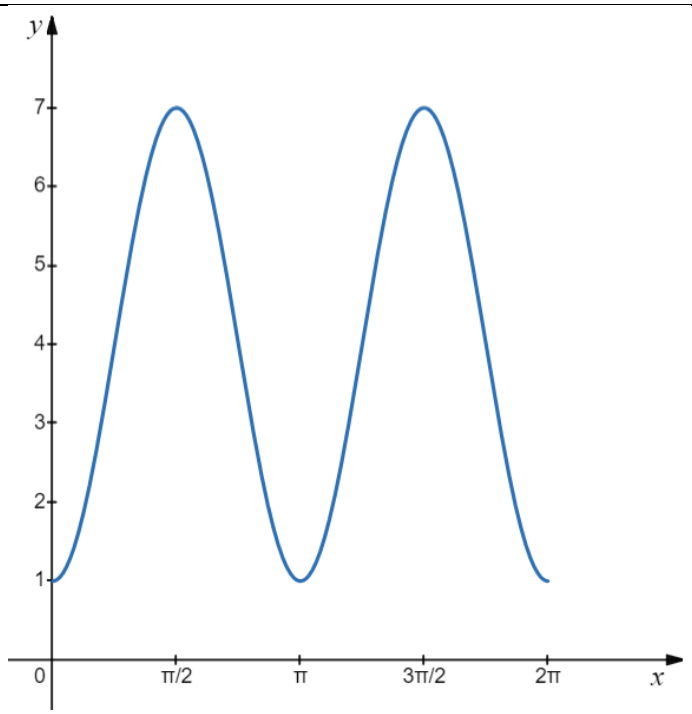
Marking Scheme AM P1 (4049/01)- Prelim 2024

Qn	Answer	Marks	Partial Marks	Guidance
1	$b^2-4ac<0$ $[2(p+2)]^2-4p(p+7)<0$ $4(p^2+4p+4)-4p^2-28p<0$ $4p^2+16p+16-4p^2-28p<0$ $-12p+16<0$ $12p-16>0$ $p>\frac{4}{3}$			
2	$y=3e^{2x}+2e^{-x}$ $\frac{dy}{dx}=6e^{2x}-2e^{-x}$ $\frac{d^2y}{dx^2}=12e^{2x}+2e^{-x}$ $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=12e^{2x}+2e^{-x}+6e^{2x}-2e^{-x}+3e^{2x}+2e^{-x}$ $=21e^{2x}+2e^{-x}$ $A=21$ $B=2$			
3(a)	Period $=180^\circ$ $(or\ \pi)$ Amplitude $=3$			

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3(b)				
4	$\frac{d^2y}{dx^2} = 3x - 2$ $\frac{dy}{dx} = \int (3x-2)dx$ $= \frac{3x^2}{2} - 2x + c$ $c = -13$ $\frac{dy}{dx} = \frac{3x^2}{2} - 2x - 13$ $y = \int \frac{3x^2}{2} - 2x - 13\ dx$ $= \frac{x^3}{2} - x^2 - 13x + d$ $-40 = \frac{(4)^3}{2} - (4)^2 - 13(4) + d$ $d = -4$ $y = \frac{x^3}{2} - x^2 - 13x - 4$			

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5(a)	$\frac{1}{2}(3\sqrt{2}+\sqrt{5})BC = \frac{16+7\sqrt{10}}{2}$ $BC = \frac{16+7\sqrt{10}}{3\sqrt{2}+\sqrt{5}}$ $= \frac{\left(16+7\sqrt{10}\right)\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{5}\right)}$ $= \frac{48\sqrt{2}-16\sqrt{5}+21\sqrt{20}-7\sqrt{50}}{13}$ $= \frac{48\sqrt{2}-16\sqrt{5}+42\sqrt{5}-35\sqrt{2}}{13}$ $= \frac{13\sqrt{2}+26\sqrt{5}}{13}$ $= \sqrt{2}+2\sqrt{5}$			
5(b)	$AC^2 = \left(3\sqrt{2}+\sqrt{5}\right)^2 + \left(\sqrt{2}+2\sqrt{5}\right)^2$ $= 18+6\sqrt{10}+5+2+4\sqrt{10}+20$ $= 45+10\sqrt{10}$			
6(a)	$f'(x) = 3\cos x + 8\sin 2x$ $\cos x > 0$ and $\sin 2x > 0$ for $0 < x < \frac{\pi}{2}$ $\therefore 3\cos x + 8\sin 2x > 0$ for $0 < x < \frac{\pi}{2}$ $f'(x) > 0$ for $0 < x < \frac{\pi}{2}$ Hence f is an increasing function			
6(b)	$\frac{dy}{dx} = 3\cos\frac{\pi}{6} + 8\sin\left(2\times\frac{\pi}{6}\right)$ $= 3\times\frac{\sqrt{3}}{2} + 8\times\frac{\sqrt{3}}{2}$ $= 11\frac{\sqrt{3}}{2}$ $\frac{dy}{dt} = \frac{11\sqrt{3}}{2}\times 2$ $= 11\sqrt{3}$			

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7(a)	General term $= \binom{10}{r}\left(\frac{2}{x^3}\right)^{10-r}\left(-x^2\right)^r$ $= \binom{10}{r}2^{10-r}\left(-1\right)^r x^{5r-30}$ when $5r-30=6$ $r=\frac{36}{5}$ Hence there is no term in x^6			
7(b)	$5r-30=0$ $r=6$ $\binom{10}{6}2^{10-6}\left(-1\right)^6=3360$ $\left(....+3360+...\right)\left(3-\frac{x^6}{8}\right)$ Coefficient of $x^6 = 3360\times\frac{-1}{8}$ $=-420$			

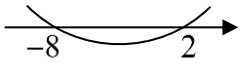
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8(a)	$\frac{4}{y}=\frac{x}{x+9}$ (similar triangles) $xy=4x+36$ $y=4+\frac{36}{x}$			
8(b)	$A=\frac{1}{2}(x+9)y$ $=\frac{1}{2}(x+9)\left(4+\frac{36}{x}\right)$ $=2x+36+\frac{162}{x}$ $\frac{dA}{dx}=2-\frac{162}{x^2}$ when $\frac{dA}{dx}=0,\quad 2-\frac{162}{x^2}=0$ $2=\frac{162}{x^2}$ $x^2=81$ $x=9$ Minimum Area = $2\times 9+36+\frac{162}{9}$ $=72\ \text{m}^2$			
9(a)	$\angle BCA=\angle ABC$ (given $AB=AC$) $\angle ABC=\angle CAF$ (Alternate segment theorem) $\therefore \angle BCA=\angle CAF$			
9(b)	$\angle OAE=\angle OAF=90^\circ$ (Radius perpendicular to tangent) $\angle BAE=\angle BCA$ (Alternate segment theorem) $\angle BAE=\angle CAF$ (Using part a) $\angle OAB=\angle OAC$ (Both 90° – Equal angles) Hence OA bisects angle BAC			
10	$\log_3(x-8)+\log_3 x=2$			

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(a)	$\log_3 x(x-8)=2$ $x(x-8)=3^2$ $x^2-8x-9=0$ $(x-9)(x+1)=0$ $x=9 \quad or \quad x=-1$ (NA) $\therefore x=9$			
10 (b)	$\log_{\frac{1}{b}} a=\dfrac{1}{\log_a\left(\dfrac{1}{b}\right)}$ $=\dfrac{1}{\log_a 1-\log_a b}$ $=\dfrac{1}{-\log_a b}$ $=\dfrac{-1}{\left(\dfrac{1}{\log_b a}\right)}$ $=\dfrac{-1}{\left(\dfrac{1}{c}\right)}$ $=-c$			
11 (a)	$16-ax-x^2=16-\left(x^2+ax\right)$			

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	$= 16 - \left[\left(x + \frac{a}{2} \right)^2 - \frac{a^2}{4} \right]$ $= 16 + \frac{a^2}{4} - \left(\frac{a}{2} + x \right)^2$ $16 + \frac{a^2}{4} = 25$ $\frac{a^2}{4} = 9$ $a^2 = 36$ $a = 6$ $b = \frac{a}{2}$ $= 3$ **Alternate solution** $16 - ax - x^2 = 25 - \left(b^2 + 2bx + x^2 \right)$ $16 - ax - x^2 = 25 - b^2 - 2bx - x^2$ B2 $16 = 25 - b^2$ $b^2 = 9$ $b = 3$			
11 (b)	$y = 25 - (3 + x)^2$ Max value of $y = 25$ Corresponding value of $x = -3$			
11 (c)	$y > 0$ $16 - 6x - x^2 > 0$ $x^2 + 6x - 16 < 0$ $(x + 8)(x - 2) < 0$  $-8 < x < 2$			
12 (a)	$\sin\left(\frac{\pi}{3} + x \right) - \sin\left(\frac{\pi}{3} - x \right)$			

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	$= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \left(\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right)$ $= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x$ $= 2\cos\frac{\pi}{3}\sin x$ $= 2\times\frac{1}{2}\times\sin x$ $= \sin x$			
12 (b)	$\sin\left(\frac{\pi}{3}+2x\right) - \sin\left(\frac{\pi}{3}-2x\right) = \sin 2x$ $\sin 2x - 2 = 4\sin 2x$ $3\sin 2x = -2$ $\sin 2x = -\frac{2}{3}$ Basic angle $= 0.72973$ $2x = \pi + 0.72973,\quad 2\pi - 0.72973$ $x = 1.94,\quad 2.78$			
13 (a)	$\lg y = (\lg b)x + \lg a$			
13 (b)	<table><tr><td>$\lg y$</td><td>0.792</td><td>0.672</td><td>0.568</td><td>0.462</td><td>0.342</td><td>0.230</td></tr></table> Straight line graph			
13 (c)	$\lg a \approx 0.9$ $a \approx 7.94$ $\lg b \approx \frac{0.3-0.5}{2.7-1.8} \approx -0.222$ $b = 0.599$			
13 (d)	when $x = 0.8$ $\lg y = 0.72$ $y = 5.23$			

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