O

GAN ENG SENG SCHOOL
Preliminary Examination 2024

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS
Paper 2

4049/02

28 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

	For Examiner's Use
Total	90

This paper consists of 18 printed pages (including the cover page).

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 (a) Given that the expression $2x^3+ax^2+bx+c$ leaves the same remainder when divided by $(x-5)$ and when divided by $(2x+1)$, show that $9a+2b=-91$. [3]

Let $F(x)=2x^3+ax^2+bx+c$

$F(5)=2(5)^3+a(5)^2+b(5)+c$ (M1)

$=250+25a+5b+c$

$F\left(-\frac{1}{2}\right)=2\left(-\frac{1}{2}\right)^3+a\left(-\frac{1}{2}\right)^2+b\left(-\frac{1}{2}\right)+c$

$=-\frac{1}{4}+\frac{1}{4}a-\frac{1}{2}b+c$

$250+25a+5b+c=-\frac{1}{4}+\frac{1}{4}a-\frac{1}{2}b+c$ (M1)

$1000+100a+20b=-1+a-2b$

$99a+22b=-1001$ (M1)

$9a+2b=-91$ (Shown)

(b) Given that the remainder is -6 when the expression $2x^3+ax^2+bx+c$ is divided by $(x-1)$, and $(x+1)$ is a factor, calculate the value of c. [4]

$F(x)=2x^3+ax^2+bx+c$

$F(1)=2(1)^3+a(1)^2+b(1)+c=-6$

$a+b+c=-8$ ...(1) (M1)

$F(-1)=2(-1)^3+a(-1)^2+b(-1)+c=0$

$a-b+c=2$ ...(2) (M1)

(1) − (2): $2b=-10$

$b=-5$ (M1)

$\Rightarrow 9a+2(-5)=-91$

$a=-9$

$\Rightarrow c=2+9-5=6$ (A1)

Marker's remarks:

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2 Mr Phua bought a new car. After t months, its value V is given by $V = 250000e^{-pt}$, where p is a constant. The value of the car after 36 months is \$180 000.

(a) Show that p is approximately 0.00913. [2]

$180000 = 250000e^{-p(36)}$ (M1)

$\frac{18}{25} = e^{-36p}$

$\ln(0.72) = -36p$

$-0.3285 = -36p$

$p = 0.00913$ (3 s.f.) (A1)

(b) Calculate the age of the car, to the nearest month, when its expected value will be \$100000. [2]

$100000 = 250000e^{-0.00913t}$ (M1)

$\frac{2}{5} = e^{-0.00913t}$

$\ln(0.4) = -0.00913t$

$t = 100.4$ months

= 100 months (nearest month) (A1)

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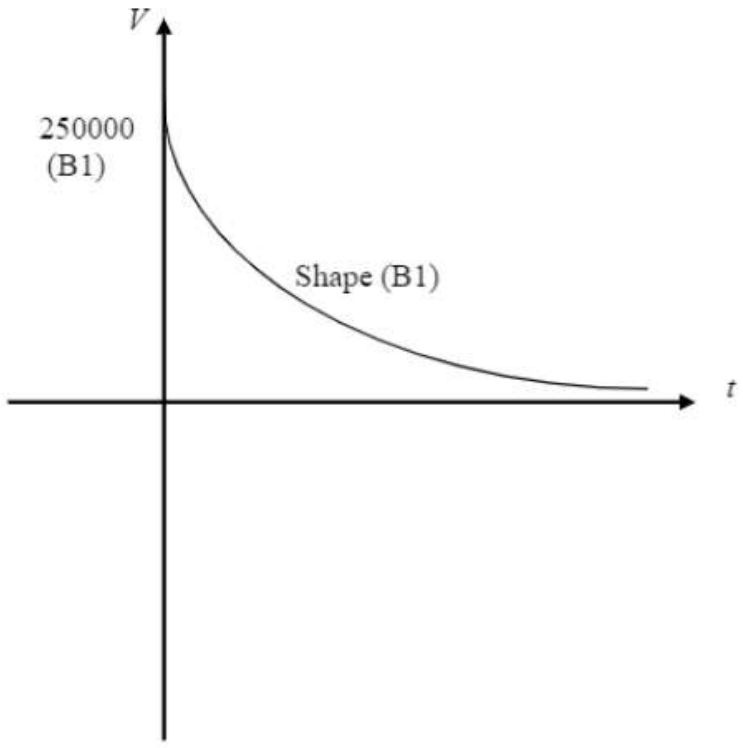
(c) 5 years after Mr Phua bought the car, a second-hand car agent offers to pay him \$130000 for his car. Would you advise Mr Phua to take up the offer. Explain your answer. [2]

Value of the car after 5 years $= 250000e^{-0.00913(60)}$

$= \$144555.12$ (M1)

No, he should not take up the offer as the value offered is far below the expected value. (A1)

(d) Sketch the graph of $V = 250000e^{-pt}$, for $t \geq 0$. [2]



Marker's remarks:

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3. The line $y=10$ and $3y+4x=32$ are tangent to a circle C at the points $(-2,10)$ and $(2,8)$ respectively.

(a) Show that the equation of C is $(x+2)^2+(y-5)^2=25$. [5]

$3y+4x=32$

$y=-\frac{4}{3}x+\frac{32}{3}$

Gradient of the $\perp$ line $=\frac{3}{4}$ (M1)

Eqn of the line passing thru' (2, 8) and centre of the circle:

$(y-8)=\frac{3}{4}(x-2)$

$y=\frac{3}{4}x+\frac{13}{2}$ (M1)

x-coord of the centre of the circle $=-2$ (B1)

$\Rightarrow y=\frac{3}{4}(-2)+\frac{13}{2}=5$ (M1)

Radius of the circle $=10-5=5$ (B1)

Eqn of the circle: $(x+2)^2+(y-5)^2=5^2$

or $(x+2)^2+(y-5)^2=25$

(b) Explain if the x-axis is tangent to C. [2]

The lowest point of C is $(-2,\ 5-5)=(-2,0)$, which is on the x-axis. (M1)

Therefore, x-axis is tangent to C. (A1)

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(c) Write down the equations of the two vertical tangents. [2]

$x = -7$ (B1)

$x = 3$ (B1)

(d) Two points P and Q lie on the circle and the length of PQ is 4 units. [2]
Calculate the shortest distance from the centre of the circle to the line PQ.

The length of the centre of the circle to P or Q = radius = 5 units

Length between mid-point of PQ to P = 2 units

Shortest distance = $\sqrt{5^2 - 2^2} = \sqrt{21} = 4.58$ units (3 s.f.) (A1)

Marker's remarks:

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4. A, B and C are the angles of a triangle. Given that $\sin(A+B)=\frac{12}{13}$.

(a) Find the exact value of $\tan C$. Show your working clearly. [3]

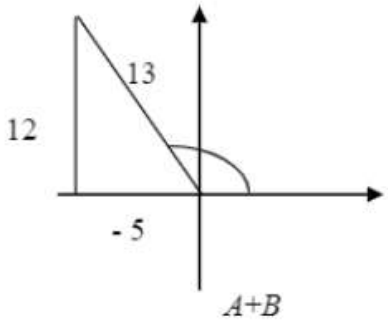
$\sin(A+B)=\frac{12}{13}$

$\Rightarrow \tan(A+B)=-\frac{12}{5}$ (M1)

$\tan C=\tan(180°-(A+B))$

$\tan C=-\tan(A+B)$ (M1)

$\tan C=-\frac{-12}{5}=\frac{12}{5}=2.4$ (A1)



Alternative solution: $(A+B)<90°$

$\Rightarrow \tan(A+B)=\frac{12}{5}$ (M1)

$\tan C=\tan(180°-(A+B))$

$\tan C=-\tan(A+B)$ (M1)

$\tan C=-\frac{12}{5}=-2.4$ (A1)

(b) Evaluate $\cos\left[(A+B)+\frac{\pi}{3}\right]$. Leave your answer in surd form. [3]

$\cos\left[(A+B)+\frac{\pi}{3}\right]=\cos(A+B)\cos\frac{\pi}{3}-\sin(A+B)\sin\frac{\pi}{3}$ (M1)

$=\left(\frac{-5}{13}\right)\left(\frac{1}{2}\right)-\left(\frac{12}{13}\right)\left(\frac{\sqrt{3}}{2}\right)$ (M1)

$=-\frac{5+12\sqrt{3}}{26}$ or $\frac{-5-12\sqrt{3}}{26}$ (A1)

Alternative solution when $(A+B)<90°$

$=\cos(A+B)\cos\frac{\pi}{3}-\sin(A+B)\sin\frac{\pi}{3}$ (M1)

$=\left(\frac{5}{13}\right)\left(\frac{1}{2}\right)-\left(\frac{12}{13}\right)\left(\frac{\sqrt{3}}{2}\right)$ (M1)

$=\frac{5-12\sqrt{3}}{26}$ (A1)

Marker's remarks:

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5. (a) Solve the equation $15(16^x)-4^x=6$. [4]

$15(16^x)-4^x=6$

$15(4^{2x})-4^x-6=0$ (M1)

$15(4^x)^2-4^x-6=0$

Let $a=4^x$

$\Rightarrow 15a^2-a-6=0$ (M1)

$(5a+3)(3a-2)=0$

$(5a+3)=0$ or $(3a-2)=0$

$a=-\frac{3}{5}$ or $a=\frac{2}{3}$

$4^x=-\frac{3}{5}$ (no solution) or $4^x=\frac{2}{3}$

(A1) $x=\log_4\left(\frac{2}{3}\right)=-0.292$ (3 s.f.) (A1)

(b) Solve the equation $\lg(2y)+2\ln(y)=2$. [4]

$\lg(2y)+2\ln(y)=2$

$\lg 2+\lg(y)+\frac{2\lg(y)}{\lg(e)}=2$

(M1) (M1)

$\lg(y)\left[1+\frac{2}{\lg(e)}\right]=2-\lg 2$ (M1)

$\lg(y)\times(5.6051)=1.69897$

$\lg(y)=0.303108$

$y=2.00959=2.01$ (3 s.f.) (A1)

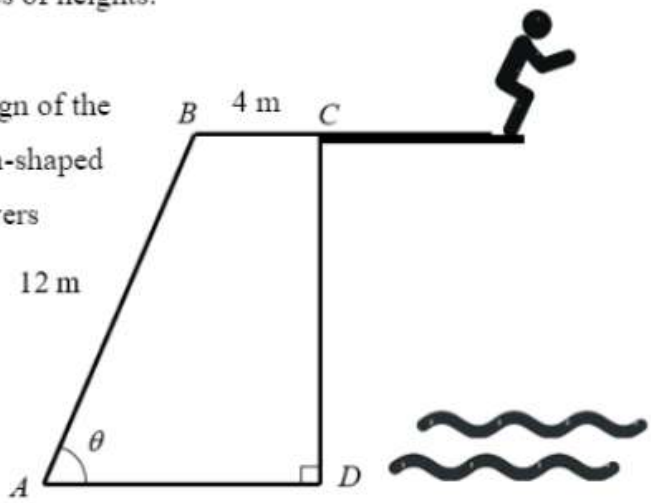
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6. A competitive platform diving competition involves one or two divers performing spins, twists and somersaults into a pool from a series of heights.

The diagram shows the cross-section of a design of the diving platform, which consists of a trapezium-shaped tower $ABCD$. AB is the stairway which the divers use to reach the platform.

Given that $\angle BAD = \theta$, where θ can vary and $\angle ADC = 90°$, the length of AB and BC are 12 m and 4 m respectively,



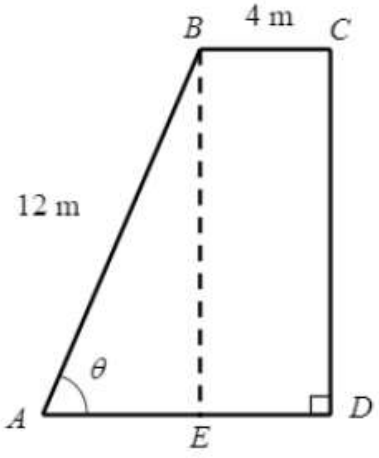
(a) show that L, the perimeter of $ABCD$, can be expressed in the form $p + q\sin\theta + r\cos\theta$. [2]

$L = AB + BC + CD + DE + EA$

$L = 12 + 4 + BE + 4 + 12\cos\theta$

$L = 12 + 4 + 12\sin\theta + 4 + 12\cos\theta$ (M1)

$L = 20 + 12\sin\theta + 12\cos\theta$ (A1)



(b) express L in the form of $p + R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$R\cos(\theta - \alpha) = 12\cos\theta + 12\sin\theta$

$R = \sqrt{12^2 + 12^2} = \sqrt{288} = 12\sqrt{2}$ (M1)

$\alpha = \tan^{-1}\left(\frac{12}{12}\right) = 45°$ (M1)

$\Rightarrow L = 20 + 12\sqrt{2}\cos(\theta - 45°)$ (A1)

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(c) State the maximum value of L, and the corresponding value of θ. [2]

Max. $L = 20 + 12\sqrt{2}$ or 37.0 m (3 s.f.) (B1)

When $\cos(\theta - 45°) = 1$

$(\theta - 45°) = 0$

$\theta = 45°$ (B1)

(d) Find the value of θ if L = 25 m. Explain why a platform with this value of L is a poor design. [4]

$L = 25$

$20 + 12\sqrt{2}\cos(\theta - 45°) = 25$ (M1)

$\cos(\theta - 45°) = \dfrac{5}{12\sqrt{2}} = 0.29463$ (M1)

$\alpha = 72.865°$

$\theta - 45° = 72.865°, \quad 287.135°$

$\theta = 117.865°, \quad 332.135°$ (M1)

θ cannot be obtuse or more than $90°$ as AB is a stairway. Otherwise, the divers cannot climb up to the platform. (A1)

Marker's remarks:

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7 (a) Prove the identity $\operatorname{cosec} 4x + \cot 4x = \cot 2x$. [3]

$$\begin{aligned}
\text{LHS} &= \frac{1}{\sin 4x} + \frac{\cos 4x}{\sin 4x} && \text{(M1)} \\
&= \frac{1+\cos 2(2x)}{\sin 2(2x)} \\
&= \frac{1+(2\cos^2 2x - 1)}{2\sin 2x \cos 2x} && \text{(M1)} \\
&= \frac{2\cos^2 2x}{2\sin 2x \cos 2x} \\
&= \frac{\cos 2x}{\sin 2x} && \text{(M1)} \\
&= \cot 2x = \text{RHS}
\end{aligned}$$

(b) Hence, find, for $-180° \le \theta \le 180°$, the values of θ for which $\operatorname{cosec} 2\theta + \cot 2\theta = -\sqrt{3}$. [4]

$$\begin{aligned}
&\operatorname{cosec} 2\theta + \cot 2\theta = -\sqrt{3} \\
&\Rightarrow \cot \theta = -\sqrt{3} && \text{(M1)} \\
&\tan \theta = -\frac{1}{\sqrt{3}} \\
&\alpha = 30° && \text{(M1)} \\
&\theta = 180° - 30°, -30° \\
&\theta = 150°, \quad -30° && \text{(A1, A1)}
\end{aligned}$$

Marker's remarks:

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8. The equation of a curve is $y = \frac{x}{e^{2x}}$

(a) Find an expression for $\frac{dy}{dx}$. [2]

$y = \frac{x}{e^{2x}} = xe^{-2x}$

$\frac{dy}{dx} = x(-2)e^{-2x} + e^{-2x}$ or $= \frac{e^{2x} - x(2)e^{2x}}{e^{4x}}$ (M1)

$= e^{-2x}(1-2x)$ $= \frac{e^{2x}(1-2x)}{e^{4x}}$

$= \frac{1-2x}{e^{2x}}$ $= \frac{1-2x}{e^{2x}}$ (A1)

(b) Given that x is changing at a constant rate of 0.18 units per second, find the rate of change of y when $x = 1$. [2]

$\frac{dx}{dt} = 0.18$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1-2x}{e^{2x}} \times 0.18$ (M1)

When $x = 1$, $\frac{dy}{dt} = \frac{1-2}{e^2} \times 0.18 = -0.0244$ units/sec (A1)

(c) Explain if the curve has more than one stationary point. [2]

$\frac{dy}{dx} = \frac{1-2x}{e^{2x}}$

As $e^{2x} > 0$, $\Rightarrow$ when $\frac{dy}{dx} = 0$, $1 - 2x = 0$

$x = \frac{1}{2}$ (M1)

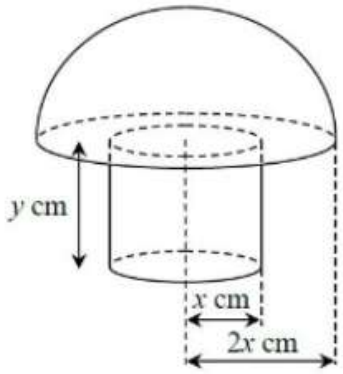
Therefore there is only one stationary value at $x = \frac{1}{2}$ (A1)

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9. The diagram shows a solid object. It is made up of a cylinder with a hemisphere above it.

The cylinder has a radius of x cm and a height of y cm. The radius of the hemisphere is $2x$ cm.



(a) Express y in terms of x if the total volume of the solid is 72π cm^3. [2]

$$72\pi = \frac{1}{2}\times\frac{4}{3}\times\pi\times(2x)^3 + \pi\times(x)^2\times y \quad \text{(M1)}$$

$$72 = \frac{16}{3}x^3 + x^2y$$

$$y = \frac{72}{x^2} - \frac{16}{3}x \qquad \text{(A1)}$$

(b) Show that the total surface area, A cm^2, of the solid is $144\pi\left(\frac{1}{x}+\frac{x^2}{108}\right)$. [2]

$$A = \frac{1}{2}\times 4\pi(2x)^2 + \pi(2x)^2 + 2\pi(x)\times y \qquad \text{(M1)}$$

$$= 12\pi x^2 + 2\pi x\left(\frac{72}{x^2} - \frac{16}{3}x\right)$$

$$= 12\pi x^2 + \frac{144\pi}{x} - \frac{32\pi x^2}{3}$$

$$= \frac{144\pi}{x} + \frac{4\pi x^2}{3} \qquad \text{(A1)}$$

$$= 144\pi\left(\frac{1}{x}+\frac{x^2}{108}\right)$$

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(c) Find the value of x for which A is a stationary value and determine the nature of the stationary value. [4]

$$A = 144\pi\left(\frac{1}{x}+\frac{x^2}{108}\right)$$

$$\frac{dA}{dx} = 144\pi\left(\frac{-1}{x^2}+\frac{x}{54}\right) \quad \text{(M1)}$$

At stationary value, $\frac{dA}{dx}=0$

$$144\pi\left(\frac{-1}{x^2}+\frac{x}{54}\right)=0$$

$$\left(\frac{-1}{x^2}+\frac{x}{54}\right)=0$$

$$x^3 = 54$$

$x = 3.78$ cm (3 s.f.) (M1)

$$\frac{d^2A}{dx^2} = 144\pi\left(\frac{2}{x^3}+\frac{1}{54}\right) \quad \text{(M1)}$$

When $x = 3.78$, $\frac{d^2A}{dx^2} = 25.1 \ (>0)$

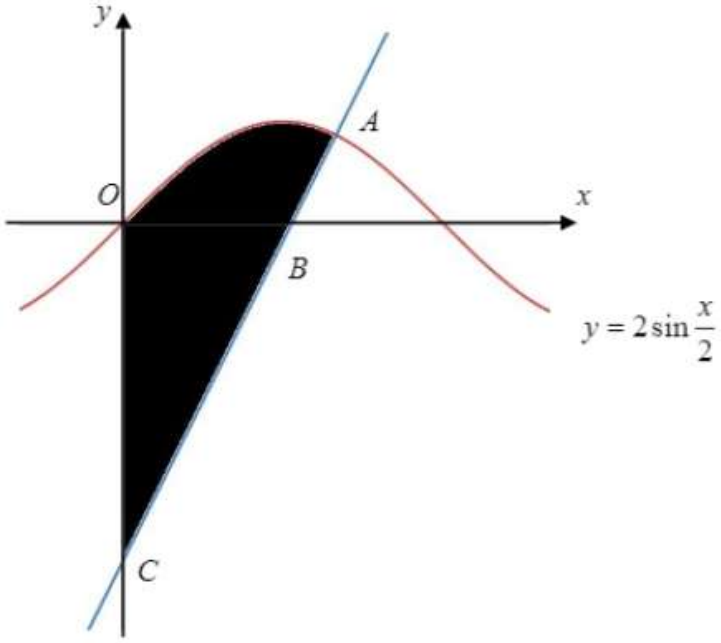
$\therefore$ the area is a minimum value.

(A1) - value must be shown

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10.



The diagram shows part of the curve $y = 2\sin\frac{x}{2}$, where A lies on the curve.

The x-coordinate of A is $\frac{4\pi}{3}$ and the normal to the curve at A meets the x-axis at B and the y-axis at C.

(a) Find the coordinates of B and of C, leaving your answers in its **exact form**. [6]

$y = 2\sin\frac{x}{2}$ $\quad \frac{dy}{dx} = \cos\frac{x}{2}$ (M1)

When $x = \frac{4\pi}{3}$ $\quad \frac{dy}{dx} = \cos\frac{4\pi}{6} = -\frac{1}{2}$

Gradient of $AC = 2$ (M1)

$y = 2\sin\frac{4\pi}{6} = \sqrt{3}$ (M1)

Equation of AC: $\left(y - \sqrt{3}\right) = 2\left(x - \frac{4\pi}{3}\right)$ (M1)

$y = 2x - \frac{8\pi}{3} + \sqrt{3}$

When $x = 0$, $\quad y = -\frac{8\pi}{3} + \sqrt{3}$ $\quad \therefore C$ is $\left(0, \sqrt{3} - \frac{8\pi}{3}\right)$

(A1) - Must be in exact value as required

When $y = 0$, $\quad 0 = 2x - \frac{8\pi}{3} + \sqrt{3}$

$x = \frac{4\pi}{3} - \frac{\sqrt{3}}{2}$ $\quad \therefore B$ is $\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}, 0\right)$

(A1) - Must be in exact value as required

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(b) Find the area of the shaded region. [6]

Area = area under the curve – area of the triangle ABD + area of triangle BOC

$$= \int_0^{\frac{4\pi}{3}} y\,dx - \frac{1}{2}\left(\sqrt{3}\right)\left(\frac{4\pi}{3} - \left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right)\right) + \frac{1}{2}\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right)\left(\sqrt{3} - \frac{8\pi}{3}\right)$$

(M1) (M1) (M1)

$$= \int_0^{\frac{4\pi}{3}} 2\sin\frac{x}{2}\,dx - \frac{1}{2}\left(\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}(3.3228)(6.6455)$$

$$= \left[-4\cos\frac{x}{2}\right]_0^{\frac{4\pi}{3}} - \frac{3}{4} + 11.041$$

(M1)

$$= 6 - \frac{3}{4} + 11.041 \qquad \text{(M1)}$$

$$= 16.291$$

$$= 16.3 \text{ units}^2 \qquad \text{(A1)}$$

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11. (a) Show that $\frac{d}{dx}\left[(2x)(3-5x)^{\frac{6}{5}}\right]=(6-22x)(3-5x)^{\frac{1}{5}}$. [3]

$$\frac{d}{dx}\left[(2x)(3-5x)^{\frac{6}{5}}\right] = (2x)\left(\frac{6}{5}\right)(3-5x)^{\frac{1}{5}}(-5)+(3-5x)^{\frac{6}{5}}(2)$$

(M1) (M1)

$$= (2)(3-5x)^{\frac{1}{5}}\left[-6x+(3-5x)\right] \quad \text{(M1)}$$

$$= (2)(3-5x)^{\frac{1}{5}}\left[-11x+3\right]$$

$$= (6-22x)(3-5x)^{\frac{1}{5}} \quad \text{(Shown)}$$

(b) Hence, evaluate $\int x(3-5x)^{\frac{1}{5}}dx$. [3]

From Part (i):

$$\frac{d}{dx}\left[(2x)(3-5x)^{\frac{6}{5}}\right] = (6-22x)(3-5x)^{\frac{1}{5}}$$

$$\left[(2x)(3-5x)^{\frac{6}{5}}\right] = \int(6-22x)(3-5x)^{\frac{1}{5}}dx$$

$$\left[(2x)(3-5x)^{\frac{6}{5}}\right] = \int 6(3-5x)^{\frac{1}{5}}dx - \int 22x(3-5x)^{\frac{1}{5}}dx \quad \text{(M1)}$$

$$\int 22x(3-5x)^{\frac{1}{5}}dx = \frac{6(3-5x)^{\frac{6}{5}}}{(-5)\left(\frac{6}{5}\right)} - (2x)(3-5x)^{\frac{6}{5}}+c \quad (c: \text{ arbitrary constant})$$

(M1)

$$= (3-5x)^{\frac{6}{5}}(-1-2x)+c$$

$$\int x(3-5x)^{\frac{1}{5}}dx = \frac{-1}{22}(1+2x)(3-5x)^{\frac{5}{6}}+d \quad (d: \text{arbitrary constant}) \quad \text{(A1)}$$

Marker's remarks:

End of paper

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