CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	**Worked Solutions**
CLASS	4
INDEX NUMBER	
CENTRE/ INDEX NO	/

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **21** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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Answer **all** the questions.

1 Two cylinders are such that the ratio of their heights is $\sqrt{7}:1$. The height of the smaller cylinder is $\dfrac{2\sqrt{7}-1}{\left(2-\sqrt{7}\right)^2}$ cm.

Without using a calculator, find the height of the larger cylinder, expressing your answer in the form $\left(a+b\sqrt{7}\right)$ cm, where a and b are integers. [4]

Height of larger cylinder $= \dfrac{2\sqrt{7}-1}{\left(2-\sqrt{7}\right)^2}\times\sqrt{7}$

$$= \frac{14-\sqrt{7}}{\left(2-\sqrt{7}\right)^2}$$

$$= \frac{14-\sqrt{7}}{4-4\sqrt{7}+7}$$

$$= \frac{14-\sqrt{7}}{11-4\sqrt{7}}\times\frac{11+4\sqrt{7}}{11+4\sqrt{7}}$$

$$= \frac{154+56\sqrt{7}-11\sqrt{7}-28}{9}$$

$$= \frac{126+45\sqrt{7}}{9}$$

$$= 14+5\sqrt{7} \text{ cm}$$

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2 The profit, \$$y$ of a company can be modelled by the equation $y = a(x-h)^2 + k$, where x is the number of goods sold and a, h and k are constants.
The company obtained the maximum profit of \$24 500 when 800 goods were sold.
The company incurred a loss of \$7 500 when no goods were sold.

(a) State the value of h and of k. [2]

Since maximum profit of \$24500 occurs when 800 goods were sold,

$$y = a(x-800)^2 + 24500$$

Therefore, $h = 800$

$k = 24500$

(b) Using the values of h and k found in part **(a)**, find the value of a. [2]

$y = a(x-800)^2 + 24500$

When $x = 0$, $y = -7500$

$-7500 = a(-800)^2 + 24500$

$640\,000\,a = -32\,000$

$a = -\frac{1}{20}$ or -0.05

(c) Find the range of the number of goods the company needed to sell to earn a profit. [2]

$y = -\frac{1}{20}(x-800)^2 + 24500$

For the company to be profitable, $y > 0$

$-\frac{1}{20}(x-800)^2 + 24500 > 0$

$(x-800)^2 - 490\,000 < 0$

$(x-100)(x-1500) < 0$

$100 < x < 1500$

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3 Solve the equation $2\log_3 p + \dfrac{1}{\log_{27} 3} - 1 = \log_3(8p+1)$. [5]

$2\log_3 p + \dfrac{1}{\log_{27} 3} - 1 = \log_3(8p+1)$

$\log_3 p^2 + \log_3 27 - \log_3 3 = \log_3(8p+1)$

$\log_3 \dfrac{27p^2}{3} = \log_3(8p+1)$

$\log_3 9p^2 = \log_3(8p+1)$

$9p^2 = 8p+1$

$9p^2 - 8p - 1 = 0$

$(p-1)(9p+1) = 0$

$p = 1$ or $p = -\dfrac{1}{9}$

(reject)

4 Solve the equation $5^{x+1} = 5^{2x-1} - 30$. [5]

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$5^{x+1} = 5^{2x-1} - 30$

$5\left(5^{x}\right) = \frac{1}{5}\left(5^{2x}\right) - 30$

$\frac{1}{5}\left(5^{2x}\right) - 5\left(5^{x}\right) - 30 = 0$

$5^{2x} - 25\left(5^{x}\right) - 150 = 0$

Let 5^{x} be y

$y^{2} - 25y - 150 = 0$

$\left(y - 30\right)\left(y + 5\right) = 0$

$y = 30$ or $y = -5$

$5^{x} = 30$ or $5^{x} = -5$ (reject)

$x \lg 5 = \lg 30$

$x = \frac{\lg 30}{\lg 5} = 2.11$ (3 s.f)

5 Given that $y = x^{3} - px^{2} - 45x + 10$ is decreasing for $-3 < x < q$, find the value of the constants p and q. [5]

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$\frac{dy}{dx} = 3x^2 - 2px - 45$

Since y is decreasing, $3x^2 - 2px - 45 < 0$

For y is decreasing for $-3 < x < q$,

$3(x+3)(x-q) < 0$

$3x^2 - 3qx + 9x - 9q < 0$

$3x^2 + (9-3q)x - 9q < 0$

Comparing coefficients,

$-9q = -45$

$q = 5$

$-2p = 9 - 3q$

$-2p = 9 - 15$

$p = 3$

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6 The graph $y = a + b\cos(cx)$ is defined for $0 \le x \le 2\pi$, where a, b and c are constants.

The graph has a period of $\frac{4\pi}{3}$ and a minimum value of 1 when $x = \frac{4\pi}{3}$.

The graph also passes through the point $\left(\frac{\pi}{3}, 3\right)$.

(a) Show that $a = 3$, $b = -2$ and $c = \frac{3}{2}$. [3]

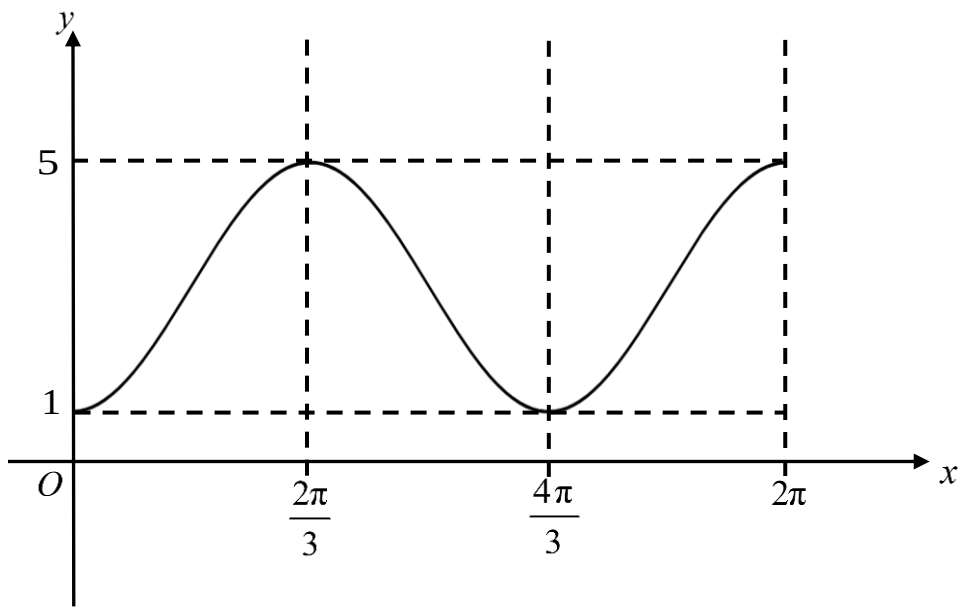
Period = $\frac{2\pi}{c} = \frac{4\pi}{3}$ ➔ $c = \frac{3}{2}$

When $x = \frac{\pi}{3}$, $y = 3$ ➔ $a = 3$

When $x = \frac{4\pi}{3}$, $y = 1$,

$1 = 3 + b$ ➔ $b = -2$

(b) Sketch the graph of $y = a + b\cos(cx)$ for $0 \le x \le 2\pi$. [2]



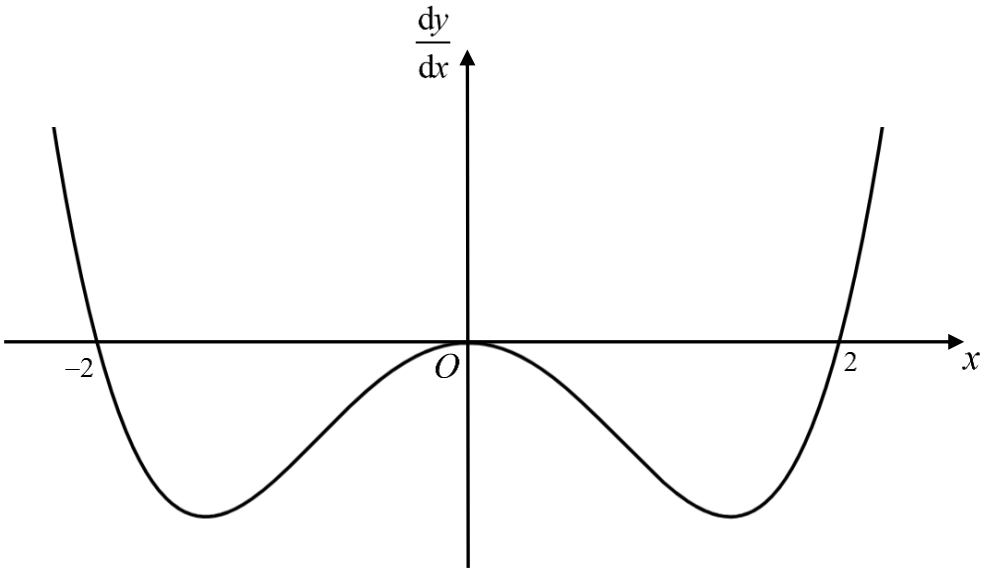
(c) Given that there are exactly 3 solutions for the equation $b\cos(cx) = k - a$, for $0 \le x \le 2\pi$, state the range of values of the constant k. [1]

$-2\cos\left(\frac{3}{2}x\right) + 3 = k$

For 3 points of intersections, $1 < k < 5$

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The graph of $\frac{dy}{dx}$ of a function $y = f(x)$ is shown in the diagram, passing through the x-axis at $(-2,0)$, $(0,0)$ and $(2,0)$.

(a) State the number of stationary points of the graph of $y = f(x)$. [1]

3

(b) State the x-coordinate of the **minimum** point of $y = f(x)$ and explain why it is the minimum point.

[2]

$x = 2$

The gradient is negative before the stationary point and is positive after the stationary point. Hence, the point is a minimum point.

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8 The curve $y = ax + \frac{b}{(2x-1)^3}$, where a and b are constants, has a stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$.

(a) Find the value of a and of b. [4]

When $x = \frac{3}{2}, y = \frac{11}{2}$

$\frac{11}{2} = \frac{3a}{2} + \frac{b}{8}$

$44 = 12a + b$

$b = 44 - 12b$ ------ (1)

$\frac{dy}{dx} = a - 3b(2x-1)^{-4}(2)$

$= a - \frac{6b}{(2x-1)^4}$

When $x = \frac{3}{2}, \frac{dy}{dx} = 0$

$a - \frac{6b}{16} = 0$

$a = \frac{3b}{8}$ ------(2)

Solving (1) and (2),

$a = 3, b = 8$

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(b) Find the *x*-coordinate of the other stationary point. [3]

$$\frac{dy}{dx} = 3 - \frac{48}{(2x-1)^4}$$

To find stationary points, $\frac{dy}{dx} = 0$

$$3 - \frac{48}{(2x-1)^4} = 0$$

$$(2x-1)^4 = 16$$

$$2x - 1 = 2 \quad \text{or} \quad -2$$

$$x = \frac{3}{2} \quad \text{or} \quad -\frac{1}{2}$$

The *x*-coordinate of the other stationary point is $-\frac{1}{2}$.

(c) Find $\frac{d^2y}{dx^2}$. [1]

$$\frac{d^2y}{dx^2} = 192(2x-1)^{-5}(2)$$

$$= \frac{384}{(2x-1)^5}$$

(d) Hence, determine the nature of the stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$. [2]

When $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} = \frac{384}{2^5}$

= 12 (> 0)

Therefore, the stationary point at $\left(\frac{3}{2}, \frac{11}{2}\right)$ is a minimum point.

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9 It is given that $y = \ln\left(\frac{2x}{5-x}\right)$, where $0 < x < 5$.

(a) If y is decreasing at a rate of 10 units/s , find the rate of change of x when $y = \ln 8$. [5]

$$\ln 8 = \ln\left(\frac{2x}{5-x}\right)$$

$$8 = \frac{2x}{5-x}$$

$$x = 4$$

$$y = \ln\left(\frac{2x}{5-x}\right)$$

$$= \ln 2x - \ln(5-x)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{5-x}$$

When $x = 4$, $\frac{dy}{dx} = \frac{5}{4}$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-10 = \frac{5}{4} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = -8$$

Rate of change of x is -8 units/s

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(b) The equation of the normal to the curve at point A is $8y = 8\ln 2 + 25 - 10x$.
Find the x-coordinate of point A. [4]

$$8y = 8\ln 2 + 25 - 10x$$

$$y = -\frac{5}{4}x + \frac{25}{8} + \ln 2$$

Gradient of normal at point $A = -\frac{5}{4}$

Gradient of tangent at point $A = \frac{4}{5}$

$$\frac{1}{x} + \frac{1}{5-x} = \frac{4}{5}$$

$$\frac{5-x+x}{x(5-x)} = \frac{4}{5}$$

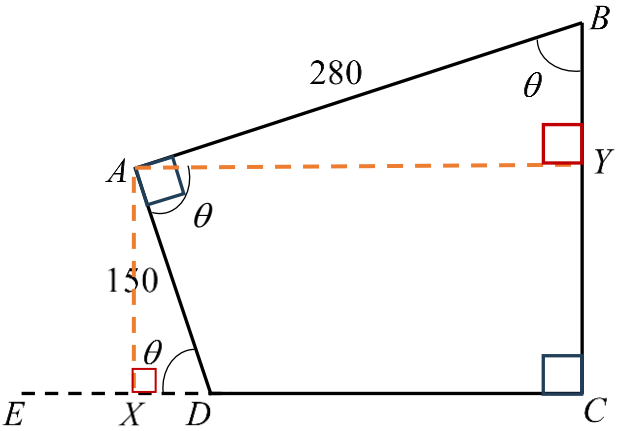
$$4x^2 - 20x + 25 = 0$$

$$(2x-5)^2 = 0$$

$$x = \frac{5}{2}$$

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The diagram shows a running track $ABCD$. D is a point on the straight line EC. It is given that $AB = 280$ m, $AD = 150$ m, angle BCD = angle $DAB = 90°$ and angle $ADE = \theta$, where $0° < \theta < 90°$.

(a) Show that L m, the length of the running track, is given by $L = p + 130\cos\theta + 430\sin\theta$, where p is a constant. [3]

$$\cos\theta = \frac{BY}{280} \qquad \cos\theta = \frac{XD}{150}$$
$$BY = 280\cos\theta \qquad XD = 150\cos\theta$$
$$\sin\theta = \frac{AY}{280} \qquad \sin\theta = \frac{AX}{150}$$
$$AY = 280\sin\theta \qquad AX = 150\sin\theta$$

Therefore, $BC = BY + AX$
$= 280\cos\theta + 150\sin\theta$

$CD = AY - XD$
$= 280\sin\theta - 150\cos\theta$

$L = AB + BC + CD + DA$

$= 280 + 280\cos\theta + 150\sin\theta + 280\sin\theta - 150\cos\theta + 150$

$= 430 + 130\cos\theta + 430\sin\theta$

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(b) Express L in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

$430 + 130\cos\theta + 430\sin\theta = 430 + R\cos(\theta - \alpha)$

$R = \sqrt{130^2 + 430^2} = 10\sqrt{2018}$ or 449.22

$\tan\alpha = \dfrac{430}{130}$

$\alpha = 73.179°$

$430 + 130\cos\theta + 430\sin\theta = 430 + 449\cos(\theta - 73.2°)$

(c) Xin Ting ran five complete laps around the track.
She claims that she ran a total of 5 km. Explain why she might be wrong. [2]

Maximum distance of one lap = 430 + 449 = 879 m

Maximum total distance ran = $5 \times 879 = 4395$ m (<5000)

Hence, it is not possible to run 5 km.

(d) Given that the running track is 700 m long, find the value of θ. [3]

$430 + 449.22\cos(\theta - 73.179°) = 700$

$\cos(\theta - 73.179°) = \dfrac{270}{449.22}$

$\theta - 73.179° = 53.055°$ or $306.945°$

$\theta = 126.234°$ or $\theta = 380.124°$

Since θ is an acute angle, $\theta = 380.124° - 360° = 20.1°$

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11 After n years, the population, P, **in thousands**, of a certain species of foxes can be modelled by an equation $P = Ae^{kn}$, where A and k are constants.

The table below shows the values of n and P.

n	5	10	15	20	25
P	6.23	4.85	3.78	2.94	2.29

(a) On the grid on the next page, plot $\ln P$ against n and draw a straight line graph. [2]

n	5	10	15	20	25
$\ln P$	1.83	1.58	1.33	1.08	0.83

(b) Use your graph to estimate the initial population of the foxes and the value of k. [4]

$\ln P = \ln\left(Ae^{kn}\right)$

$\ln P = \ln A + \ln e^{kn}$

$\ln P = \ln A + kn$

From the graph, vertical intercept = 2.08

$\ln A = 2.08$

$A = 8.00$ (3 sf) ➔ Initial population is 8000

$$\text{Gradient} = \frac{2.08 - 1.08}{0 - 20}$$
$$= -0.05 \quad \rightarrow \quad k = -0.05$$

(c) If this model for the population remains valid, after how many whole years would the population of the foxes first decreased to 1000? [3]

$P = 8e^{-0.05n}$

$8e^{-0.05n} = 1$

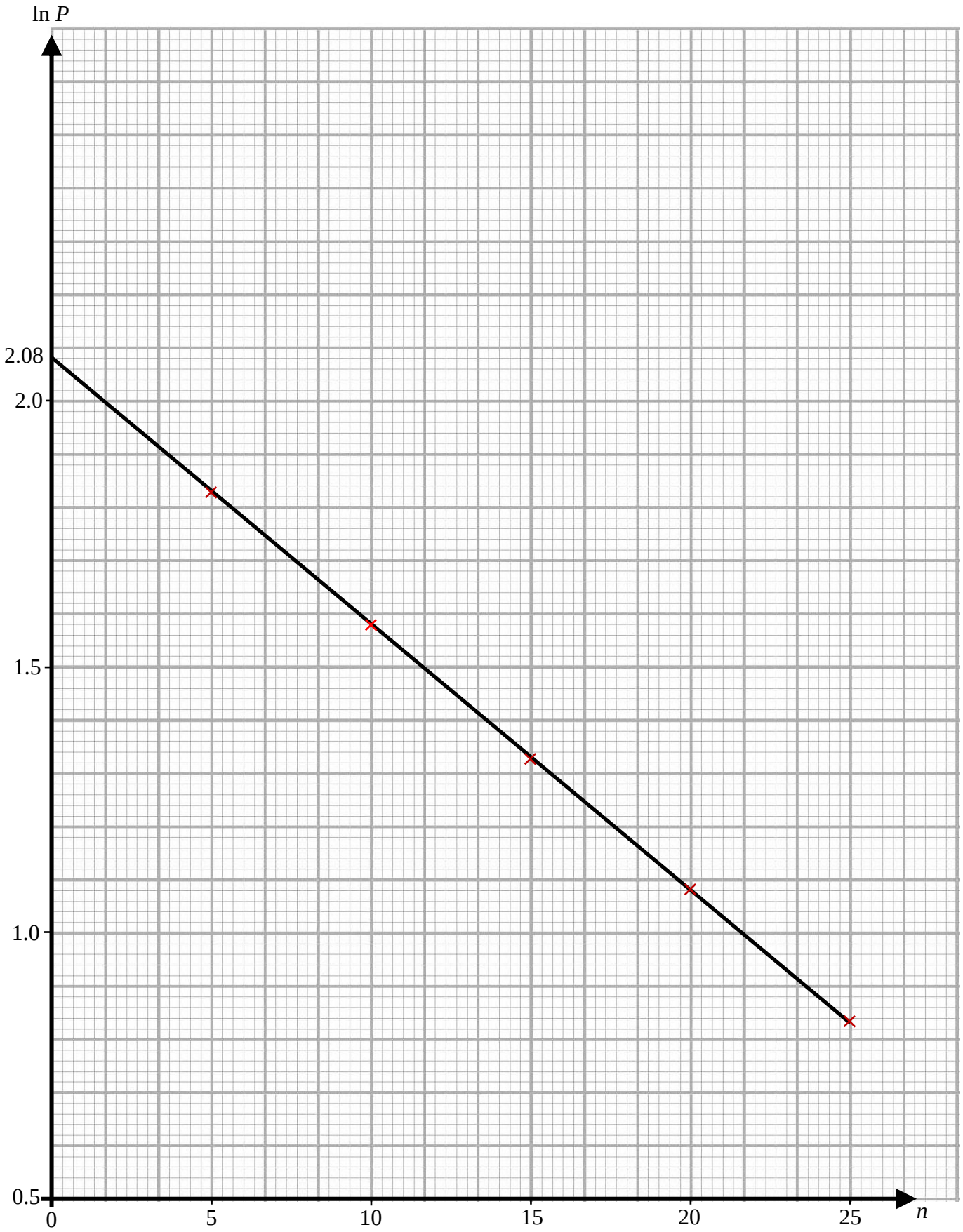
$e^{-0.05n} = \frac{1}{8}$

$-0.05n = \ln\left(\frac{1}{8}\right)$

$n = 41.6$

After **<u>42</u>** years, the population would have first decreased to 1000.

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12 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{3x^2}\right)=\frac{1}{3x^3}-\frac{2\ln 2x}{3x^3}$. [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{3x^2}\right)$$

$$=\frac{\left(3x^2\right)\left(\frac{2}{2x}\right)-\left(6x\right)\left(\ln 2x\right)}{9x^4}$$

$$=\frac{3x-\left(6x\right)\left(\ln 2x\right)}{9x^4}$$

$$=\frac{3x\left(1-2\ln 2x\right)}{9x^4}$$

$$=\frac{1-2\ln 2x}{3x^3}$$

$$=\frac{1}{3x^3}-\frac{2\ln 2x}{3x^3}$$

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(b) Integrate $\frac{\ln 2x}{x^3}$ with respect to x. [3]

$\int \frac{1}{3x^3} - \frac{2\ln 2x}{3x^3}\ \mathrm{d}x = \frac{\ln 2x}{3x^2} + c,$ where c is an arbitrary constant.

$\frac{2}{3}\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = \int \frac{1}{3x^3}\ \mathrm{d}x - \frac{\ln 2x}{3x^2} + c_1$

$\frac{2}{3}\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = -\frac{1}{6x^2} - \frac{\ln 2x}{3x^2} + c_1$

$\int \frac{\ln 2x}{x^3}\ \mathrm{d}x = -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + c_2$

(c) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{2}, 1\right)$ and is such that $\mathrm{f}'(x) = \frac{\ln 2x}{x^3}$, find $\mathrm{f}(x)$. [3]

$\mathrm{f}(x) = \int \frac{\ln 2x}{x^3}\ \mathrm{d}x$

$= -\frac{1}{4x^2} - \frac{\ln 2x}{2x^2} + c$

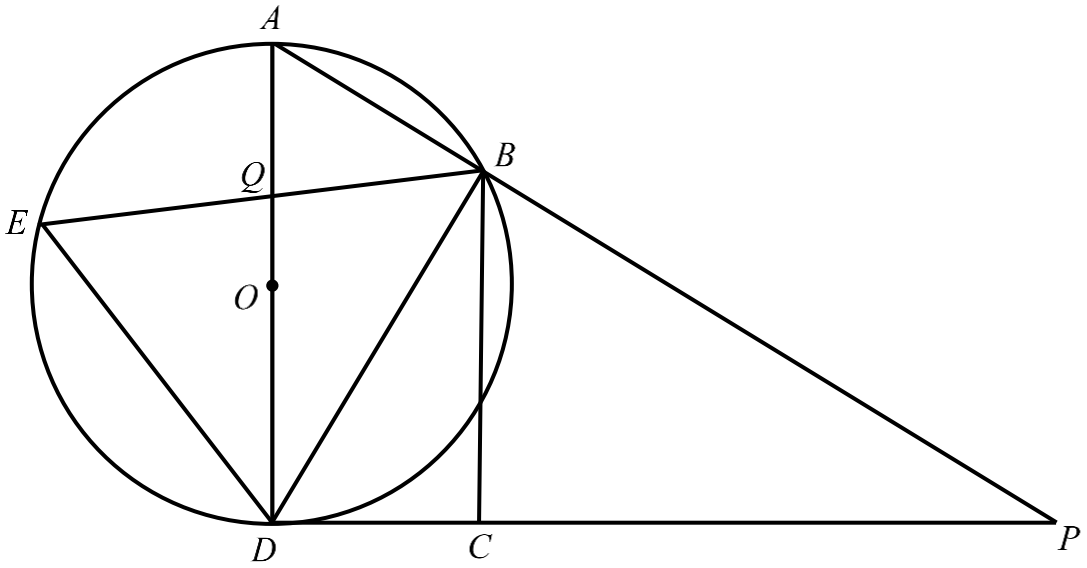
$1 = -\frac{1}{4\left(\frac{1}{2}\right)^2} - 0 + c$

$c = 2$

Equation of curve: $y = 2 - \frac{1}{4x^2} - \frac{\ln 2x}{2x^2}$

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The diagram shows a circle, centre *O*. Points *A*, *B*, *D* and *E* lie on the circle.
The line *AD* is a diameter of the circle and *DCP* is the tangent of the circle at *D*.
The lines *EB* and *AD* intersect at point *Q*.
The chord *AB* is produced to meet the tangent *DCP* at *P*.
It is given that angle *CBP* = angle *DEB*.

(a) Show that angle $BCD = 90°$. [4]

$\angle ABD = 90°$ (right $\angle$ in semicircle)

Let $\angle CDB = x$

$\angle DEB = \angle CDB = x$ (tangent chord theorem)

Since $\angle CBP = \angle DEB$

$\angle CBP = x$

$\angle DBC = 180° - 90° - x$ (adj $\angle$s on str line)
$= 90° - x$

$\angle BCD = 180° - x - (90° - x)$ ($\angle$ sum of Δ)
$= 90°$

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(b) Show that $DP^2 = BP^2 + DA \times BC$. [4]

$\angle ABD = \angle DCB = 90°$

$\angle CDB = \angle DAB$ (tangent chord theorem)

Therefore, Triangle ABD is similar to triangle DCB
(all corresponding angles are equal)

$$\frac{BD}{CB} = \frac{AD}{BD}$$

$BD^2 = DA \times BC$

Since $BD^2 = DP^2 - BP^2$ (Pythagoras' Theorem),

$DP^2 - BP^2 = DA \times BC$

$DP^2 = BP^2 + DA \times BC$

End of Paper

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