height l is πrl.] [2]

(b) Water is poured into the container such that r increases at a constant rate. Given that it takes 30 seconds to completely fill up the empty container, write down the rate at which r increases. [1]

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(c) Hence calculate the rate at which S increases when the container is one-eighth filled. [4]

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6 A curve has equation $y = \dfrac{30(2x-1)^3}{e^{3x}}$, where $x > 0$.

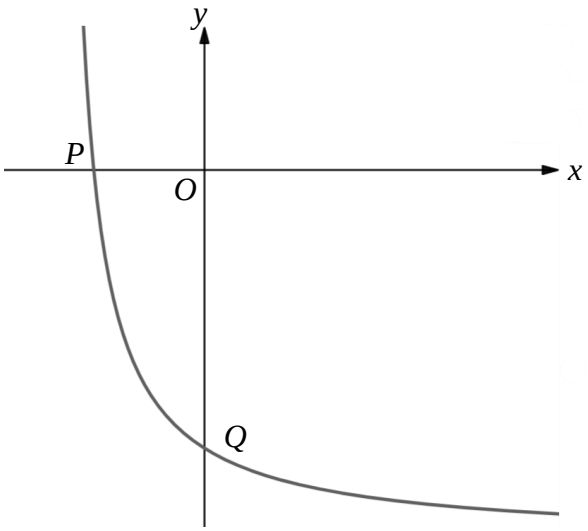
(a) Find the x-coordinates of the stationary points of the curve. [5]

(b) Determine the nature of each of the stationary points. [3]

(c) Using your answer in **part (b)**, infer and write down the set of values of x for which y is an increasing function. [2]

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7 The diagram shows part of the curve $y = \frac{40}{x+8} - 20$, where $x > -8$. The curve intersects the x-axis and y-axis at P and Q respectively. The tangent to the curve at R is parallel to line PQ.



(a) Show that the coordinates of point R are $(-4, -10)$. [5]

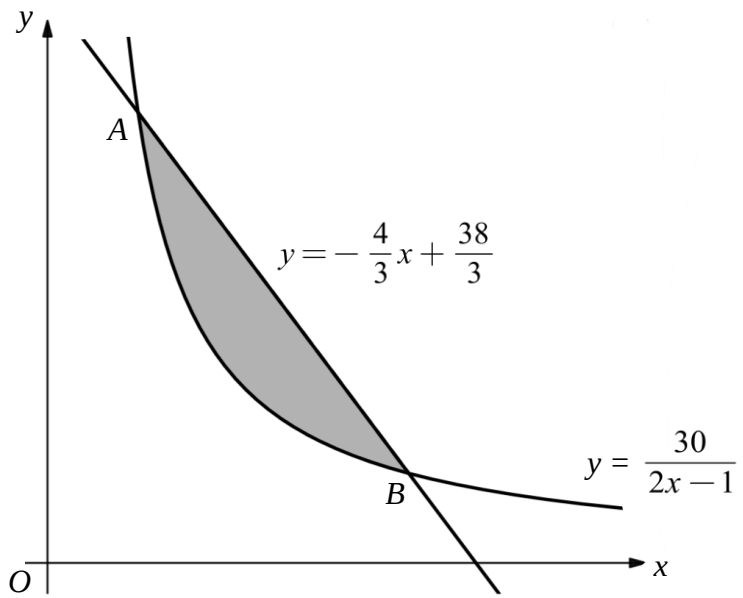
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(b) Given that the point S has coordinates $(-2, -5)$, find the area of the quadrilateral $PRQS$. [2]

(c) Determine, with reason, whether $PRQS$ is a parallelogram. [3]

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8 The diagram shows part of the curve $y = \frac{30}{2x-1}$ and the line $y = -\frac{4}{3}x + \frac{38}{3}$. The curve intersects the line at the points A and B.



(a) Find the coordinates of the points A and B. [3]

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(b) Find the area of the shaded region. [4]

(c) Show that the area bounded by the curve, the y-axis and the lines $y = 2$ and $y = 6$ can be expressed in the form $15\ln p + q$, where p and q are constants to be determined. [3]

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9 The function f is given by $\mathrm{f}(x) = x^3 - x^2 + ax - 1$.

It is given that $\frac{\mathrm{f}(x)}{x-1} = \mathrm{Q}(x) + \frac{3}{x-1}$, where $\mathrm{Q}(x)$ is a quadratic function.

(a) Explain how you can use Remainder Theorem to show that $a = 4$. [3]

(b) By using long division, divide $\mathrm{f}(x)$ by $x^2 + 1$.

Hence, express $\frac{\mathrm{f}(x)}{x^2+1}$ in the form $px + q + \frac{rx}{x^2+1}$, where p, q and r are constants to be determined. [2]

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(c) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln(x^2+1)$. [1]

(d) Hence, using your results from **part (b) and (c)**, find $\int \frac{\mathrm{f}(x)}{x^2+1}\,\mathrm{d}x$. [2]

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10 The equation of a circle, C_1, with centre A, is $x^2 + y^2 - 6x + 8y - 75 = 0$.

(a) Find the radius of the circle and the coordinates of its centre, A. [4]

(b) A second circle , C_2, with centre B, has radius 12 units.

The equation of the perpendicular bisector of AB is $y = -\frac{1}{2}x + 5$.

Find the equation of the second circle, C_2. [5]

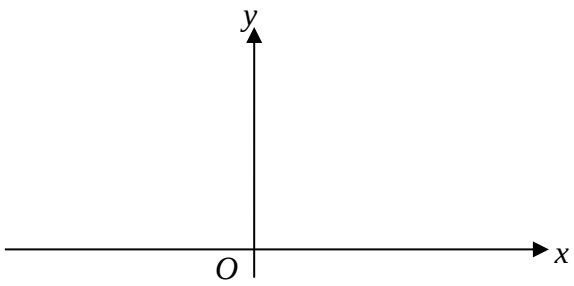
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(c) P is a point on the perpendicular bisector of AB, where P is **not** the midpoint of A and B.

Explain whether triangle PAB is an isosceles triangle. [1]

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11 **(a)** Sketch the graph of $y = \mathrm{e}^{-x}$ on the given axes. Label any axial intercepts. [1]



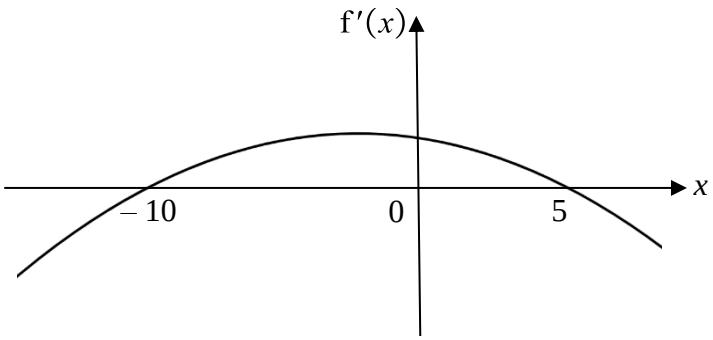
(b) The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = \mathrm{e}^{2x+1} + \mathrm{e}^{-x}$.

(i) Explain why the curve has no stationary points. [2]

(ii) The curve passes through the point (ln 4, 8e). Without the use of a calculator, find an expression for $\mathrm{f}(x)$. Show all working clearly. [6]

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12 The diagram shows the graph of $y = \mathbf{f}'(x)$.



(a) Write down the range of values of *x* for which the **function f** decreases as *x* increases. [1]

(b) The above function f is given by $\mathrm{f}(x) = 12k^2x - 2x^3 - 3kx^2$, where *k* is a positive constant. By solving a suitable inequality, use the answer in **part (a)** to find the value of the constant *k*. [4]

(c) Explain how you can use the above graph of $y = \mathrm{f}'(x)$ to determine the value of *x* for which f(*x*) is a maximum value. [1]

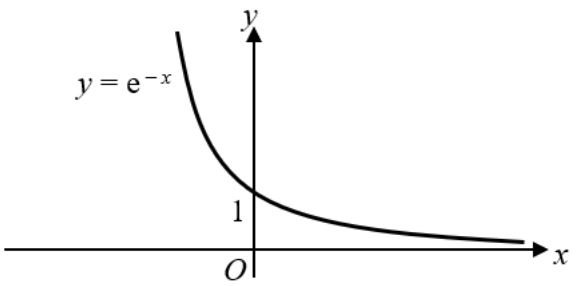
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ANSWER KEY

1	$\frac{4}{x}+\frac{2}{x^2}-\frac{1}{x+3}$
2(a)(i)	$-\frac{\pi}{3}$
2(a)(ii)	$-\frac{\pi}{2}\leqslant \sin^{-1}x\leqslant\frac{\pi}{2}$
2(b)	$\frac{3}{10}-\frac{2}{5}\sqrt{3}$
3(a)	$\frac{1}{2}<k<1$
3(b)	Since $\frac{1}{2}<k<1$, **coefficient of x^2 = – k < 0.** The curve lies entirely below the axis, therefore it is **not possible** for $-kx^2-2x+2p-3$ to be positive for all x.
4(a)	$8.5-2\left(x-\frac{5}{2}\right)^2$
4(b)	Since the **<u>maximum value of the rider is 8.5 m</u>** and 8.5 < 10, the rider cannot reach a height of 10 m.
4(c)	$h=9.7-2(x-2.4)^2$
5(b)	$\frac{1}{5}$ cm/s
5(c)	2π cm/s (or 6.28 cm/s to 3 s.f.)
6(a)	$x=\frac{1}{2}$ or $x=\frac{3}{2}$
6(b)	Stationary point at $x=\frac{1}{2}$ is a **point of inflexion** and stationary point at $x=\frac{3}{2}$ is a **maximum point**.
6(c)	$0<x<\frac{3}{2},\ x\neq\frac{1}{2}$
7(b)	30 units^2
7(c)	Since $m_{PR}=m_{QS}$ and $m_{RQ}=m_{PS}$, (or any other suitable reason) $PRQS$ is a parallelogram.
8(a)	$A(2, 10)$ and $B(8, 2)$
8(b)	11.9 units^2
8(c)	15ln3 + 2
9(a)	Since the **remainder is 3** when f(x) is divided by x – 1, we let f(1) = 3 1 – 1 + a – 1 = 3 a = 4 (shown)

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9(b)	$x-1+\dfrac{3x}{x^2+1}$
9(c)	$\dfrac{2x}{x^2+1}$
9(d)	$\dfrac{x^2}{2}-x+\dfrac{3}{2}\ln(x^2+1)+c$
10(a)	$A\,(3,-4)$ and radius = 10 units
10(b)	$(x-9)^2+(y-8)^2=144$
10(c)	Since ***P* is equidistant from *A* and *B***, i.e. $PA = PB$, the triangle is isosceles.
11(a)	
11(b)(i)	Since $e^{2x+1} > 0$ and $e^{-x} > 0$ for all real values of x, $f'(x) \neq 0$, therefore the curve has no stationary points.
11(b)(ii)	$f(x)=\dfrac{e^{2x+1}}{2}-e^{-x}+\dfrac{1}{4}$
12(a)	$x < -10$ or $x > 5$
12(b)	$k = 5$
12(c)	From the graph, $f'(x)$ changes from positive to negative at $x = 5$, therefore $f(x)$ is a maximum value when $x = 5$. [1st derivative test] Or We look for the point on the graph where $f'(x) = 0$ and gradient is negative (i.e. $f''(x) < 0$), therefore $f(x)$ is a maximum value when $x = 5$. [2nd derivative test]

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