Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS **4052/01**
PAPER 1

14 August 2023 Monday 2 hours 15 minutes

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2023 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATION

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES:

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen. You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

Note that all the diagrams in this paper are not drawn to scale.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use				
Category	Accuracy	Notations	Others	*Marks Deducted*
Question No.				

Total Marks
90

This question paper consists of **23** printed pages (including this cover page) and **1** blank page.

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Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

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Answer **all** the questions.

1 Solve $7x = 18 + 3x$.

Answer $x =$ …….. [1]

---

2 **(a)** Calculate $\dfrac{26.18^3}{\sqrt{4.52 - 0.4^2}}$.

Write your answer correct to 5 significant figures.

Answer …… [1]

(b) Write your answer to **part (a)** in standard form.

Answer …… [1]

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3 **(a)** Express 784 as the product of prime factors.

Answer ……[1]

(b) It is given that a and b are prime numbers.

Find the smallest values of a and b such that $784 \times \frac{a}{b}$ is a perfect cube.

Answer $a =$ ……[1]

$b =$ ……[1]

---

4 Expand and simplify $(w+5)(1-w)$.

Answer ……[2]

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5 The bar graph below shows the results of a survey conducted on the service quality of a hotel.



(a) Find the percentage of respondents who answered 'Strongly Satisfied' and 'Satisfied'.

Answer ……% [1]

(b) Suggest the use of another statistical diagram to represent the results of the survey conducted, that can show the relative size of a part in relation to the whole.

Answer …… [1]

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6 Find the largest integer that satisfies $2y - 3 < 4$.

Answer ……. [2]

---

7 P is directly proportional to Q^3.

When $Q = 2$, $P = 64$.

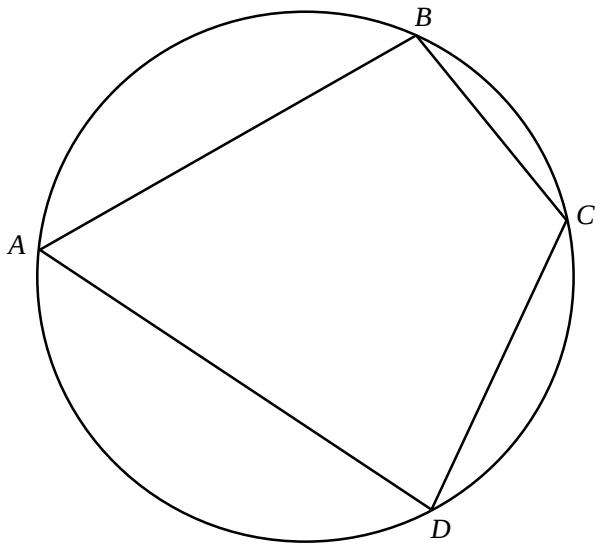
When the value of Q is halved, the value of P changes by a factor of m.

Find the value of m.

Answer $m =$ ……. [2]

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8 The diagram shows a quadrilateral playground *ABCD*.
A circular fence is constructed around the playground such that the vertices, *A*, *B*, *C* and *D* of the playground touch the circumference of the fence.



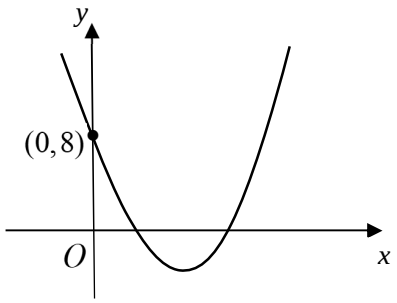
(a) Construct the perpendicular bisector of *AB*. [1]

(b) Construct the bisector of angle *ADC*. [1]

(c) A sand pit is to be constructed inside the circular fence but outside the quadrilateral playground. The sand pit is nearer to *AD* than *CD* and nearer to *B* than *A*.
Shade the region for the sand pit to be constructed. [1]

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9 The diagram below shows the graph of $y = 3(x-h)^2 - 4$.



(a) Find the value of *h*.

Answer $h =$ ……[2]

(b) Explain why the graph of $y = 3(x-h)^2 + 1$ does not cut the *x*-axis.

Answer

……

……

…… [1]

---

10 A group of six students took a Mathematics quiz and the marks were recorded below.

8 10 9 13 10 9

(a) Calculate the standard deviation.

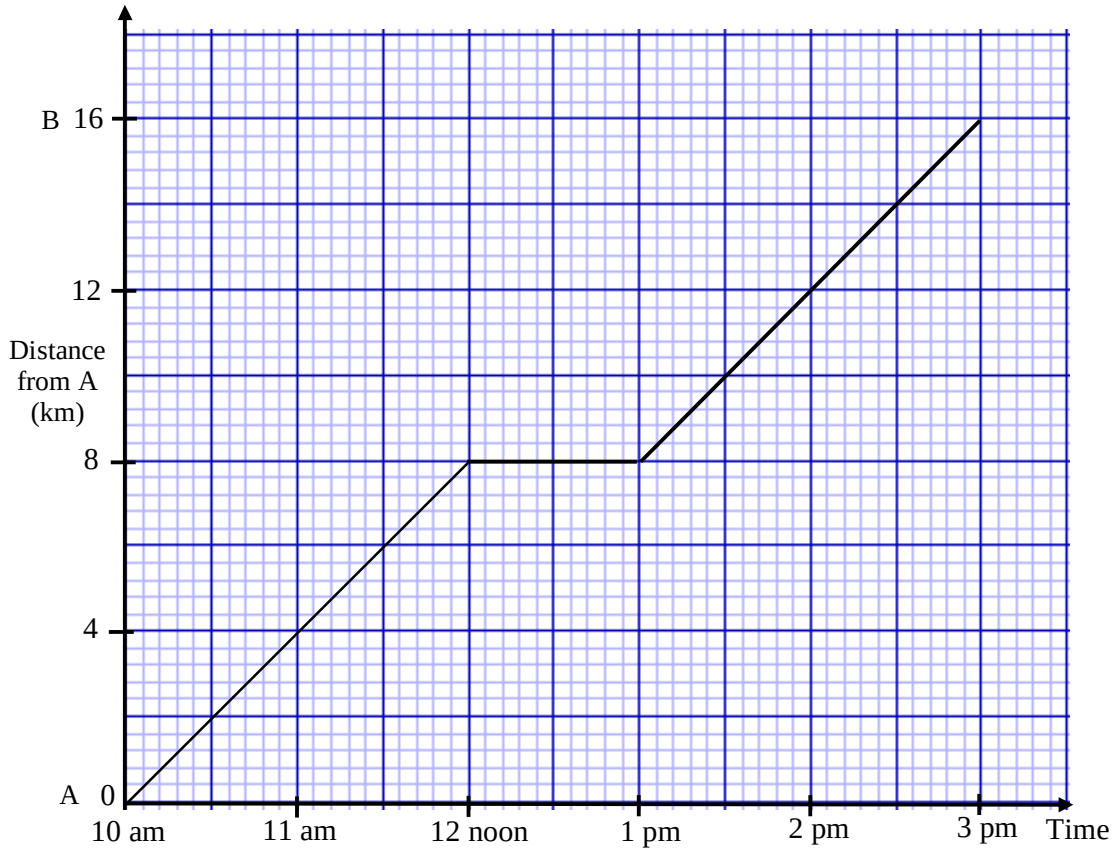
Answer ……[1]

(b) **Two** other students also took the quiz, and their marks were recorded.
Given that the mean mark obtained by the eight students was 10 and the mode was also 10, find the marks of these two students.

Answer …… and …… [2]

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11



The distance-time graph shows the journey Tan took to run from town A to B.

(a) Find the distance Tan ran in the first two hours.

Answer ……km [1]

(b) Calculate the average speed, in m/s, for the whole journey Tan ran.

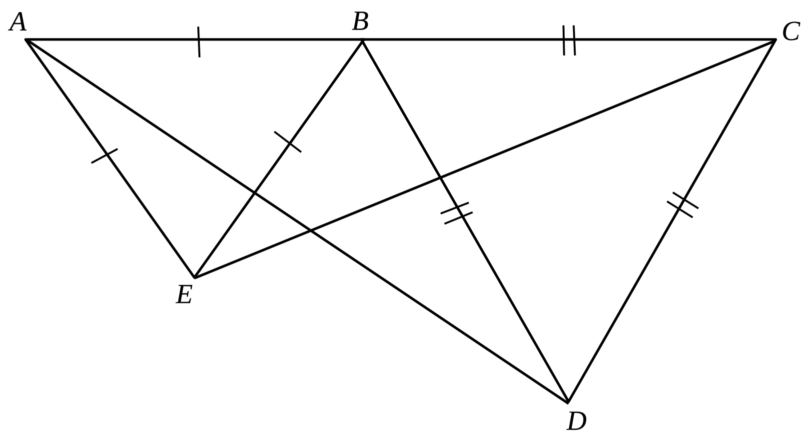
Answer ……m/s [2]

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12 Simplify $\dfrac{2y^2+y-3}{4y^2-9}$.

Answer ……………………………………… [3]

13



In the diagram, *ABC* is a straight line and triangles *ABE* and *BCD* are equilateral triangles.

Show that triangle *ABD* and triangle *EBC* are congruent.
Give a reason for each statement you make.

Answer
……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

…………………………………………………………………………………………………… [3]

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14 The first three terms in a sequence of numbers, $T_1,\ T_2,\ T_3,\ ...$ are given below.

$$T_1 = 1 - \frac{1}{2}$$
$$T_2 = \frac{1}{2} - \frac{1}{3}$$
$$T_3 = \frac{1}{3} - \frac{1}{4}$$
$$\vdots$$

(a) Write down T_4.

Answer ……[1]

(b) Show that the total sum of $T_1, T_2,\ T_3,\ \ldots, T_n$ in the above sequence is $1 - \frac{1}{n+1}$.

Answer

[2]

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15 A, B and C are points $(-1, 0)$, $(3, 8)$ and $(2, 1)$ respectively.

(a) Find the length of AB.

Answer $AB =$ ……. units [2]

(b) Find the equation of the line that passes through B and has the same gradient as AC.

Answer ……. [2]

---

16 **(a)** Find the interior angle of a regular 18-sided polygon.

Answer …….° [2]

(b) An n-sided polygon has two of its exterior angles at 45° and 75°.
If the remaining exterior angles are each 20°, calculate the value of n.

Answer $n =$ ……. [2]

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17 **(a)** Simplify $\left(\frac{a^{-6}}{b^{9}}\right)^{\frac{1}{3}}$ and leave your answer in positive index notation.

Answer ……[2]

(b) Given that $2^{4x} \div 2^{x} = \sqrt[3]{2}$, find x.

Answer $x =$……[2]

---

18 **(a)** Given that $m^2 - 8mn + 16n^2 = 0$, find the value of $\frac{m}{n}$.

Answer ……[2]

(b) Factorise completely $3ac - 7c + 18ab - 42b$.

Answer ……[2]

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19 A florist sells three types of bouquets, Bliss, Love and Commitment.
The number of stalks for each type of flower in each type of bouquet is shown in the table.

		Type of Flower			
		Rose	Lily	Gerbera	Sunflower
Type of Bouquet	Bliss	2	0	7	3
	Love	3	1	5	1
	Commitment	8	2	4	0

(a) Represent the above information in a 3×4 matrix, **F**.

Answer $\mathbf{F} = \begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix}$ [1]

(b) The cost of each stalk of Rose, Lily, Gerbera and Sunflower is \$6, \$7.80, \$2.50 and \$3 respectively.

(i) Represent this information in a 4×1 column matrix, **H**.

Answer $\mathbf{H} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$ [1]

(ii) Evaluate the matrix $\mathbf{J} = \mathbf{F}\,\mathbf{H}$.

Answer $\mathbf{J} =$ [1]

(iii) State what the elements of **J** represent.

Answer

……

……

……

…… [1]

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20 Box *X* contains 5 balls numbered 2, 3, 4, 7 and 9.
Box *Y* contains another 5 balls numbered 1, 5, 6, 8, and 10.
In a game, Ming drew a ball at random from each box, and the sum of both numbers is obtained.

(a) Complete the possibility diagram below.

Box *Y*

Box *X*	+	1	5	6	8	10
	2	3			10	12
	3		8			
	4			10		
	7				15	
	9	10	14			19

[2]

(b) Find the probability that

(i) the sum of both numbers is an odd number,

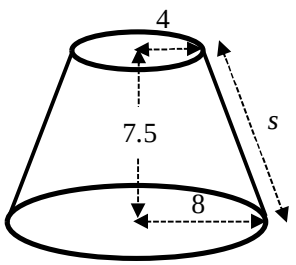
Answer ……..…..…..…..…..…..…..... [1]

(ii) the sum is a multiple of **one** of the two numbers drawn.

Answer ……..…..…..…..…..…..…..... [1]

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21 The upper part of a solid wooden right circular cone was cut off leaving the frustum as shown in the diagram. The frustum has top radius 4 cm, base radius 8 cm and height 7.5 cm.



(a) Show that the slant height, s, is 8.5 cm.

Answer

[1]

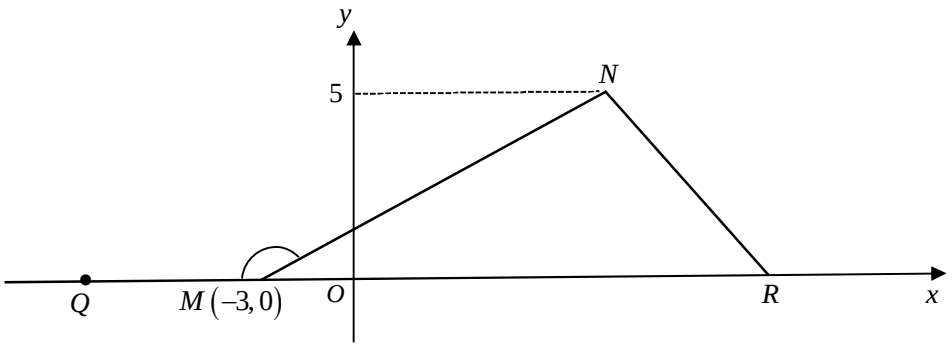
(b) Find the curved surface area of the frustum.

Answer ……..…… cm^2 [3]

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22 In triangle *MNR*, point *M* is $(-3, 0)$ and $\sin \angle NMR = \frac{5}{13}$.

Q is a point on the negative *x*-axis.



(a) Express the following as a fraction

(i) $\cos \angle NMQ$,

Answer ……..…..…..…..…..…..…..…..... [2]

(ii) $\tan \angle NMR$.

Answer ……..…..…..…..…..…..…..…..... [1]

(b) The area of triangle *MNR* is 50 square units.

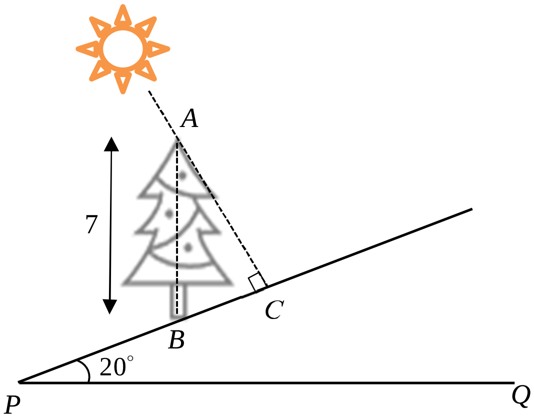
Find the coordinates of *R*.

Answer (.....,.....) [2]

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23 The diagram below shows a tree *AB* of height 7 m that stands vertically on a slope inclined at 20° with the horizontal *PQ*.

At a particular time in the morning, the tree casts a shadow, *BC*, on the slope.
AC is perpendicular to the slope.

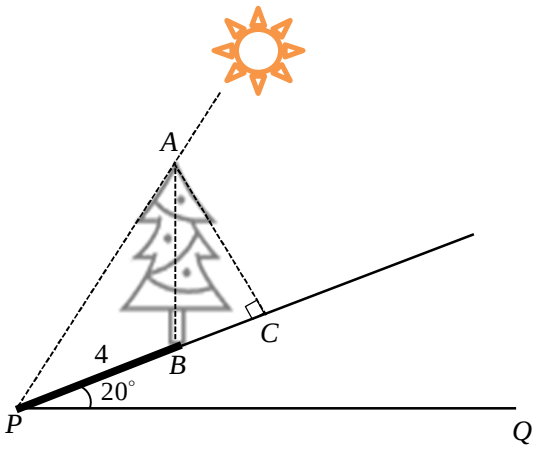


(a) Calculate the length of the shadow, *BC*.

Answer …………………………….. m [2]

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After some time, the sun goes into a position as shown below.



(b) If the shadow, *BP*, of the tree on the slope is 4 m, find the angle that the sunray makes with the horizontal *PQ*.

Answer ……° [3]

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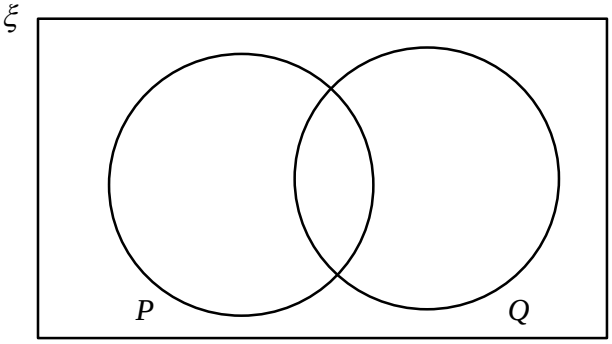
24 **(a)** $\xi = \{\text{integers } x : 1 \le x \le 12\}$

$P = \{\text{prime numbers}\}$

$Q = \{\text{multiples of 3}\}$

(i) Represent the above information on the Venn diagram shown in the answer space below.

Answer



[2]

(ii) List the elements in $\left(P' \cap Q'\right) \cup \left(P \cap Q\right)$.

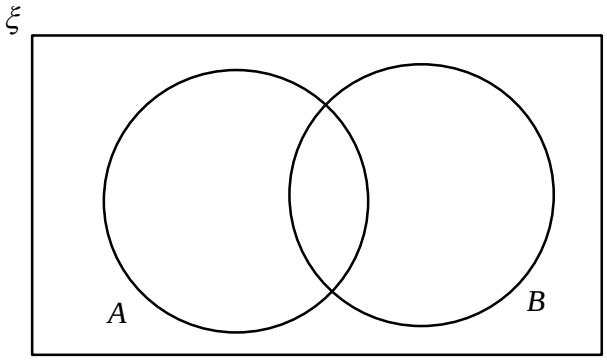
Answer ……[1]

(iii) $R = \{x : x \text{ is a multiple of } 6\}$

Use set notation to describe the relationship between *Q* and *R*.

Answer ……[1]

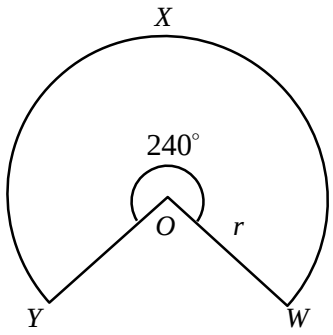
(b) On the Venn diagram, shade the region which represents the set $A \cap B'$.



[1]

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25 *OWXY* is a sector of a circle, centre *O*, of radius *r* cm and reflex angle 240°.



The sector *OWXY* has an area of 150π cm^2.

(a) Express 240° in terms of π radians.

Answer ……rad [2]

(b) Show that $r = 15$.

Answer

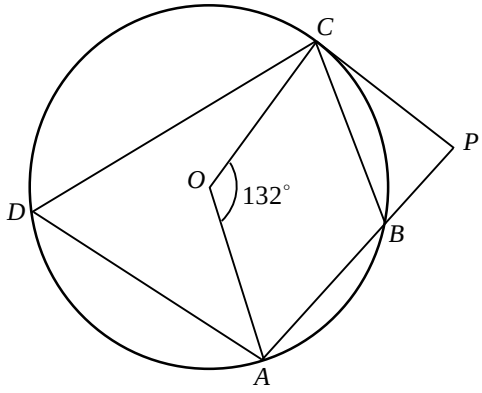
[2]

(c) The radii, *OW* and *OY*, are joined together to form a cone.
Find the base radius of the cone.

Answer ……cm [2]

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26



In the diagram above, O is the centre of the circle, such that angle $COA = 132^\circ$.
PC is a tangent to the circle at C and PBA is a straight line.

By giving a reason for each step of your working, find

(a) $\angle CDA$,

Answer ……° [2]

(b) $\angle CBP$.

Answer ……° [2]

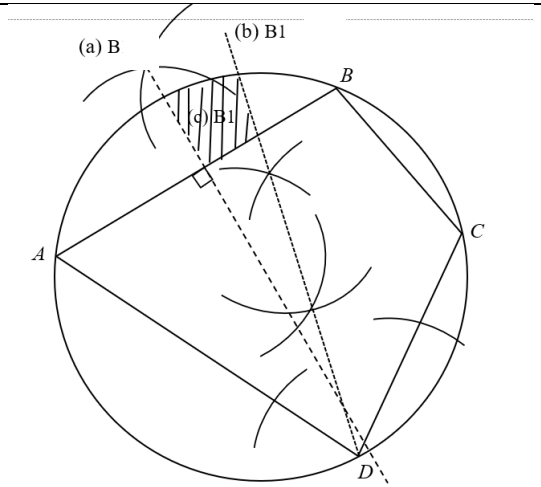
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(c) If the radius of the circle is 3.55 cm, calculate the area of triangle *AOC*.

Answer ……cm^2 [2]

END OF PAPER

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1	$x = \frac{9}{2}$
2a	8593.4
2b	8.5934×10^3
3a	$784 = 2^4 \times 7^2$
3b	$a = 7$ and $b = 2$
4	$-w^2 - 4w + 5$
5a	56.25%
5b	Pie chart
6	3
7	$m = \frac{1}{8}$
8	
9a	$h = 2$
9b	**Either one** 1. The minimum point of the graph $y = 3(x-h)^2 + 1$ is $(h, 1)$ or $(2, 1)$.
	2. The equation $(x-h)^2 = -\frac{1}{3}$ has no solution for x
10a	1.57
10b	10 and 11
11a	8 km
11b	$\frac{8}{9}$ m/sec or 0.889 m/sec
12	$\frac{y-1}{2y-3}$
13	1) $AB = EB$ (sides of an equilateral triangle / given) 2) $BD = BC$ (sides of an equilateral triangle / given) 3) $\measuredangle ABD = 180^\circ - 60^\circ$ (adj. $\angle$ on a st. line) $= \measuredangle EBC$ $= 120^\circ$ $\therefore$ triangle ABD is congruent to triangle EBC (SAS)
14a	$T_4 = \frac{1}{4} - \frac{1}{5}$
14b	$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$
15a	8.94 units
15b	$y = \frac{1}{3}x + 7$
16a	160°
16b	$n = 14$
17a	$\frac{1}{a^2 b^3}$
17b	$x = \frac{1}{9}$
18a	$\frac{m}{n} = 4$

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18b $(c+6b)(3a-7)$

19a $\mathbf{F}=\begin{pmatrix}2 & 0 & 7 & 3\\ 3 & 1 & 5 & 1\\ 8 & 2 & 4 & 0\end{pmatrix}$

19bi $\mathbf{H}=\begin{pmatrix}6\\ 7.80\\ 2.50\\ 3\end{pmatrix}$

19bii $\begin{pmatrix}38.50\\ 41.30\\ 73.60\end{pmatrix}$

19biii The elements of **J** represent the total cost of the four types of flowers - Rose, Lily, Gerbera and Sunflower in bouquet Bliss, Love and Commitment respectively.

20a

+	1	5	6	8	10
2	3	7	8	10	12
3	4	8	9	11	13
4	5	9	10	12	14
7	8	12	13	15	17
9	10	14	15	17	19

20bi $\frac{13}{25}$

20bii $\frac{2}{5}$

21a Show

21b $320\ \text{cm}^2$

22ai $-\frac{12}{13}$

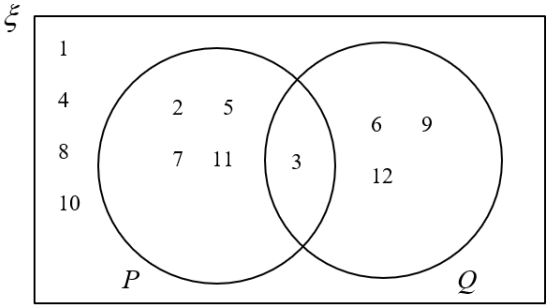
22aii $\frac{5}{12}$

22b Coordinates of $R=(17,\ 0)$

23a 2.39 m

23b 65.8°

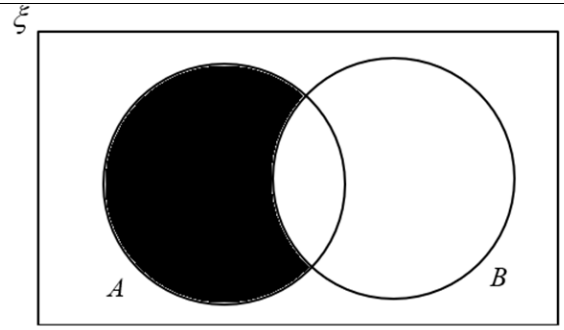
24ai



24aii $(P'\cap Q')\cup(P\cap Q)=\{1,3,4,8,10\}$

24aiii $R\subset Q$

24b



25a $\frac{4}{3}\pi$ rad

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25b	Show
25c	10
26a	$\angle CDA = 132^\circ \div 2$ $= 66^\circ$ ($\angle$ at the centre = twice $\angle$ at circumference)
26b	$\angle CBA = 180^\circ - 66^\circ$ $= 114^\circ$ ($\angle$s in opp. segment) $\angle CBP = 180^\circ - 114^\circ$ (adj. $\angle$s on a st. line) $= 66^\circ$
26c	4.68 cm^2

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