Marking Scheme
2024 Sec 4 Express PRELIM AM Paper 1

			Marker's Comments
1(a)	$y = -2x^2 + 3x + 5$ $= -2\left[x^2 - \frac{3}{2}x\right] + 5 - - -$ M1 $= -2\left[\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 5 - - -$ M1 $= -2\left(x - \frac{3}{4}\right)^2 + \frac{18}{16} + 5$ $= -2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8} - - -$ A1	**3 marks**	
(b)	For all real values of x, $\left(x - \frac{3}{4}\right)^2 \geq 0 - - -$ M1 $-2\left(x - \frac{3}{4}\right)^2 \leq 0$ $-2\left(x - \frac{3}{4}\right)^2 + \frac{49}{8} \leq \frac{49}{8} - - -$ A1 $\therefore x = \frac{3}{4}, y_{max} = \frac{49}{8}$	**2 marks**	
	Alternative method $y = -2\left(x - \frac{3}{4}\right)^2 + k$ When $x = \frac{3}{4}, y = -2(0) + k$ $= k$ $\frac{dy}{dx} = -4\left(x - \frac{3}{4}\right)$ When $\frac{dy}{dx} = 0 - - -$ M1 $x = \frac{3}{4}$ $\frac{d^2y}{dx^2} = -4(< 0) - - -$ A1 $\therefore y$ is a maximum value, k	**2 marks**	

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2(a)	$64-(x+1)^3$ $=4^3-(x+1)^3$ $=[4-(x+1)][4^2+4(x+1)+(x+1)^2]$ ---M1 $=(3-x)(16+4x+4+x^2+2x+1)$ $=(3-x)(x^2+6x+21)$ ---A1	**2 marks**	
(b)	$(3-x)(x^2+6x+21)=15(3-x)$ $(3-x)(x^2+6x+21)-15(3-x)=0$ $(3-x)(x^2+6x+21-15)=0$ $(3-x)(x^2+6x+6)=0$ ---M1 $(3-x)=0$ or $(x^2+6x+6)=0$ $x=3$ or $x=\dfrac{-6\pm\sqrt{36-4(6)}}{2}$ $x=3$ or $x=\dfrac{-6\pm\sqrt{12}}{2}$ $x=3$ or $x=\dfrac{-6\pm2\sqrt{3}}{2}$ $x=3$ or $x=-3\pm\sqrt{3}$ ---A1,0	**2 marks**	
3.	$\dfrac{x+7}{(x^2-9)(x-3)}$ $=\dfrac{x+7}{(x+3)(x-3)(x-3)}$ $\dfrac{x+7}{(x+3)(x-3)^2}=\dfrac{A}{(x+3)}+\dfrac{B}{(x-3)}+\dfrac{C}{(x-3)^2}$ ----M1 $A(x-3)^2+B(x+3)(x-3)+C(x+3)=x+7$ ----M1 Sub $x=3$ $6C=10$ $C=\dfrac{5}{3}$ ----A1 Sub $x=-3$ $36A=4$ $A=\dfrac{1}{9}$ ---A1 Sub $x=0$ $7=9A-9B+3C$ $9\left(\dfrac{1}{9}\right)-9B+3\left(\dfrac{5}{3}\right)=7$ $B=-\dfrac{1}{9}$ ---A1 $\dfrac{x+7}{(x-3)(x-3)(x+3)}=\dfrac{1}{9(x+3)}-\dfrac{1}{9(x-3)}+\dfrac{5}{3(x-3)^2}$ ----A1	**6 marks**	

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4(a)	$y = x^3 + kx^2 + kx + 8$ $\frac{dy}{dx} = 3x^2 + 2kx + k$ – – – M1 Let $\frac{dy}{dx} > 0$ $3x^2 + 2kx + k > 0$ Let discriminant < 0 $(2k)^2 - 4(3)(k) < 0$ – – – M1 $4k^2 - 12k < 0$ $k^2 - 3k < 0$ $k(k-3) < 0$ $0 < k < 3$ – – – A1	**3 marks**	
(b)	$\frac{dy}{dx} = 3x^2 + 2kx + k$ $\frac{dy}{dx} = 0$ $3x^2 + 2kx + k = 0$ Let discriminant = 0 $(2k)^2 - 4(3)(k) = 0$ $4k^2 - 12k = 0$ $k^2 - 3k = 0$ } M1 $k(k-3) = 0$ $k = 0$ or $k = 3$ – – – A1	**2 marks**	
5(a)	$\left(2x^2 + \frac{3}{x}\right)^9$ $T_{r+1} = \binom{9}{r}(2x^2)^{9-r}\left(\frac{3}{x}\right)^r$ $= \binom{9}{r}(2)^{9-r}(x^2)^{9-r}(3)^r x^{-r}$ $= \binom{9}{r}(2)^{9-r}(3)^r x^{18-3r}$ – – – M1 Let $18 - 3r = 0$ $r = 6$ – – – A1 $T_7 = \binom{9}{6}(2)^3(3)^6$ $= 489888$ – – – A1	**3 marks**	

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5(b)	$(2-3x)^5$ $= 2^5 + \binom{5}{1}(2)^4(-3x) + \binom{5}{2}(2)^3(-3x)^2 + \ldots.$ $= 32 - 240x + 720x^2 + \ldots$ ----B2,1,0	**2 marks**	
(c)	$(2-3x)^5(1-3x)^n$ $= (32 - 240x + 720x^2 + \ldots)(1-3x)^n$ $= (32 - 240x + 720x^2 + \ldots)\left(1 + \binom{n}{1}(-3x) + \ldots\right)$ ---M1 Coefficient of $x = 32\binom{n}{1}(-3x) - 240x$ ---M1 $32 \times n \times (-3x) - 240x = -720x$ $-96nx - 240x = -720x$ $-96n = -720 + 240$ $n = 5$ ---A1	**3 marks**	
6(a)	$\frac{dy}{dx} = 36(2x+1)^{-2}$ $y = \int 36(2x+1)^{-2}\,dx$ $= 36 \times \frac{(2x+1)^{-1}}{-1(2)} + c$ (c : constant) $= \frac{-18}{(2x+1)} + c$ ---M1 Sub $\left(\frac{1}{2}, 2\right)$ $2 = \frac{-18}{(1+1)} + c$ ---M1 $2 = -9 + c$ $c = 11$ $y = \frac{-18}{(2x+1)} + 11$ ---A1	**3 marks**	
(b)	$\frac{dy}{dx} = 36(2x+1)^{-2}$ At $x = 1$ $\frac{dy}{dx} = \frac{36}{3^2}$ $= 4$ ---M1 $\tan\theta = 4$ $\theta = 76.0^\circ$ ---A1	**2 marks**	

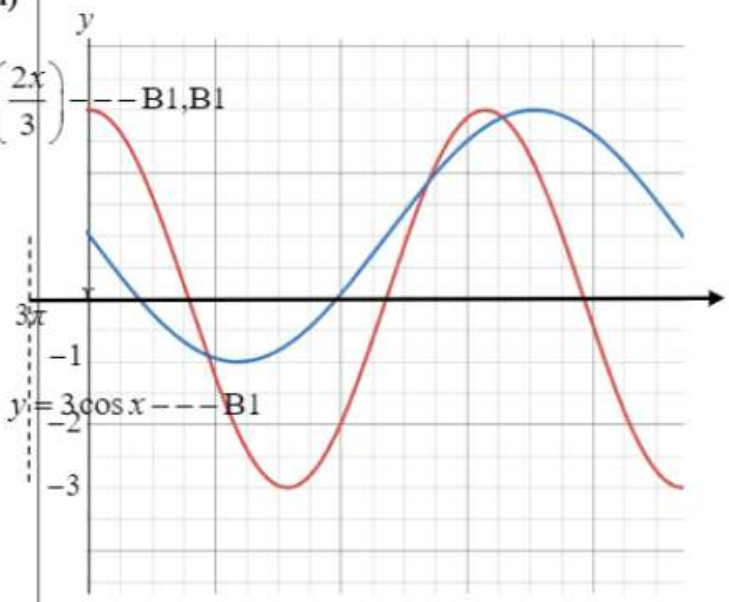
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7	$y = \ln\left(\frac{x+1}{x-1}\right)$ $y = \ln(x+1) - \ln(x-1)$ $\frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x-1}$ ---M1 $2y = 3x + 2$ $y = \frac{3}{2}x + 1$ Gradient of normal at $A = \frac{3}{2}$ $\therefore$ Gradient of tangent $= -\frac{2}{3}$ ---M1 $\frac{1}{x+1} - \frac{1}{x-1} = -\frac{2}{3}$ $\frac{x-1-(x+1)}{(x+1)(x-1)} = \frac{-2}{3}$ $\frac{-2}{(x+1)(x-1)} = \frac{-2}{3}$ ---M1 $x^2 - 1 = 3$ $x^2 = 4$ $x = 2$ or $x = -2$ $\therefore x = 2$ ---A1 At $x = 2, y = \ln\left(\frac{2+1}{2-1}\right)$ $y = \ln 3$ ---A1 $P(2, \ln 3)$ Equation of tangent $y = -\frac{2}{3}x + c$ Sub $(2, \ln 3)$ $\ln 3 = -\frac{2}{3}(2) + c$ ---M1 $c = \ln 3 + \frac{4}{3}$ $y = -\frac{2}{3}x + \ln 3 + \frac{4}{3}$ ---A1		

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	At $x = 0$ $y = \ln 3 + \frac{4}{3}$ $Q(0, \ln 3 + \frac{4}{3})$ Area of triangle POQ $= \frac{1}{2} \times 2 \times \left(\ln 3 + \frac{4}{3} \right)$ ---M1 $= \ln 3 + \frac{4}{3}$ $= 2.43$ ---A1	**9 marks**	
8(a)	$OQ = OP = OR = OT$ (radii of a circle) ---M1 $\angle QOP = \angle ROT$ (vertically opp angles) ---M1 ΔQOP is congruent to ΔROT (SAS congruency) ---A1 Hence, $TR = PQ$ (Shown)	**3 marks**	
(b)	$\angle TUR = \angle PUT$ (Common angle) $\angle PTU = \angle TRU$ (Tangent-chord theorem) ---M1 ΔUTR is similar to ΔUPT (AA similarity) ---M1 $\frac{TP}{RT} = \frac{UT}{UR} = \frac{PU}{TU}$ $RT = QP$ (From (i)) $\frac{TP}{QP} = \frac{UT}{UR}$ ---M1 $QP \times UT = TP \times UR$ (Shown)	**3 marks**	
9(a)	$D\left(6.5 + \frac{7.5}{3} \times 2, 8 + \frac{6}{3} \times 2 \right)$ ---M1 $= D(11.5, 12)$ ---A1 $M(6.5, 0)$ ---B1 $\left(\frac{x + 8.5}{2}, \frac{y + 4}{2} \right) = (6.5, 0)$ ---M1 $x = 4.5, y = -4$ $B(4.5, -4)$ ---A1	**5 marks**	

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(b)	$A(-1,2), B(4.5,-4), C(8.5,4), D(11.5,12)$ $\text{Area} = \frac{1}{2}\begin{bmatrix} -1 & 4.5 & 8.5 & 11.5 & -1 \\ 2 & -4 & 4 & 12 & 2 \end{bmatrix}$ $= \frac{1}{2}\left[4+4.5(4)+8.5(12)+2(11.5)-\left(2(4.5)-4(8.5)+4(11.5)+12(-1)\right)\right]$ ----M1 $= \frac{1}{2}(147-9)$ $= \frac{138}{2}$ $= 69 \text{ units}^2$ ----A1	**2 marks**	
10(a)	$1-2\sin\left(\frac{2x}{3}\right)$ ---B1,B1  B1 - $y = 3\cos x$, shape, 1.5 oscillations over 3π 3 and -3 to be marked on y axis π, 2π and 3π to be marked on x axis	**B1 Shape** **B1 Shifting curve vertically up by 1 unit**	
(b)(i)	3	**1 mark**	
(b) (ii)	9	**1 mark**	

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11(a)	Area of cross-sectional area of prism $=6\times\frac{1}{2}\times\left(5+\sqrt{3}\right)^2\sin 60^\circ$ ---M1 $=3\times\left(25+10\sqrt{3}+3\right)\times\frac{\sqrt{3}}{2}$ $=3\left(28+10\sqrt{3}\right)\times\frac{\sqrt{3}}{2}$ ---M1 $=\frac{3}{2}\left(28\sqrt{3}+30\right)$ $=42\sqrt{3}+45$ ---A1	**3 marks**	
(b)	Volume of the prism $=3\left(32\sqrt{3}+138\right)\text{cm}^3$ $3\left(14\sqrt{3}+15\right)\times h=3\left(32\sqrt{3}+138\right)$ $\left(14\sqrt{3}+15\right)\times h=\left(32\sqrt{3}+138\right)$ $h=\frac{32\sqrt{3}+138}{14\sqrt{3}+15}\times\frac{14\sqrt{3}-15}{14\sqrt{3}-15}$ ---M1 $=\frac{32\sqrt{3}\left(14\sqrt{3}-15\right)+138\left(14\sqrt{3}-15\right)}{\left(14\sqrt{3}\right)^2-15^2}$ $=\frac{448(3)-480\sqrt{3}+1932\sqrt{3}-2070}{\left(14\sqrt{3}\right)^2-15^2}$ ---M1 $=\frac{1452\sqrt{3}-726}{363}$ $=4\sqrt{3}-2$ ---A1	**3 marks**	
12(a)	$v=\frac{3}{t+2}-\frac{t+2}{3}$ Let $v=0$ $\frac{3}{t+2}-\frac{t+2}{3}=0$ ---M1 $(t+2)^2=9$ $t+2=3$ or $t+2=-3$ $t=1$ or $t=-5(\text{rej})$ ---A1	**2 marks**	

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(b)	$s = \int \frac{3}{t+2} - \frac{t+2}{3}\, dt$ $= 3\ln(t+2) - \frac{t^2}{6} - \frac{2}{3}t + c$ ---M1 At $t = 0, s = 0$ $3\ln(0+2) + c = 0$ $c = -3\ln 2$ $s = 3\ln(t+2) - \frac{t^2}{6} - \frac{2}{3}t - 3\ln 2$ --A1	**2 marks**	
(b)	At $t = 1$ $s = 3\ln 3 - \frac{1}{6} - \frac{2}{3} - 3\ln 2$ $= 0.383$ m ----A1 At $t = 2$ $s = 3\ln 4 - \frac{4}{6} - \frac{4}{3} - 3\ln 2$ $= 0.0794$ ---A1 Distance travelled $=0.383+(0.383-0.0794)$ $= 0.687$ m ---A1	**5 marks**	
(c)	$a = \frac{dv}{dt}$ $= \frac{-3}{(t+2)^2} - \frac{1}{3}$ ---M1 At $t = 1$ $a = -\frac{3}{9} - \frac{1}{3}$ $= -\frac{2}{3} < 0$ ---A1 Since $a < 0$, the velocity of the particle is decreasing.	**2 marks**	
13(a)	$\log_a 125 - 3\log_a \sqrt{b} + \log_a c = 3$ $\log_a 125 - \log_a b^{\frac{3}{2}} + \log_a c = 3$ $\log_a \frac{125c}{b^{\frac{3}{2}}} = 3$ ---M1 $a^3 = \frac{125c}{b^{\frac{3}{2}}}$ ---M1 $a = \frac{5\sqrt[3]{c}}{\sqrt{b}}$ ---A1	**3 marks**	

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(b)	$5^{n+1}-4(5^n)+5^{n-1}$ $=5^{n-1}(5^2)-4(5^{n-1})(5)+5^{n-1}---\text{M1}$ $=5^{n-1}(5^2-20+1)---\text{M1}$ $=6(5^{n-1})---\text{A1}$ Alternative Method $5\times5^n-4(5^n)+\frac{5^n}{5}---\text{M1}$ $=\frac{1}{5}(25(5^n)-20(5^n)+5^n)$ $=\frac{6}{5}(5^n)---\text{M1}$ $=6(5^{n-1})---\text{A1}$ Since the expression has a factor of 6 which is divisible by 2, hence $5^{n+1}-4(5^n)+5^{n-1}$ is divisible by 2 for all positive integers of n	**3 marks**	
14	Refer to solution on the graph paper.	**8 marks**	

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