CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/2

Paper 2 **23 Aug 2024**
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 The equation of a curve is $y = 5\sin^2\left(x - \frac{\pi}{6}\right)$, where $0 \le x \le \frac{\pi}{2}$.

(a) Given that y is decreasing at a rate of 0.3 units per second, find the rate of change of x at $x = \frac{5\pi}{12}$. [3]

$y = 5\sin^2\left(x - \frac{\pi}{6}\right)$

$\frac{dy}{dx} = 10\sin\left(x - \frac{\pi}{6}\right)\cos\left(x - \frac{\pi}{6}\right)$ **A1 –** Correct derivative

At $x = \frac{5\pi}{12}$,

$\frac{dy}{dx} = 10\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = 5$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ **M1 –** Correct use of connected rates of change with substitution

$-0.3 = 5 \times \frac{dx}{dt}$

$\frac{dx}{dt} = -0.06$ units/s **A1**

(b) The normal to the curve at $x = \frac{5\pi}{12}$ intersects the vertical axis at $(0, k)$.
Find the exact value of k. [3]

Gradient of normal $= -\frac{1}{5}$ **FTA1 –** Correct gradient of normal

At $x = \frac{5\pi}{12}$, $y = 5\sin^2\left(\frac{\pi}{4}\right) = \frac{5}{2}$ **M1 –** for finding value of y

$\therefore y - \frac{5}{2} = -\frac{1}{5}\left(x - \frac{5\pi}{12}\right)$

$y = -\frac{1}{5}x + \frac{5}{2} + \frac{\pi}{12}$

At horizontal axis, $x = 0$

$\therefore k = \frac{5}{2} + \frac{\pi}{12}$ **A1**

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2 (a) Find the values of x and y which satisfy the equations

$$8^x - 2^{-y} = 0,$$
$$\left(\sqrt{125^x}\right)^y = \frac{1}{\sqrt{5}}.$$

[3]

$8^x - 2^{-y} = 0$

$2^{3x} = 2^{-y}$

$y = -3x$

and

$\left(\sqrt{125^x}\right)^y = \frac{1}{\sqrt{5}}$

$5^{\frac{3xy}{2}} = 5^{-\frac{1}{2}}$

$\frac{3xy}{2} = -\frac{1}{2}$

$3xy = -1$

M1 – Correct simplification of equations

Hence, solving simultaneous equations

$3x(-3x) = -1$

$9x = 1$

$x = \frac{1}{3} \quad or \quad -\frac{1}{3}$

$y = -1 \quad or \quad 1$

M1 – Eliminating one variable using substitution

A1

When $x = \frac{1}{3}, \quad y = -1$, $x = -\frac{1}{3}, \quad y = 1$

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(b) Show that the equation $3\left(2^{x+2}\right)-1=35\left(2^{-x}\right)$ has only one solution and find its value correct to 2 significant figures. [5]

$3\left(2^{x+2}\right)-1=35\left(2^{-x}\right)$

$12\left(2^{x}\right)-1=\frac{35}{2^{x}}$

let $y=2^{x}$

$12y-1=\frac{35}{y}$

M1 – Correct substitution to obtain a quadratic equation

$12y^{2}-y-35=0$

$(4y-7)(3y+5)=0$

M1 – Factorisation

$y=\frac{7}{4}\quad or\quad y=-\frac{5}{3}$

$2^{x}=\frac{7}{4}\quad or\quad 2^{x}=-\frac{5}{3}\,(rej)$

A1

$x\lg 2=\lg\frac{7}{4}$

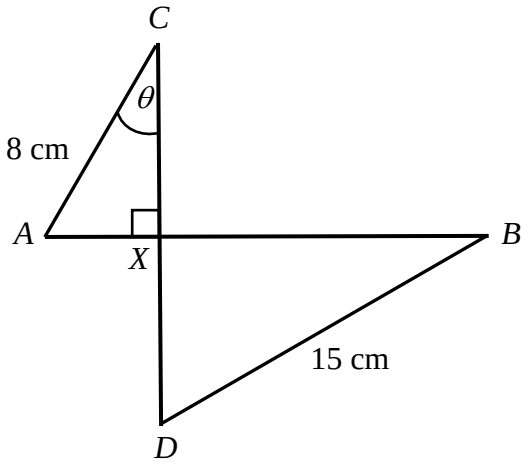
M1 – Using logarithm to solve

$x=\frac{\lg\frac{7}{4}}{\lg 2}=0.81$

A1 – Answer to 2s.f.

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3



The diagram shows two perpendicular lines *AB* and *CD* which intersect at *X*. The points *A, B, C* and *D* lie on the circumference of a circle. *AC* = 8 cm, *BD* = 15 cm, and angle *ACD* equals to $\theta°$.

(a) Show that the length of *AB* is $8\sin\theta + 15\cos\theta$. [2]

As *A, B, C* and *D* lie on the circumference of a circle,

Angle $ABD = \theta°$

$AX = 8\sin\theta$
$XB = 15\cos\theta$ **M1**

$\therefore AB = AX + XB = 8\sin\theta + 15\cos\theta \quad (shown)$ **AG1**

(b) Express *AB* in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [4]

$R = \sqrt{8^2 + 15^2} = 17$ **M1, A1**

$\alpha = \tan^{-1}\frac{15}{8} = 61.9°$ **M1**

$\therefore 8\sin\theta + 15\cos\theta = 17\sin(\theta + 61.9°)$ **A1**

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(c) Find the value(s) of θ if AB = 16 cm. [3]

$17\sin(\theta + 61.9275°) = 16$

$\sin(\theta + 61.9275°) = \frac{16}{17}$

$\alpha = \sin^{-1}\frac{16}{17} = 70.250°$ **M1 –** Finding basic angle

$\theta + 61.9275° = 70.250° \quad or \quad 180° - 70.250°$

$\theta = 8.3226° \quad or \quad 47.8225°$

$\theta = 8.3° \quad or \quad 47.8°$ **A1, A1**

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4 A calculator must not be used in this question.

It is given that $\dfrac{\cos(A+B)}{\cos(A-B)} = \dfrac{1}{3}$.

(a) Show that $\tan A \tan B = \dfrac{1}{2}$. [3]

$\dfrac{\cos A\cos B - \sin A\sin B}{\cos A\cos B + \sin A\sin B} = \dfrac{1}{3}$

$3\cos A\cos B - 3\sin A\sin B = \cos A\cos B + \sin A\sin B$

$2\cos A\cos B = 4\sin A\sin B$

$\dfrac{\sin A\sin B}{\cos A\cos B} = \dfrac{2}{4}$

$\tan A\tan B = \dfrac{1}{2}$

M1 – Correct use of addition formulae

M1 – Cross multiplying and simplifying

M1 – Dividing by cos*A*cos*B* to obtain expression

(b) If $\tan A = 2+\sqrt{3}$, find an expression for $\tan B$, in the form $a+b\sqrt{3}$, where a and b are constants. [3]

$\left(2+\sqrt{3}\right)\tan B = \dfrac{1}{2}$

$\tan B = \dfrac{1}{2\left(2+\sqrt{3}\right)} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{2-\sqrt{3}}{2(4-3)}$

$= \dfrac{2-\sqrt{3}}{2}$

$= 1 - \dfrac{1}{2}\sqrt{3}$

M1 – Rationalising denominator

M1 – Correct use of difference of two squares to simplify denominator

A1

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(c) Hence, express $\sec^2 B$ in the form $c + d\sqrt{3}$, where c and d are constants. [3]

$\tan^2 B$ $= \left(1 - \frac{1}{2}\sqrt{3}\right)^2$ $= 1 - \sqrt{3} + \frac{3}{4} = \frac{7}{4} - \sqrt{3}$	**FTB1 –** Correct squaring of tan B
$\sec^2 B = 1 + \tan^2 B = \frac{11}{4} - \sqrt{3}$	**M1 A1**

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5 The equation of a circle is $x^2 + y^2 - 6x + 16y + 48 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

$(x^2 - 6x) + (y^2 + 16y) + 48 = 0$	**M1 –** Use of completing the square
$(x-3)^2 - 9 + (y+8)^2 - 64 + 48 = 0$	**A1 –** for correct $(x-3)^2$
$(x-3)^2 + (y+8)^2 = 25 = 5^2$	**A1 –** for correct $(y+8)^2$
Radius = 5 units Coordinates of center = $(3,-8)$	**FTA1**
OR	
$2g = -6$ and $2f = 16$ $-g = 3$ and $-f = -8$ $\therefore C(3,-8)$	**B1, B1**
$r = \sqrt{3^2 + (-8)^2 - 48} = 5$	**M1, A1**

(b) The point $A(0,-4)$ lies on the circle.
Given that *AB* is a diameter of the circle, find the coordinates of *B*. [2]

Let $B(x,y)$. As centre is the midpoint of a diameter	
$3 = \frac{x + (0)}{2}$	
$x = 6$	**M1 –** Using midpoint or proportions
$-8 = \frac{y + (-4)}{2}$	
$y = -12$	
$\therefore B(6,-12)$	**A1**

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(c) A line with equation $y = mx$, where $m > 0$, does not intersect the circle. A is the point on the circle closest to the line.

(i) Find the value of m. [2]

Equation of line passing through A and centre of circle:

$gradient = \frac{-4-(-8)}{0-3} = \frac{4}{-3}$ **M1 –** for finding equation of line passing through the centre

$y + 4 = -\frac{4}{3}x$

$y = -\frac{4}{3}x - 4$

$\therefore m = \frac{3}{4}$ **A1**

(ii) Hence, find the coordinates of the point on the line that is closest to the circle. [2]

$\frac{3}{4}x = -\frac{4}{3}x - 4$ **M1** – Equating the lines to solve

$x = -\frac{48}{25}$

$y = \frac{3}{4}\left(-\frac{48}{25}\right) = -\frac{36}{25}$

$\therefore \left(-\frac{48}{25}, -\frac{36}{25}\right)$ **A1**

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6 It is given that $f(x) = 4x^p + qx^2 - 3x + 1$, where p and q are constants, has a factor of $x - 1$ and leaves a remainder of -33 when divided by $x + 2$.

(a) Find the values of p and q. [3]

Using factor theorem,

$0 = 4(1)^p + q - 3 + 1$

$0 = 4 + q - 3 + 1$

$q = -2$

M1 – Correct use of factor theorem

Using remainder theorem,

$-33 = 4(-2)^p + 4(-2) + 6 + 1$

$-32 = 4(-2)^p$

$-8 = (-2)^p$

$p = 3$

M1 – Correct use of remainder theorem

A1

(b) Using the values of p and q found in part **(a)**, solve the equation $f(x) = 0$ completely, leaving non-integer roots in their simplest surd form. [4]

Using factor theorem,

$4x^3 - 2x^2 - 3x + 1 = 0$

$(x-1)(4x^2 + 2x - 1) = 0$

$\therefore x = 1 \quad or$

$4x^2 + 2x - 1 = 0$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$

$= \dfrac{-2 \pm \sqrt{20}}{8} = \dfrac{-2 \pm 2\sqrt{5}}{8}$

$= \dfrac{-1 + \sqrt{5}}{4} \quad or \quad \dfrac{-1 - \sqrt{5}}{4}$

M1 – Correct Factorisation

M1 – Correct use of quadratic formula

A1, A1

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Name: _____ Class: _____ ( )

7 (a) Prove the identity $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} = -2\cot 2\theta$. [3]

$$LHS = \frac{1-2\cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{-\left(2\cos^2\theta - 1\right)}{\frac{1}{2}\left(2\sin\theta\cos\theta\right)}$$
$$= \frac{-\cos 2\theta}{\frac{1}{2}\sin 2\theta}$$
$$= \frac{-2\cos 2\theta}{\sin 2\theta}$$
$$= -2\cot 2\theta = RHS \quad (shown)$$

M1 – Correct use of double angle formulae for $\sin 2\theta$

M1 – Correct use of double angle formulae for $\cos 2\theta$

M1 – Correct use of $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(b) Hence, solve the equation $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta} + \tan 2\theta + 1 = 0$, for $0° \le \theta \le 90°$. [5]

$$-2\cot 2\theta + \tan 2\theta + 1 = 0$$
$$\frac{-2}{\tan 2\theta} + \tan 2\theta + 1 = 0$$
$$\left(\tan 2\theta\right)^2 + \tan 2\theta - 2 = 0$$
$$\left(\tan 2\theta + 2\right)\left(\tan 2\theta - 1\right) = 0$$

$\tan 2\theta = -2$	*or*	$\tan 2\theta = 1$
Basic Angle α	*or*	Basic Angle α
$= \tan^{-1} 2$	*or*	$= \tan^{-1} 1$
$= 63.435°$	*or*	$= 45°$
$2\theta = 116.565°, 296.565°$	*or*	$2\theta = 45°, 225°$
$\theta = 58.3°, 148.3°(rej)$	*or*	$\theta = 22.5°, 112.5°(rej)$

$\therefore \theta = 22.5°, 58.3°.$

M1 – Correctly forming quadratic equation

M1 – Correctly solving quadratic equation

M1 – Correctly finding basic angle for $\tan 2\theta = -2$

A1, A1

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8 The blood alcohol concentration, C mg/L, in a person t minutes after he consumes a bottle of wine can be modelled by the formula

$$C = 1250\left(e^{kt} - e^{-0.1t} \right).$$

(a) In Singapore, a driver can be charged with drink driving if he drives when his blood alcohol concentration exceeds 800mg/L. When Jonathan consumes a bottle of wine, his blood alcohol concentration will only fall to 800mg/L after three hours.

Show that $k = -0.0025$ when corrected to 2 significant figures, and find his blood alcohol concentration after 1 hour. [4]

$$800 = 1250\left(e^{180k} - e^{-0.1(180)}\right)$$

$$\frac{800}{1250} = e^{180k} - e^{-18}$$

$$e^{180k} = \frac{800}{1250} + e^{-18}$$

$$180k = \ln\left(\frac{800}{1250} + e^{-18}\right)$$

$$k = -0.0024793$$

M1 – Separating constants and variable

M1 – Correctly using logarithms to solve for k

AG1

When $t = 60$,

$$C = 1250\left(e^{-0.0024793(60)} - e^{-0.1(60)}\right)$$

$$= 1074.123$$

$$= 1070 \text{ mg/L}$$

A1

Using $k = -0.0025$,

(b) Find the rate of change of Jonathan's blood alcohol concentration after 1 hour. [2]

$$\frac{dC}{dt} = 1250\left(-0.0025e^{-0.0025t} + 0.1e^{-0.1t}\right)$$

B1

When $t = 60$,

$$\frac{dC}{dt} = 1250\left(-0.0025e^{-0.0025(60)} + 0.1e^{-0.1(60)}\right)$$

$$= -2.3799$$

$$= -2.38 \text{ mg/Lmin}$$

B1

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(c) After consumption of alcohol, the blood alcohol concentration will rise to a peak before decreasing slowly over time. Explain why the blood alcohol concentration found in part **(a)** is not the peak level, and find the peak blood alcohol concentration level. [5]

When $t = 60$,

$\frac{dC}{dt} = -2.38 \neq 0$

Hence blood alcohol concentration is not at peak. **B1**

$\frac{dC}{dt} = 1250\left(-0.0025e^{-0.0025t} + 0.1e^{-0.1t}\right) = 0$ **M1** – equating to 0 to solve

$-0.0025e^{-0.0025t} + 0.1e^{-0.1t} = 0$

$0.0025e^{-0.0025t} = 0.1e^{-0.1t}$

$\frac{e^{-0.0025t}}{e^{-0.1t}} = \frac{0.1}{0.0025}$ **M1** – Correctly solving using logarithms

$e^{0.0975t} = 40$

$0.0975t = \ln 40$

$t = 37.835$ mins **A1**

$\Rightarrow C = 1250\left(e^{-0.0025(37.835)} - e^{-0.1(37.835)}\right)$

$= 1108.756$

$= 1110$ mg/L **A1**

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9 **(a)** The equation of a curve is $y = \ln\sqrt{\dfrac{x^2+1}{2x+1}}$.

Show that $\dfrac{dy}{dx} = \dfrac{x}{x^2+1} - \dfrac{1}{2x+1}$. [4]

$y = \ln\sqrt{\dfrac{x^2+1}{2x+1}}$

$= \dfrac{1}{2}\left[\ln\left(x^2+1\right) - \ln\left(2x+1\right)\right]$

B1 – Simplifying Logarithms using power rule

B1 – Simplifying Logarithms using quotient rule

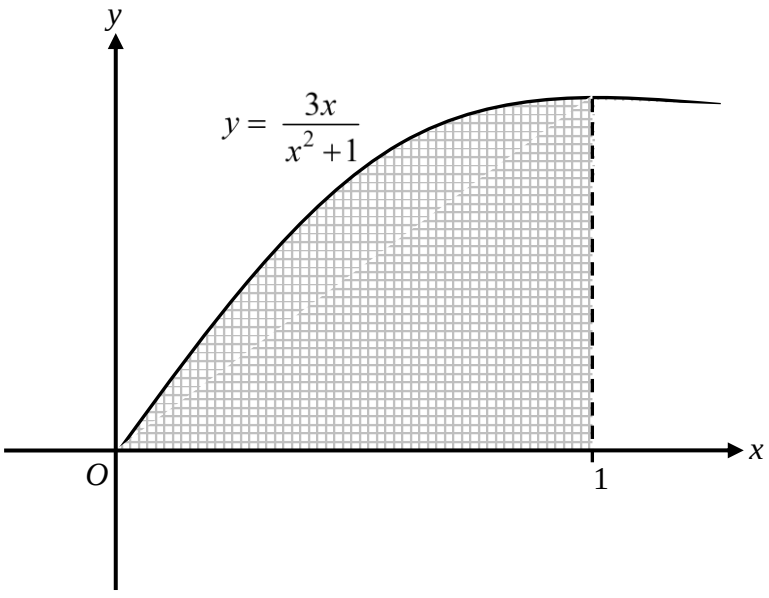
$\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{2x}{x^2+1} - \dfrac{2}{2x+1}\right)$

$= \dfrac{x}{x^2+1} - \dfrac{1}{2x+1}$ $(shown)$

A2,1 – Correct derivatives

(b) The diagram below shows part of the graph of $y = \dfrac{3x}{x^2+1}$.

Using the result from part **(a)**, find the area of the shaded region bounded by the curve, the x- axis and the line $x = 1$. Express your answer in the form $a \ln b$, where a and b are constants. [6]



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Continuation of working space for Question 9.

By reverse differentiation

$$\int \frac{x}{x^2+1} - \frac{1}{2x+1}\ \mathrm{d}x = \ln\sqrt{\frac{x^2+1}{2x+1}} + c$$

$$\int \frac{x}{x^2+1}\,\mathrm{d}x = \ln\sqrt{\frac{x^2+1}{2x+1}} + \frac{1}{2}\ln(2x+1) + c$$

B1 – Correct identification of reverse differentiation

B1 – $\frac{1}{2}\ln(2x+1)$

Area of shaded region

$$= \int_0^1 \frac{3x}{x^2+1}\,\mathrm{d}x$$

$$= 3\int_0^1 \frac{x}{x^2+1}\,\mathrm{d}x$$

$$= 3\left[\ln\sqrt{\frac{x^2+1}{2x+1}} + \frac{1}{2}\ln(2x+1)\right]_0^1$$

$$= 3\left[\left(\ln\sqrt{\frac{2}{3}} + \frac{1}{2}\ln 3\right) - 0\right]$$

$$= 3\left(\frac{1}{2}\ln\frac{2}{3} + \frac{1}{2}\ln 3\right)$$

$$= \frac{3}{2}\ln\left(\frac{2}{3}\times 3\right)$$

$$= \frac{3}{2}\ln 2$$

B1 – Correct definite integral

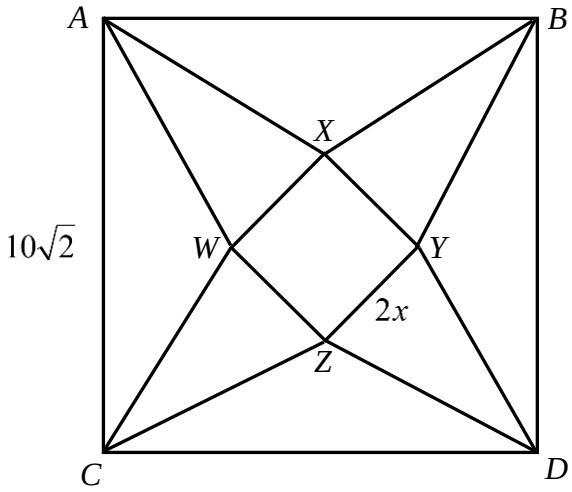
M1 – substitution into a logarithmic function to find value

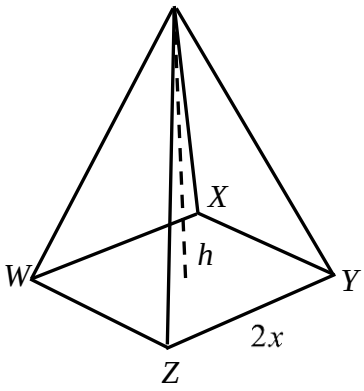
M1– combining into single fraction using logarithm laws

A1 – Simplified form

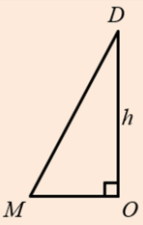
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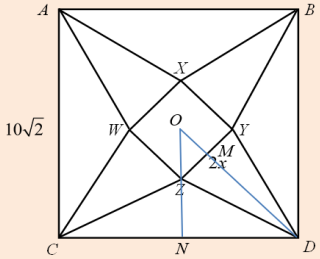
10 In the diagram below, *ABCD* is a square paper with side $10\sqrt{2}$ cm. The net of a regular pyramid with square base *WXYZ* was cut from the paper. *AB* is parallel to *WY* and the base of the pyramid has sides $2x$ cm.





(a) By expressing the perpendicular height, *h* cm, of the pyramid in terms of *x*, show that $V = \frac{8}{3}x^2\sqrt{25-5x}$. [4]





Let centre of square paper be *O* and midpoint of *YZ* and *CD* be *M* and *N* respectively.

$OM = \frac{1}{2}XY = x$

$ON = ND = \frac{1}{2}AC = 5\sqrt{2}$

$OD = \sqrt{\left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2} = \sqrt{100} = 10$

M1 – Finding *OD* or *AD*

$\Rightarrow MD = 10 - x$

$h = \sqrt{MD^2 - OM^2}$

$= \sqrt{(10-x)^2 - x^2}$

M1 – Using Pythagoras Thm to find *h*

$= \sqrt{100 - 20x}$

A1 – $h = \sqrt{100-20x}$

$V = \frac{1}{3}(2x)^2\sqrt{100-20x}$

$= \frac{1}{3}\left(4x^2\right)2\sqrt{25-5x}$

AG1

$= \frac{8}{3}x^2\sqrt{25-5x}$ *(shown)*

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(b) Given that x can vary, find the value of x for which the volume of the pyramid is stationary. [6]

$$\frac{dV}{dx} = \frac{16}{3}x\sqrt{25-5x} + \frac{8}{3}x^2\left(\frac{-5}{2\sqrt{25-5x}}\right)$$

At stationary point,

$$0 = \frac{16}{3}x\sqrt{25-5x} + \frac{8}{3}x^2\left(\frac{-5}{2\sqrt{25-5x}}\right)$$

$$\frac{16}{3}x\sqrt{25-5x} = \frac{8}{3}x^2\left(\frac{5}{2\sqrt{25-5x}}\right)$$

$$\frac{16}{3}x(25-5x) = \frac{20}{3}x^2$$

$$20x - 5x^2 = 0$$

$$5x(4-x) = 0$$

$$x = 0\ (rej) \quad or \quad x = 4$$

B1, B1 – Correctly using product rule

M1 – Finding stationary point by equating to 0

M1 – removing surds in denominator through cross multiplication or combining into a single fraction

M1 – Correct simplification

A1

(c) Determine whether this value of x gives a maximum or minimum value for the volume of the pyramid. [2]

By first derivative test,

x	3.99	4	4.01
$\frac{dV}{dx}$	+	0	-
Shape	/	—	\

When $x = 4$, Volume is a **maximum**.

M1 – First derivative test

A1

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