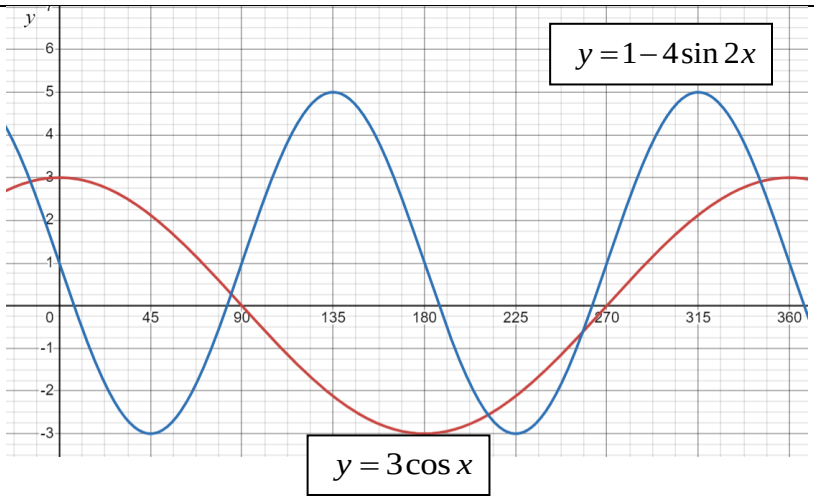
Qns	Suggested Solutions	Remarks
1	$\frac{-3x^2+12x-1}{(3x+1)(x-1)^2}=\frac{A}{3x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ $-3x^2+12x-1=A(x-1)^2+B(3x+1)(x-1)+C(3x+1)$ When $x=1$, $-3+12-1=C(4)$ $C=2$ When $x=-\frac{1}{3}$, $-3\left(-\frac{1}{3}\right)^2+12\left(-\frac{1}{3}\right)-1=A\left(-\frac{4}{3}\right)^2$ $A=-3$ When $x=1,\ A=-3,\ C=2$ $-1=(-3)(-1)^2+B(1)(-1)+(2)(0+1)$ $B=0$ $\frac{-3x^2+12x-1}{(3x+1)(x-1)^2}=-\frac{3}{3x+1}+\frac{2}{(x-1)^2}$	[M1] [M1] [M1] [M1] [A1]
2(a)(i)	$3\cos x$ Amplitude = 3 Period = 360°	 [B1]
2(a)(ii)	$1-4\sin 2x$ Amplitude = 4 $\text{Period}=\frac{360^\circ}{2}$ $=180^\circ$	 [B1] [B1]

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2(b)		[M1] Correct shape [M1] Smoothness [M1] Intersection
2(c)	4 solutions	[B1]
3(a)(i)	$\left(2-\frac{x}{3}\right)^7 = 2^7 + \binom{7}{1}(2)^6\left(-\frac{x}{3}\right) + \binom{7}{2}(2)^5\left(-\frac{x}{3}\right)^2 + \ldots$ $= 128 - \frac{448}{3}x + \frac{224}{3}x^2 + \ldots$	[M1] [A1]
3(a)(ii)	$(p+x)^2\left(2-\frac{x}{3}\right)^7 = \left(p^2 + 2px + x^2\right)\left(128 - \frac{448}{3}x + \frac{224}{3}x^2 + \ldots\right)$ $= \frac{224}{3}p^2x^2 - \frac{896}{3}px^2 + 128x^2 + \ldots$ $= \left(\frac{224}{3}p^2 - \frac{896}{3}p + 128\right)x^2 + \ldots$ $\frac{224}{3}p^2 - \frac{896}{3}p + 128 = -\frac{32}{3}p^2$ $256p^2 - 896p + 384 = 0$ $2p^2 - 7p + 3 = 0$ $(p-3)(2p-1) = 0$ $p = 3$ $p = \frac{1}{2}$ (rejected)	[M1] [M1] [M1] [A1]
3(b)	$\left(x^2 - \frac{1}{2x}\right)^{17}$	

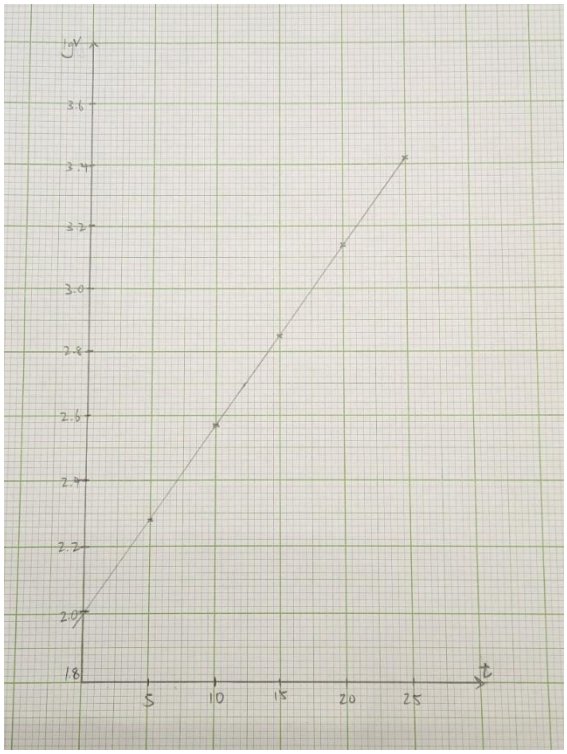
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	$T_{r+1} = \binom{17}{r}\left(x^2\right)^{17-r}\left(-\frac{1}{2x}\right)^r$	[M1]
	$= \binom{17}{r}\left(x^{34-2r}\right)(-1)^r\left(\frac{1}{2}\right)^r\left(\frac{1}{x}\right)^r$	
	$= \binom{17}{r}(-1)^r\left(\frac{1}{2}\right)^r x^{34-3r}$	[M1]
	Let $x^{34-3r} = x^0$	[M1]
	$34-3r = 0$ $r = \frac{34}{3}$ $= 11\frac{1}{3}$ Since r is <u>not an integer</u>, there is no independent term.	[A1]
4(i)	$y = ax^2 + 4x + 3a - 7 \cdots (1)$ $y = 2ax - 5 \cdots (2)$ Sub (2) into (1)	
	$ax^2 + 4x + 3a - 7 = 2ax - 5$	
	$ax^2 + (4-2a)x + 3a - 2 = 0$	[M1]
	$b^2 - 4ac = 0$	
	$(4-2a)^2 - 4a(3a-2) = 0$	[M1]
	$16 - 16a + 4a^2 - 12a^2 + 8a = 0$	
	$-8a^2 - 8a + 16 = 0$	
	$a^2 + a - 2 = 0$	
	$(a+2)(a-1) = 0$	[M1]
	$a = -2, a = 1$	[A1]
4(ii)	Since gradient is positive, $a = 1$. $y = x^2 + 4x - 4 \cdots (1)$ $y = 2x - 5 \cdots (2)$ Sub (2) into (1)	

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	$x^2 + 4x - 4 = 2x - 5$ $x^2 + 2x + 1 = 0$ $(x+1)^2 = 0$ $x = -1$ Sub $x = -1$, $y = -2 - 5$ $y = -7$ $Q(-1, -7)$	[M1] [A1]
5(i)		

t (years)	5	10	15	20	25
V (in thousands \$)	192	370	714	1374	2642
lgV	2.28	2.57	2.85	3.14	3.42



[G1] Axes

[G1] Points plotted correctly

[G1] Straight line passing through all plotted points.

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5(ii)	From graph $\lg V = 2$ $V = 10^2$ $V = 100$ The initial value of investment is \$100 000.	[M1] [A1]
5(iii)	$V = V_o k^t$ $\lg V = t \lg k + \lg V_o$ From graph, Gradient $= \frac{3.08 - 2.4}{19 - 7} = \frac{17}{300}$ $\lg k = \frac{17}{300}$ $k = 1.13937$ $k \approx 1.14$	 [M1] [M1] [A1]
5(iv)	Increase by 50% → investment is at \$150 000 $V = 150$ $\lg V \approx 2.18$ From graph, when $\lg V = 2.18$, $t = 3.25$ years	 [M1] [A1]
6(a)	$2x(x-3) \geq 3 - 5x$ $2x^2 - 6x \geq 3 - 5x$ $2x^2 - x - 3 \geq 0$ $(x+1)(2x-3) \geq 0$ $x \leq -1$ or $x \geq 1\frac{1}{2}$	[M1] Expansion [M1] Factorisation [A1]
6(b)	$2y + k = x \cdots (1)$ $y = 2x + \frac{6}{x} \cdots (2)$ Sub (2) into (1)	

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	$2\left(2x+\frac{6}{x}\right)+k=x$ $4x+\frac{12}{x}+k=x$ $4x^2+12+kx=x^2$ $3x^2+kx+12=0$ $b^2-4ac<0$ $k^2-4(3)(12)<0$ $k^2-144<0$ $(k+12)(k-12)<0$ $-12<k<12$ Therefore, the greatest value of k is 11.	[M1] [M1] [M1] [A1]
7(a)	$y=2x^2-6x+9$ $=2\left(x^2-3x\right)+9$ $=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+9$ $=2\left(x-\frac{3}{2}\right)^2+\frac{9}{2}$ Since $\left(\frac{3}{2},\frac{9}{2}\right)$ is a <u>minimum point, it is not possible</u> for the curve to have a value smaller than $\frac{9}{2}$.	[M1] [A1]
7(b)	$y=3x+5\cdots(1)$ $y=2x^2-6x+9\cdots(2)$ Sub (1) into (2) $2x^2-6x+9=3x+5$ $2x^2-9x+4=0$ $(x-4)(2x-1)=0$ $x=4, x=\frac{1}{2}$	[M1]

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	Sub $x = 4$ $y = 3(4) + 5$ $= 17$ $(4,17)$	[M1]
	Sub $x = \frac{1}{2}$ $y = 3\left(\frac{1}{2}\right) + 5$ $= \frac{13}{2}$ $\left(\frac{1}{2}, \frac{13}{2}\right)$	
	Length PQ $= \sqrt{\left(4 - \frac{1}{2}\right)^2 + \left(17 - \frac{13}{2}\right)^2}$ $= \sqrt{\frac{490}{4}}$ $= \frac{\sqrt{490}}{2}$ or $\frac{7\sqrt{10}}{3}$ or $\frac{7}{2}\sqrt{10}$	[M1] [A1]
8(a)	$y = 27 + \frac{8}{(x-2)^3}$ $\frac{dy}{dx} = -24(x-2)^{-4}$ $= -\frac{24}{(x-2)^4}$ Since $\frac{dy}{dx} \neq 0$ for all $x \neq 2$, the curve does not have a stationary point.	[M1] [M1] [A1]
8(b)	Sub $x = 0$	

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	$y = 27 + \frac{8}{(-2)^3}$ $= 26$ $B(0, 26)$ Sub $y = 0$ $0 = 27 + \frac{8}{(x-2)^3}$ $-27 = \frac{8}{(x-2)^3}$ $-27(x-2)^3 = 8$ $(x-2)^3 = -\frac{8}{27}$ $x - 2 = -\frac{2}{3}$ $x = 1\frac{1}{3}$ $A\left(1\frac{1}{3}, 0\right)$	[B1] [B1]
8(c)	Sub $x = 0$ $\frac{dy}{dx} = -\frac{24}{(-2)^4}$ $= -\frac{3}{2}$ Gradient of normal $= \frac{2}{3}$ $y - 26 = \frac{2}{3}(x - 0)$ $y = \frac{2}{3}x + 26$	[M1] [M1] [A1]
8(d)	Sub $y = 0$	

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	$\frac{2}{3}x = -26$ $x = -39$ Area of triangle BOC $= \frac{1}{2} \times 39 \times 26$ $= 507 \text{ units}^2$	[M1] [A1]
9(i)	$\angle PAB = \angle BCA$ (alternate segment Thm) $\angle ABC = 90^\circ$ (angle in semi-circle) $\angle ABP = 90^\circ$ (angle on a straight line) $\therefore \angle ABP = \angle ABC = 90^\circ$ $\therefore \Delta BAP$ is similar to ΔBCA (AA)	[M1] [A1]
9(ii)	Let $\angle AKH = x$, $\angle KAP = 90^\circ$ (tangent perpendicular to radius) $\angle APH = 90^\circ - x$ (sum of angles in triangle) $\angle HPB = 90^\circ - x$ (angle bisector) $\angle ABC = 90^\circ$ (angle in semi-circle) $\angle ABP = 90^\circ$ (angle on a straight line) $\angle BHP = 180^\circ - 90^\circ - \left(90^\circ - x\right)$ (sum of angles in triangle) $= x$ $\angle AHK = x$ (vertically opposite angles) $\therefore$ Since $\angle AHK = \angle AKH, AH = AK$ (isosceles triangle)	 [M1] [A1]
9(iii)	Since ΔBHP is similar to ΔAKP $\frac{AK}{BH} = \frac{KP}{HP}$ $\Rightarrow \frac{AH}{BH} = \frac{KP}{HP}$ ($AK = AH$ from (ii)) $\Rightarrow AH \times HP = KP \times BH$ (Proved)	[M1] [A1]

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10	$y = \frac{e^{2x}}{1-x}$ $\frac{dy}{dx} = \frac{2e^{2x}(1-x)+e^{2x}}{(1-x)^2}$ $= \frac{3e^{2x}-2xe^{2x}}{(1-x)^2}$ $= \frac{e^{2x}(3-2x)}{(1-x)^2}$ Since $x > \frac{3}{2}$, $(1-x)^2 > 0$ $\frac{1}{(1-x)^2} > 0$ $\frac{(3-2x)}{(1-x)^2} < 0$ $\frac{e^{2x}(3-2x)}{(1-x)^2} < 0$ $\frac{dy}{dx} < 0$ Since $\frac{dy}{dx} < 0$, y is always decreasing.	[M1] [M1] [M1] [M1] [A1]
11	$\frac{d^2y}{dx^2} = 36e^{3x} - e^{-x}$ $\frac{dy}{dx} = \int 36e^{3x} - e^{-x}dx$ $= \frac{36}{3}e^{3x} + e^{-x} + c$ $= 12e^{3x} + e^{-x} + c$ Sub $\frac{dy}{dx} = 14, x = 0$ $14 = 12 + 1 + c$ $c = 1$	[M1] [M1] [M1]

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	$\frac{dy}{dx} = 12e^{3x} + e^{-x} + 1$ $y = \int 12e^{3x} + e^{-x} + 1 dx$ $y = 4e^{3x} - e^{-x} + x + d$ Sub $x = 0, y = 5$ $5 = 4 - 1 + 0 + d$ $d = 2$ $y = 4e^{3x} - e^{-x} + x + 2$	[M1] [M1] [M1] [A1]
12(i)	When $t = 0$, $v = 4\sin^2\left(\frac{0}{2}\right) - 1 = -1$ m/s $\therefore$ initial velocity = –1 m/s	[B1]
12(ii)	When acceleration, $a = 2$ m/s^2 $\frac{dv}{dt} = 8\sin\left(\frac{t}{2}\right) \times \cos\left(\frac{t}{2}\right) \times \frac{1}{2}$ $4\sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right) = 2$ $2\sin t = 2$ $\sin t = 1$ $t = \frac{\pi}{2}$ $\therefore$ when $a = 2$ m/s^2, $t = \frac{\pi}{2}$ and $v = 4\sin^2\left(\frac{\pi}{4}\right) - 1$ $v = 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1$ m/s	[M1] [M1] [M1] [A1]
12(iii)	Instantaneous rest $\Rightarrow$ $v = 0$ $\Rightarrow 4\sin^2\left(\frac{t}{2}\right) - 1 = 0$ $\Rightarrow \sin\left(\frac{t}{2}\right) = \pm\frac{1}{2}$ $\Rightarrow \frac{t}{2} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \ldots,$ For first instantaneous rest, $\frac{t}{2} = \frac{\pi}{6}$ $\Rightarrow t = \frac{\pi}{3}$ *or* 1.05 second	[M1] [M1] [A1]

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12(iv)	$\cos t = 1 - 2\sin^2\left(\frac{t}{2}\right)$ $2\sin^2\left(\frac{t}{2}\right) = 1 - \cos t$ $4\sin^2\left(\frac{t}{2}\right) = 2 - 2\cos t$ $S = \int 4\sin^2\left(\frac{t}{2}\right) - 1 dt$ $= \int 2 - 2\cos t - 1 dt$ $= \int 1 - 2\cos t dt$ $= t - 2\sin t + c$ When $t = 0$, $S = 0$, $c = 0$ When $t = 2$ $S = 2 - 2\sin(2)$ ≈ 0.18140 When $t = 3$ $S = 3 - 2\sin(3)$ ≈ 2.7177 Distance travelled during the 3rd second $2.7177 - 0.18140 \approx 2.54$ m	[M1] Double angle formula [M1] Finding S either $t = 2$ or $t = 3$ [A1]

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