Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Monday 21 August 2023

ADDITIONAL MATHEMATICS
PAPER 2

4049/02

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

90

This question paper consists of 17 printed pages and 3 blank pages.

Downloaded from SingaKids.com

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

Downloaded from SingaKids.com

1. **(i)** A prism has a volume of $\left(3x^2+8x+1\right)$ cm^3 and a cross-sectional area of $\left(x^2+2x+1\right)$ cm^2. Write down an expression for the height of the prism, in the form $A+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$. [4]

(ii) Find $\int A+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}\,\mathrm{d}x$. [3]

Downloaded from SingaKids.com

2. A circle C passes through the point $P(4,-3)$ and has the same centre as the circle $x^2+y^2+4x-2y-1=0$.

(i) Find the equation of the circle C. [3]

(ii) Find the equation of the tangent to the circle C at the point $P(4,-3)$. [2]

(iii) Another point $Q(4, q)$ which lies on the circle C, is the same distance from the y-axis as the point P. Find the coordinates of the point Q. [2]

Downloaded from SingaKids.com

3. **(i)** Given that $y = (x-1)\sqrt{4x+1}$, show that $\frac{dy}{dx} = \frac{6x-1}{\sqrt{4x+1}}$. [3]

(ii) Hence, evaluate $\int_1^2 \frac{6x-5}{\sqrt{4x+1}}\,dx$. [4]

Downloaded from SingaKids.com

4. **(a)** Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$.

[3]

(b) John models the height of sea water level, H metres, on a particular day by the equation $H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$, $0 \le t < 12$, where t hours is the number of hours after midday. Using this model, calculate

(i) the maximum possible height of sea water level and the value of t, to 2 decimal places, when it occurs. [3]

Downloaded from SingaKids.com

4 **(b)** **(ii)** the first time when the height of the sea water is 7 metres, leaving your answer correct to the nearest minute. [4]

Downloaded from SingaKids.com

5. The temperature, θ °C, of an oven t minutes after it was switched on is given by $\theta = 300 - 280e^{-0.05t}$, $t \geq 0$.

(i) State the initial temperature of the oven. [1]

(ii) Find the value of t when the temperature of the oven reaches 160 °C. [2]

(iii) Determine, with justification, the maximum temperature that this oven can approach. [2]

Downloaded from SingaKids.com

5. The temperature, θ °C, of another oven t minutes after it was switched on is given by $\theta = 250 - 230e^{-0.1t}$, $t \geq 0$.

(iv) Assuming that both ovens are switched on at the same time, find the time when both ovens will have the same temperature since they were switched on. [5]

Downloaded from SingaKids.com

6. **(i)** Prove that $\sin(A+B)\sin(A-B)=(\sin A+\sin B)(\sin A-\sin B)$. [3]

(ii) Given that $\sin A=\frac{1}{2}$, find all the values of B for $-\pi \le B \le \pi$ that satisfy the equation $\sin(A+B)\sin(A-B)=0$. [4]

Downloaded from SingaKids.com

7. **(a)** Find the range of values of k for which $y=(k-6)x^2-8x+k$ cuts the $x-$axis at two distinct points and has a minimum point. [4]

(b) Given that the line $y=5x+c$ is a tangent to the curve $y=2x^2+bx$, show that c is never positive. [4]

Downloaded from SingaKids.com

8. The table below shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{a}{x+b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

(i) Draw the graph of x against $\frac{1}{y}$, using a scale of 5 cm to 1 unit on the x-axis and a scale of 2 cm to 0.1 unit on the $\frac{1}{y}$-axis on the graph paper in the next page. [3]

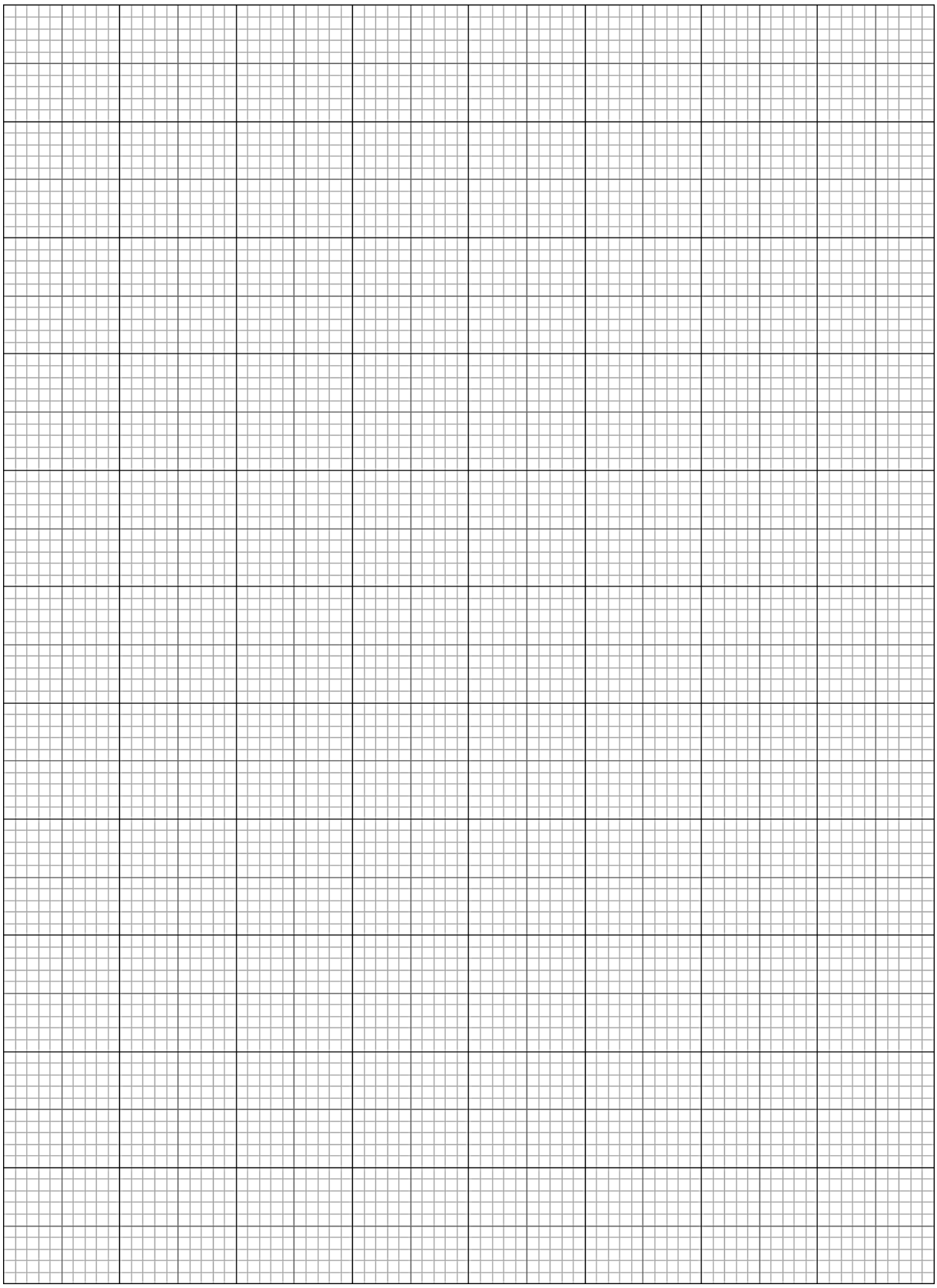
(ii) Use your graph to estimate the value of a and of b. [4]

An alternative method for obtaining a straight line graph for the equation $y = \frac{a}{x+b}$ is to plot y against xy.

(iii) Using answers of a and b in part **(ii)** estimate the gradient and vertical intercept of the graph of y plotted against xy.

[3]

Downloaded from SingaKids.com



Downloaded from SingaKids.com

9. A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v cm/s, is given by $v = 3t^2 - 7t + 2$.

(i) Find the value of t at the instant when the acceleration is -1 cm/s^2. [2]

(ii) Find the values of t at which the particle is instantaneously at rest. [2]

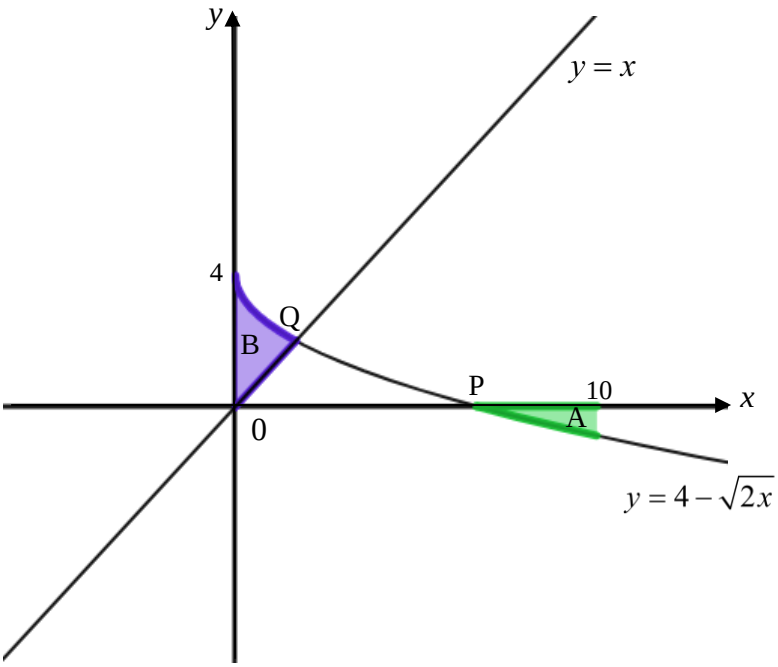
Downloaded from SingaKids.com

9. **(iii)** Find the total distance travelled by the particle during the first 2 seconds. [3]

(iv) Find the time(s) when the particle returns to O. [3]

Downloaded from SingaKids.com

10. The diagram shows part of the curve $y = 4 - \sqrt{2x}$ and the line $y = x$. The curve cuts the x – axis at the point P and the line intersects the curve at the point Q.



(i) Show that the x- coordinate of the point P is 8. [2]

(ii) Find the coordinates of the point Q. [3]

Downloaded from SingaKids.com

10. **(iii)** Find the area of the region A, bounded by the curve $y = 4 - \sqrt{2x}$, the x axis and the line $x = 8$ and $x = 10$. [3]

(iv) Find the area of the region B, bounded by the straight line $x - y = 0$, the curve $y = 4 - \sqrt{2x}$ and the y – axis. [4]

(v) If $\int_k^{10} \left(4 - \sqrt{2x}\right) \mathrm{dx} = 0$, where $2 < k < 8$, explain what this result implies about the curve $y = 4 - \sqrt{2x}$. [2]

End of Paper

Downloaded from SingaKids.com

Answer Key

Qn 1 (i)	Height, $h = 3 + \frac{2}{x+1} - \frac{4}{(x+1)^2}$	6(ii)	$\angle B = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$
(ii)	$3x + 2\ln(x+1) + \frac{4}{(x+1)^2} + C$	7(a)	$\therefore 6 < k < 8$
Qn 2(i)	$(x+2)^2 + (y-1)^2 = 52$ or $x^2 + y^2 + 4x - 2y - 47 = 0$	(b)	since $(b-5)^2 \geq 0$ $-8c \geq 0$ $c \leq 0$
(ii)	$y = \frac{3}{2}x - 9$	8(ii)	b = - vertical intercept =0.8 Range: {0.74 to 0.84 } a = gradient = 7.27 Range: {7.0 to 7.4}
(iii)	Q(4, 5)	(iii)	$y = -\frac{1}{b}xy + \frac{a}{b}$ Gradient = - 1.25 Vertical intercept = 9.025 Range: {9.025 to 9.1}
Qn 3(ii)	1.47	9 (i)	t = 1
Qn 4(a)	$2\sin\theta - 1.5\cos\theta = \frac{5}{2}\sin(\theta - 0.644)$	(ii)	$t = \frac{1}{3}$ or t = 2
4(b)(i)	t = 4.41	(iii)	Total distance = $2\frac{17}{27}$ or 2.63
4(b)(ii)	First time = 14 06 or 2.06 pm	(iv)	t = 0 (NA), 2.78 or 0.719
Qn 5 (i)	$20°C$	10(i)	x = 8
(ii)	t = 13.9 mins	(ii)	$\therefore Q(2,2)$
(iii)	As t increase, $e^{-0.05t}$ approaches zero. Hence, maximum temperature that the oven can reach = $300°C$	(iii)	A = 0.481
(iv)	t = 30.5 s	(iv)	$3\frac{1}{3}$
		(v)	The area bounded by the curve x = k, x = 8 and x-axis lies above the x axis and it is equal to the area of region A, which is area bounded by the curve x = 8, x = 10 and x-axis. This is area below the x axis.

Downloaded from SingaKids.com

BLANK PAGE

Downloaded from SingaKids.com

BLANK PAGE

Downloaded from SingaKids.com