| NAME | | INDEX NO. | | CLASS | |
|---|---|---|---|---|---|

# NORTHLAND SECONDARY SCHOOL
## PRELIMINARY EXAMINATION
## Secondary 4 Express / 5 Normal Academic

# ADDITIONAL MATHEMATICS

**4049/02**

Paper 2

**28 August 2024**

**2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

| For Examiner's Use |
|---|
| 90 |

This document consists of **19** printed pages and **1** blank page.

**[Turn over**

Downloaded from SingaKids.com

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Downloaded from SingaKids.com

1 Express $\frac{1-8x-3x^2}{(x-1)(2x^2+3)}$ in partial fractions. [5]



Downloaded from SingaKids.com

2 **(a)** Find the smallest integer $c$ for which the line $y = 7x + c$ intersects the curve $y = 2x^2 - 4$ at two distinct points. [4]

Downloaded from SingaKids.com

**(b)** Find the range of values of $k$ for which the curve $y = kx^2 + 2(2k-5)x + 9k$ lies below the $x$-axis. [4]



Downloaded from SingaKids.com

3 (a) Prove the identity $\dfrac{1-\cos x}{\sin x - \operatorname{cosec} x + \cot x} = \tan x$. [4]

Downloaded from SingaKids.com

**(b)** Hence solve the equation $\dfrac{1-\cos 2x}{\sin 2x - \operatorname{cosec} 2x + \cot 2x} = -3$ for $0° < x < 180°$. [3]

**(c)** Show that there are no solutions to the equation $\dfrac{1-\cos x}{\sin x - \operatorname{cosec} x + \cot x} = \tan 2x$ for $0° < x < 180°$. [2]



Downloaded from SingaKids.com

4 **(a)** The equation $\log_2 x + \log_8 x = \log_4 2$ has the solution $x = 2^m$. Find the value of $m$. [4]

**(b)** Sketch the graph of $y = \log_3 x$. [2]

Downloaded from SingaKids.com

**(c)** Explain why the equation $\log_5(2x-11)-\log_5(x-4)=1$ has no real solutions. [4]

**5** **(a)** The function f is defined by $f(x)=e^{x^2+x}$ where $x>0$. Explain, with working, whether f is an increasing or a decreasing function. [3]



Downloaded from SingaKids.com

**(b)** The equation of a curve is $y = \frac{x^2}{x+3}$.

**(i)** Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

Downloaded from SingaKids.com

**(ii)** Find an expression for $\frac{d^2y}{dx^2}$ and hence determine the nature of these stationary points. [3]



Downloaded from SingaKids.com

6 The points $P$ and $Q$ both lie on a circle and have coordinates $(2, 7)$ and $(-6, 1)$ respectively. The line with equation $y+2x+4=0$ is a normal to the circle.

**(a)** Find the equation of the perpendicular bisector of $PQ$. [4]

Downloaded from SingaKids.com

**(b)** Find the equation of the circle. [5]

**(c)** Find the exact value of the coordinates of the point on the circle which is furthest from the $y$-axis. [2]



Downloaded from SingaKids.com

7 A particle moves in a straight line, such that its velocity, $v$ m/s, $t$ seconds after passing a fixed point $O$, is given by $v = -0.3(4-t)^2 + 1.2$.

**(a)** Find the acceleration of the particle when it first comes to instantaneous rest. [4]

After 1 second, its displacement, $s$ m, is 2.5 m from $O$.

**(b)** Find the initial displacement of the particle. [3]

Downloaded from SingaKids.com

**(c)** Explain clearly why the total distance travelled by the particle in the interval $t = 0$ to $t = 7$ is **not** obtained by finding the value of $s$ when $t = 7$. [2]

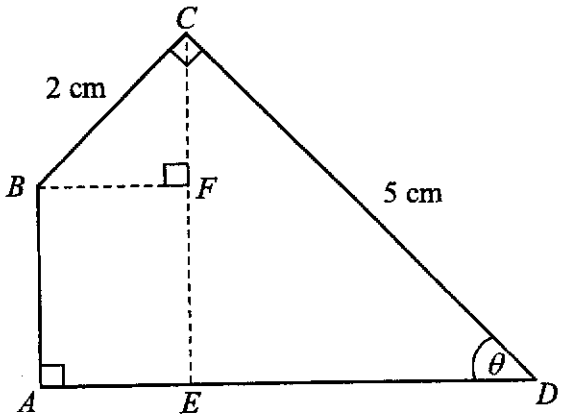
**(d)** Find the total distance travelled by the particle in the interval $t = 0$ to $t = 7$. [3]



Downloaded from SingaKids.com

**8** **(a)** On the same axes, sketch, for $0 \le x \le 2\pi$, the graphs of $y = \sin 2x$ and $y = 1 - 3\cos x$. [3]

**(b)**




The diagram shows a quadrilateral in which $CFE$ and $AED$ are straight lines. $BC = 2$ cm and $CD = 5$ cm. Angle $BCD$ = angle $BFC$ = angle $BAE = 90°$. Angle $CDE = \theta$, where $\theta$ is an angle in degree.

**(i)** Show that the perimeter of the quadrilateral is $3\cos\theta + 7\sin\theta + 7$. [3]

Downloaded from SingaKids.com

**(ii)** Express $3\cos\theta + 7\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $\alpha$ is acute. [3]

The perimeter of the quadrilateral is 13 cm.

**(iii)** Find the angle $\theta$ and hence find the total area of the quadrilateral. [3]



Downloaded from SingaKids.com

9 **(a)** Express $\frac{3x}{3x+1}$ in the form $a+\frac{b}{3x+1}$ where $a$ and $b$ are constants, and hence find $\int \frac{3x}{3x+1}\,dx$.

[4]

**(b)** Given that $y = x\ln(3x+1)$, find an expression for $\frac{dy}{dx}$. [3]

Downloaded from SingaKids.com

**(c)** Using the results from parts **(a)** and **(b)**, show that $\int_0^4 \ln(3x+1)\,dx = a\ln 13 + b$, where $a$ and $b$ are constants to be found. [5]



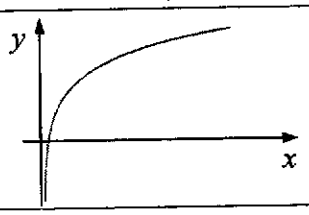
Downloaded from SingaKids.com

**BLANK PAGE**

Downloaded from SingaKids.com

| Mark Scheme for 2024 S4E5N Add Math Prelim Paper 2 | | | | |
|---|---|---|---|---|
| 1 | $\frac{1-8x-3x^2}{(x-1)(2x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{2x^2+3}$ | M1 | | Realising the form of the partial fractions |
| | $1-8x-3x^2 = A(2x^2+3)+(x-1)(Bx+C)$ | M1 | | Realising the need to eliminate denominator |
| | When $x=1$, $1-8(1)-3(1)^2 = A[2(1)^2+3]$ | | | |
| | $-10=5A \rightarrow A=-2$ | | | |
| | When $x=0$, $1=-2[2(0)^2+3]+(0-1)(C)$ | | | |
| | $1=-6-C \rightarrow C=-7$ | | | |
| | When $x=2$, $1-8(2)-3(2)^2=-2[2(2)^2+3]+[2B-7]$ | | | |
| | $-27=-22+2B-7 \rightarrow B=1$ | | | |
| | $\frac{-2}{x-1}+\frac{x-7}{2x^2+3}$ | A3, 2, 1 | | −1 for each error inc. final answer |
| | | | [5] | |
| 2a | $2x^2-7x-4-c=0$ | M1 | | Eliminates $y$ or $x$ |
| | $b^2-4ac=(-7)^2-4(2)(-4-c)$ | M1 | | Uses the discriminant |
| | $49+32+8c>0$ | | | |
| | $c>-10.125$ | A1 | | Accept $c>-\frac{81}{8}$ |
| | smallest integer $c=-10$ | A1 | | |
| 2b | $y=kx^2+2(2k-5)x+9k$ | | | |
| | $b^2-4ac=(4k-10)^2-4(k)(9k)$ | M1 | | Uses the discriminant |
| | $16k^2-80k+100-36k^2<0$ | | | |
| | $-20k^2-80k+100<0 \rightarrow k^2+4k-5>0$ | | | |
| | $(k+5)(k-1)>0$ | | | |
| | $k<-5$ or $k>1$ | A2 | | A1 for each |
| | $k<0 \rightarrow k<-5$ | B1 | | s.o.i. when $k>1$ rejected |
| | | | [8] | |
| 3a | $\frac{1-\cos x}{\sin x-\operatorname{cosec} x+\cot x}$ | | | |
| | $\frac{1-c}{s-\frac{1}{s}+\frac{1}{t}}$ | B1 | | cosec and cot all correct |
| | $\frac{1-c}{\frac{s^2-1+c}{s}} \rightarrow \frac{s(1-c)}{s^2-1+c}$ | M1 | | Correct algebra |
| | $\frac{s(1-c)}{-c^2+c}$ | M1 | | Uses $s^2+c^2=1$ |
| | $\frac{s(1-c)}{c(1-c)}=t$ | A1 | | All correct |
| 3b | $\tan 2x=-3$ | M1 | | Uses part (a) and replaces $x$ with $2x$ |
| | $\alpha=\tan^{-1}(3)$ | M1 | | For finding basic angle |
| | $2x=180°-\alpha,\ 360°-\alpha$ | | | |
| | $x=54.2°, 144.2°$ | A1 | | Accept 54.21… and 144.21… |
| 3c | $t=\frac{2t}{1-t^2}$ | | | |

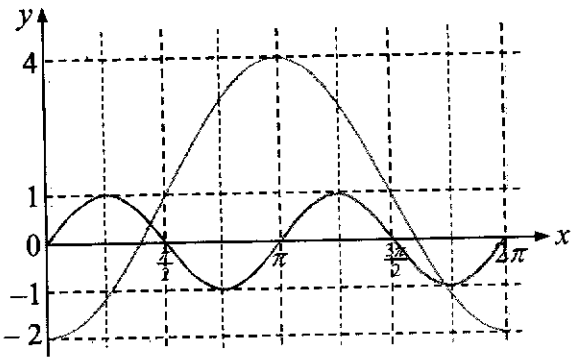
Downloaded from SingaKids.com

| | | | | |
|---|---|---|---|---|
| | $t-t^3=2t$ | | | |
| | $t^3+t=0 \rightarrow t(t^2+1)=0$ | | | |
| | $t=0$ (not valid for $0°<x<180°$) | B1 | | Must include and then reject $t=0$ |
| | or $t^2=-1$ not possible. No solution | B1 | | Realises there is no solution to $t^2=-1$ |
| | **Alternative Answer:** | | | |
| | Correct sketch of $y=\tan x$ and $y=\tan 2x$ | B1 | | |
| | No point of intersection for $0°<x<180°$ and so no solution to the equation $\tan x=\tan 2x$ | B1 | | |
| | | | **[9]** | |
| **4a** | $\log_2 x+\frac{\log_2 x}{\log_2 8}=\frac{\log_2 2}{\log_2 4}$ | M1 | | Change of base |
| | $\log_2 x+\frac{\log_2 x}{3}=\frac{1}{2}$ | B1 | | For '$=\frac{1}{2}$' |
| | $\log_2 x+\frac{1}{3}\log_2 x=\frac{1}{2} \rightarrow \log_2 x=\frac{3}{8}$ | M1 | | Making $\log_2 x$ the subject |
| | $x=2^{\frac{3}{8}} \rightarrow m=\frac{3}{8}$ | A1 | | |
| **4b** |   | B2 | | B1 for curvature of decreasing gradient observed<br>B1 for graph close to asymptote observed with $x$-intercept 1 |
| **4c** | $\log_5(\frac{2x-11}{x-4})=1$ | M1 | | Combine to single log |
| | $\frac{2x-11}{x-4}=5$ | | | |
| | $2x-11=5(x-4) \rightarrow x=3$ | A1 | | |
| | If $x=3$, $\log_5(2x-11)=\log_5(-5)$ or $\log_5(x-4)=\log_5(-1)$ | M1 | | Substitute into log expression |
| | Does not exist $\rightarrow$ No solutions | A1 | | Correct argument and conclusion |
| | | | **[10]** | |
| **5a** | $f'(x)=e^{x^2+x}(2x+1)$ | B1 | | |
| | Since $x>0, 2x+1>0$ and $e^{x^2+x}>0$ | M1 | | Argues correctly |
| | $f'(x)>0 \rightarrow$ increasing | A1 | | $f'(x)>0$ or $e^{x^2+x}(2x+1)>0$ must be seen |
| **5bi** | $\frac{(x+3)(2x)-x^2(1)}{(x+3)^2}=\frac{x^2+6x}{(x+3)^2}$ | B2, 1 | | B1 for unsimplified |
| | $\frac{x^2+6x}{(x+3)^2}=0 \rightarrow x=0,\ x=-6$ | M1 | | Sets to 0 and solves |
| | (0, 0) and (−6, −12) | A1 A1 | | SR1 answers not in coordinate form |
| **5bii** | $\frac{18}{(x+3)^3}$ | B1 | | |
| | $x=0 \rightarrow \frac{d^2y}{dx^2}=\frac{2}{3}>0 \rightarrow$ minimum point | DB1 | | Correct $\frac{d^2y}{dx^2}$ value and conclusion |

Downloaded from SingaKids.com

| | | | | |
|---|---|---|---|---|
| | $x=-6 \rightarrow \frac{d^2y}{dx^2}=-\frac{2}{3}<0 \rightarrow$ maximum point | DB1 | | Correct $\frac{d^2y}{dx^2}$ value and conclusion |
| | | | [11] | |
| 6a | $m_{PQ}=\frac{7-1}{2-(-6)}=\frac{3}{4}$ | B1 | | |
| | $M_{PQ}=(\frac{2+(-6)}{2},\frac{7+1}{2})=(-2,\ 4)$ | B1 | | |
| | Gradient of Perpendicular $=-\frac{4}{3}$ | M1 | | Uses $m_1m_2=-1$ |
| | $y-4=-\frac{4}{3}(x-(-2))$ | | | |
| | $y=-\frac{4}{3}x+\frac{4}{3}$ | A1 | | |
| 6b | $-\frac{4}{3}x+\frac{4}{3}=-2x-4$ | M1 | | Realises the need to use sim eqns |
| | $x=-8$ | | | |
| | $(-8,12)$ | A1 | | |
| | $\sqrt{(-8-2)^2+(12-7)^2}$ | M1 | | Uses distance formula correctly with $P$ or $Q$ |
| | $\sqrt{125}$ | A1 | | |
| | $(x+8)^2+(y-12)^2=125$ | B1√ | | √ for their radius and centre |
| 6c | $(-8-\sqrt{125},12)$ | B2√ | | B1 for each, √ for their radius and centre |
| | | | [11] | |
| 7a | $v=-0.3(4-t)^2+1.2$ | | | |
| | $a=0.6(4-t)$ | B1 | | |
| | When $v=0 \rightarrow (4-t)^2=4$ | M1 | | Sets to 0 |
| | $t=2,\ t=6$ | A1 | | |
| | $a=0.6(4-2)=1.2\ \text{m/s}^2$ | A1 | | |
| 7b | $s=\int -0.3(4-t)^2+1.2\ dt$ | M1 | | Realises need to integrate |
| | $s=0.1(4-t)^3+1.2t\ \ (+c)$ | A1 | | |
| | When $t=1,\ s=2.5,\ c=-1.4 \rightarrow$ $s=0.1(4-t)^3+1.2t-1.4$ | | | Use $t=1$ and $s=3.9$ to find $c$ |
| | When $t=0, s=5\text{m}$ | A1 | | |
| 7c | Particle turned during $t=0$ to $t=7$ | B1 | | |
| | When $t=7$, it only gives distance from $O$ | B1 | | |
| 7d | When $t=2,\ s=1.8$ or When $t=6,\ s=5$ OR distance $=\left\|\int_0^2 v\ dt\right\|=3.2\text{m}$ or $\int_2^6 v\ dt=3.2\text{m}$ | M1 | | For finding displacement at either time it turns |
| | When $t=7,\ s=4.3$ OR distance $=\left\|\int_6^7 v\ dt\right\|=0.7\text{m}$ | M1 | | For finding displacement at ending time |
| | distance $=(5-1.8)+(5-1.8)+(5-4.3)=7.1\text{m}$ | A1 | | |
| | | | [12] | |

Downloaded from SingaKids.com

| | | | | |
|---|---|---|---|---|
| **8a** |   | B3 | | B1 for two complete sine cycles with correct amplitude and intersections at $x$-axis<br><br>B1 for one negative cosine cycle<br><br>B1 for cosine cycle with correct amplitude and intersections at axis of rotation |
| **8bi** | $AB = CE - CF = 5\sin\theta - 2\cos\theta$ | B1 | | |
| | $AD = ED + BF = 5\cos\theta + 2\sin\theta$ | B1 | | |
| | Perimeter $= 5\sin\theta - 2\cos\theta + 5\cos\theta + 2\sin\theta + 7$<br>$\rightarrow 3\cos\theta + 7\sin\theta + 7$ | B1 | | Answer was given – so all working must be correct |
| **8bii** | $R = \sqrt{3^2 + 7^2} = \sqrt{58}$ | B1 | | |
| | $\tan\alpha = \frac{7}{3} \rightarrow \alpha = 66.8°$ | M1 | | For finding $\alpha$ |
| | $\sqrt{58}\cos(\theta - 66.8°)$ | A1 | | |
| **8biii** | $\sqrt{58}\cos(\theta - 66.8°) + 7 = 13$ | | | |
| | $\cos(\theta - 66.8°) = \frac{6}{\sqrt{58}}$ | | | |
| | $\alpha = 38.0°$ | M1 | | For finding $\alpha$ |
| | $\theta - 66.8° = -\alpha,\ \alpha$ | | | |
| | $\theta = 28.8°,\ 104.8°$ (rejected) | A1 | | Accept 28.78… |
| | 6.75 cm$^2$ | A1 | | |
| | | | [12] | |
| **9a** | $\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$ | B1 | | |
| | $\int \frac{3x}{3x+1}\,dx = x - \frac{1}{3}\ln(3x+1) + c$ | B3, 2, 1 | | −1 for each error in the integration – needs +c |
| **9b** | $\frac{dy}{dx} = \ln(3x+1) + x \times \frac{1}{3x+1} \times 3$ | M1 B1 A1 | | Uses product formula. B1 '$\frac{1}{3x+1}$'. A1 '×3' |
| | $= \ln(3x+1) + \frac{3x}{3x+1}$ | | | |
| **9c** | $\int \ln(3x+1) + \frac{3x}{3x+1}\,dx - \int \frac{3x}{3x+1}\,dx$<br>$\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$<br><br>OR<br><br>$\ln(3x+1) = \frac{dy}{dx} - \frac{3x}{3x+1}$<br>$\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$ | M1 | | Attempts to use the result of part (b) |

Downloaded from SingaKids.com

| | | | | |
|---|---|---|---|---|
| | $x\ln(3x+1)-\int\frac{3x}{3x+1}\,dx$<br>$\rightarrow x\ln(3x+1)-\int 1-\frac{1}{3x+1}\,dx$ | M1 | | Realises the need to use part (a) |
| | $[x\ln(3x+1)]_0^4-\left[x-\frac{1}{3}\ln(3x+1)\right]_0^4$ | A1 | | All correct |
| | $[4\ln 13-0]-\left[4-\frac{1}{3}\ln 13\right]$ | M1 | | Use definite integral formula on $x\ln(3x+1)$ or antiderivative from part (a) |
| | $\frac{13}{3}\ln 13-4$ | A1 | | For both values |
| | | | [12] | |

Downloaded from SingaKids.com

Downloaded from SingaKids.com