Kent Ridge Secondary School
Secondary 4 Express/5 Normal Academic Preliminary Examination 2024
Add Math Prelim 2024 P1 Mark scheme

Qn	Solutions	Marks
1a	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $1 = \frac{2\tan A}{1-\tan^2 A}$ $1-\tan^2 A = 2\tan A$ $\tan^2 A + 2\tan A - 1 = 0$ $\tan A = \frac{-2 \pm \sqrt{4+4}}{2}$ $= -1+\sqrt{2}$	M1 A1 M1 A1
1b	$\sec^2 A = 1 + \tan^2 A$ $= 1 + \left(-1+\sqrt{2}\right)^2$ $= 1+1-2\sqrt{2}+2$ $= 4-2\sqrt{2}$	M1 A1
2a	Gradient $L_1 = -2$ Gradient $L_2 = \frac{1}{2}$ $-3 = \frac{1}{2}(2) + c$ $c = -4$ $B(0,-4)$	M1 M1 A1
2b	$(4-2)^2 + (k+3)^2 = 25$ $(k+3)^2 = 21$ $k = \pm\sqrt{21} - 3$	M1 M1 or apply quad formula to general eqn A1
3a	Min -2 and max 2	B1
3b	Min -3 + 1 = -2, max = 3+1 = 4	B1
3c	f(x) period or shape correct g(x) period or shape correct f(x) fully correct g(x) fully correct	B1 B1 B1 B1
3d	3	B1
4a	$(a-x^2)^6 = a^6 - 6a^5x^2 + \binom{6}{2}a^4x^4 + \ldots$ $= a^6 - 6a^5x^2 + 15a^4x^4 + \ldots$	B1,B1,B1
4b	$\left(\frac{1}{x^2} + 2 + x^2\right)(a-x^2)^6$ $\left(\frac{1}{x^2} + 2 + x^2\right)(a^6 - 6a^5x^2 + 15a^4x^4)$ Term independent of x: $-6a^5 + 2a^6 = 0$ $-6 + 2a = 0$ $a = 3$ Term in x^2: $b = 15a^4 - 12a^5 + a^6 = -972$	M1 M1 A1 M1,A1
5a	$f'(x) = \frac{(x-3)-(x+1)}{(x-3)^2}$	M1

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Qn	Solutions	Marks
	$$f'(x) = \frac{-4}{(x-3)^2}$$ Since $(x-3)^2 > 0, x \neq 3$ **$f'(x)$ is always negative or** **Gradient of f is always negative**, so it is a decreasing function	A1 B1 B1
5b	$$f(x) = 1 + \frac{4}{x-3}$$ $$\int 1 + \frac{4}{x-3}\, dx = x + 4\ln(x-3) + c$$	M1 M1, M1, A1
6a	75℃	B1
6b	$75e^{-0.02t} = 65$ $e^{-0.02t} = \frac{65}{75}$ $-0.02t = \ln\left(\frac{65}{75}\right)$ $t = \frac{\ln\left(\frac{65}{75}\right)}{-0.02} = 7.16$ min	 M1 M1 A1
6c	$63e^{15k} = 54.2$ $k = \frac{\ln\left(\frac{54.2}{63}\right)}{15} = -0.01003 = -0.01$	M1 M1, A1
6d	$$75e^{-0.02t} = 63e^{-0.01t}$$ $$\frac{e^{-0.02t}}{e^{-0.01t}} = \frac{63}{75}$$ $$e^{-0.01t} = \frac{63}{75}$$ $t = \frac{\ln\left(\frac{63}{75}\right)}{-0.01} = 17.4$ minutes	M1 M1 A1
7a	$$x\left(\frac{1}{x}\right) + \ln x$$ $$= 1 + \ln x$$	M1 – diff ln x correctly A1
7b	$$f(x) = \ln x - x + c$$ $$0 = \ln 1 - 1 + c$$ $$c = 1$$ $$f(x) = \ln x - x + 1$$	M1 M1 A1
8a	$2x^2 - 8x = -x^2 - 4x - 3$ $3x^2 - 4x + 3 = 0$ Discriminant = $(-4)^2 - 4(3)(3) = -20$ Since **discriminant < 0, there are no real roots** to the simultaneous equations. The 2 curves do not intersect	M1 M1 A1
8b	$$2(x^2 - 4x) = 2[(x-2)^2 - 4]$$ $$= 2(x-2)^2 - 8$$ $-(x^2 + 4x + 3) = -[(x+2)^2 - 1]$$ $$= -(x+2)^2 + 1$$ Sketch min curve with TP (2,-8) passing through O Sketch max curve with TP (-2,1) passing through (0,-3) Non intersecting	M1 M1 A1 B1 B1

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Qn	Solutions	Marks
9a	$\frac{r}{x} = \frac{5}{25}$ $r = \frac{x}{5}$ Volume of liquid $= \frac{1}{3}\pi(5)^2(25) - \frac{1}{3}\pi\left(\frac{x}{5}\right)^2(x)$ $= \frac{1}{3}\pi(625 - \frac{x^3}{25})$	M1 B1
9b	$\frac{dV}{dx} = \frac{1}{3}\pi(-\frac{3x^2}{25})$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $= \frac{1}{3}\pi(-\frac{3x^2}{25})\left(-\frac{1}{2}t\right)$ $x = \int -\frac{1}{2}t\, dt$ $x = -\frac{1}{2}\left(\frac{t^2}{2}\right) + c$ Sub t = 0, x = 25 $c = 25$ $x = -\frac{t^2}{4} + 25$ Sub x = 2 $-\frac{t^2}{4} + 25 = 2$ $t^2 = 92$ $t = \sqrt{92}$ $\frac{dV}{dt} = \frac{1}{3}\pi\left(-\frac{3(2)^2}{25}\right)\left(-\frac{1}{2}\sqrt{92}\right) = 2.41\ cm^3/s$	M1 M1 M1 A1 A1 A1
10a	$BC = 8\sin\theta$ $AC = 8\cos\theta$ Area $= \frac{1}{2}(8\sin\theta)(8\cos\theta)$ $= 32\sin\theta\cos\theta$ $= 16\sin 2\theta$	M1 – either BC or AC found M1 A1
10b	Max = 16	B1
10c	Perimeter = AB + BC + CA = $8 + 8\sin\theta + 8\cos\theta$ $= 8(1 + \sin\theta + \cos\theta)$	B1
10d	$\sin\theta + \cos\theta = \sqrt{2}\sin(\theta + 45°)$ Max perimeter = $8(1 + \sqrt{2})$	B1, B1 B1
11(a)	$f(0) = -4 + b = 0$ $b = 4$ $f(3) = \frac{1}{3}(3)^3 + a(3)^2 - 20(3) - 4 + 4 = -51$ $a = 0$	M1 A1 M1 A1
11(b)	$f(x) = \frac{1}{3}x^3 - 20x$ $f'(x) = x^2 - 20 = 0$ $x = \pm\sqrt{20} = \pm 2\sqrt{5}$	 M1 A1

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Qn	Solutions	Marks
	$f''(x) = 2x$ When $x = 2\sqrt{5}$, $f''(x) > 0$, it is a minimum point When $x = -2\sqrt{5}$, $f''(x) < 0$, it is a maximum point	M1 A1 A1
12(a)	$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{2}{3}$ $3 \sin A \cos B - 3 \cos A \sin B = 2 \sin A \cos B + 2 \cos A \sin B$ $\sin A \cos B = 5 \cos A \sin B$ $\frac{\sin A \cos B}{\cos A \cos B} = \frac{5 \cos A \sin B}{\cos A \cos B}$ $\tan A = 5 \tan B$	M1 M1 B1
12(b)	$\frac{\sin(45° - \theta)}{\sin(45° + \theta)} = \frac{2}{3}$ $A = 45°, B = \theta$ $\tan 45° = 5 \tan \theta$ $\tan \theta = \frac{1}{5}$ $\alpha = \tan^{-1}\left(\frac{1}{5}\right) = 11.30°$ $\theta = 11.30, 180 + 11.30 = 11.3°, 191.3°$	M1 M1 A1 A1

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