NAME: _____ **( )** **CLASS:** _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Date: 24 August 2023 **Duration: 2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		**Question Number**		
Presentation	□ 1 □ 2			
Rounding off	□ 1		Parent's/Guardian's Signature	**90**

Setter: Mdm Haliza

This question paper consists of <u>21</u> printed pages including the cover page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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Answer **all** the questions.

1 Without using a calculator, find the exact value of

(i) $\text{cosec}\,\theta$, given that θ is acute and $\cos\theta = \frac{3}{4}$, [1]

(ii) $\cos 30° (\tan 45° + \sin 60°)$. [2]

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2 Find the set of values of a for which the curve $y = 2x^2 + 7$ lies entirely above the line $y = ax - 3$. [4]

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3 The equation of a curve is $y = Ae^{\frac{1}{2}x} + Be^{-\frac{1}{2}x}$, and that $\frac{dy}{dx} + \frac{3}{2}y = 2e^{\frac{1}{2}x} - 5e^{-\frac{1}{2}x}$.

Find the value of each of the constants A and B. [5]

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4 Water is leaking from a container at a rate of 8 cm^3/s. If the volume of water, V cm^3, in the container is given by $V = \frac{3}{2}\left(h^2 + 8h\right)$ where h is the depth of the water, in cm, remaining in the container, find the rate of change of h when the volume of water is 13.5 cm^3. [5]

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5 A certain radioactive material, radium-226, decomposes according to the formula $A = A_0 e^{kt}$ where A is the remaining mass in grams, after decomposition, A_0 is the original mass in grams, t is the time in years and k is a constant. A radioactive substance is often described in terms of its half-life, which is the time required for half the material to decompose.

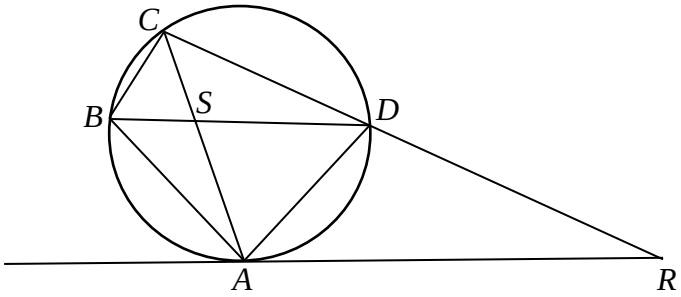
(i) Given that after 400 years, a sample of radium-226 has decayed to 84.1% of its original mass, show that $k = -0.000433$, rounded off to 3 significant figures. [2]

(ii) Hence, find the half-life of radium-226, to the nearest whole number. [2]

(iii) If a sample of radium-226 has an initial mass of 100 grams, what is the remaining mass after 3200 years? [2]

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6 In the diagram, A, B, C and D are points on the circle. The tangent at A meets CD produced at R. The chords AC and BD intersect at S. The line BSD bisects angle ABC.



Prove that

(i) $\angle DAR = \angle CAD$, [3]

(ii) ΔRAD is similar to ΔRCA, [2]

(iii) $RA^2 = RC \times RD$. [1]

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7 **(i)** Explain why there is no constant term in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^8$. [2]

(ii) Show that the coefficient of x^{-6} in the expansion of $\left(x^3 - \frac{1}{x^2}\right)^8 \left(1 + x^5\right)$ is 20. [4]

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8 The equation of a curve is $y = 2x^2 - 4x + 9$.

(i) By expressing $2x^2 - 4x + 9$ in the form $a(x+b)^2 + c$, where a, b and c are constants, find the coordinates of the stationary point on the curve. [2]

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8 **(ii)** The line $y = 3x + 3$ intersects the curve at points A and B. Find the value of h for which the distance AB can be expressed as $\sqrt{h}$. [4]

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9 The equation of a curve is $y = x - \frac{2x+1}{1-2x}$.

(i) State the value of x for which y is not defined. [1]

(ii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

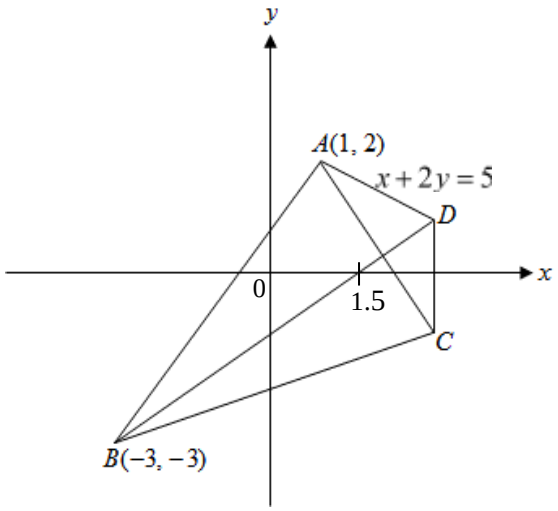
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9 **(iii)** Find the coordinates of the stationary points of the curve. [3]

(iv) Using the second derivative test, find the nature of each stationary point. [3]

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10 Solutions to this question by accurate drawing will not be accepted.



The diagram, not drawn to scale, shows a kite *ABCD*, where *A* is (1, 2) and *B* is $(-3,-3)$. The line *BD* cuts the *x*-axis at *x* = 1.5. The equation of the line *AD* is $x+2y=5$. Find the

(i) coordinates of *D*, [4]

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10 **(ii)** equation of AC, [2]

(iii) coordinates of C. Hence, find the area of $ABCD$. [4]

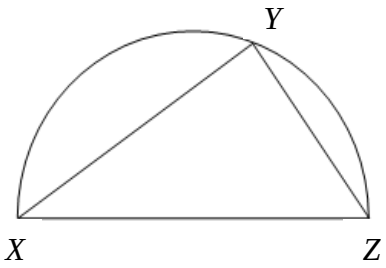
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11 **(a)** Show that there is no solution for the equation $9^{x-1} = 3^x - 8$. [3]

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11 **(b)** The diagram shows a semicircle XYZ with XZ as the diameter, $XY = \left(\sqrt{50} + \sqrt{2}\right)$cm and $YZ = \left(\sqrt{28} - \sqrt{2}\right)$cm.



(i) Show that $XZ^2 = 102 - 4\sqrt{14}$. [2]

(ii) Express $\tan \angle YXZ$ in the form $a\sqrt{14} + b$, where a and b are constants. [3]

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12 A computer animation shows a cartoon giraffe moving in a straight line so that t seconds after it passes a fixed point O, its velocity, v cm s^{-1}, is given by $v = pt - qt^2$, where p and q are constants.

(i) Given that the giraffe attains a maximum speed of 48 cm s^{-1} after 2 seconds, show that the values of p and q are 48 and 12 respectively. [5]

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12 **(ii)** Explain clearly why the total distance travelled by the giraffe in the interval $t = 0$ to $t = 7$ is not obtained by finding the value of the displacement, s, when $t = 7$. [1]

(iii) Find the total distance travelled by the giraffe in the first 7 seconds. [5]

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13 **(i)** It is given that $\mathrm{f}(x) = 3\sin\left(\frac{x}{2}\right) + 1$.

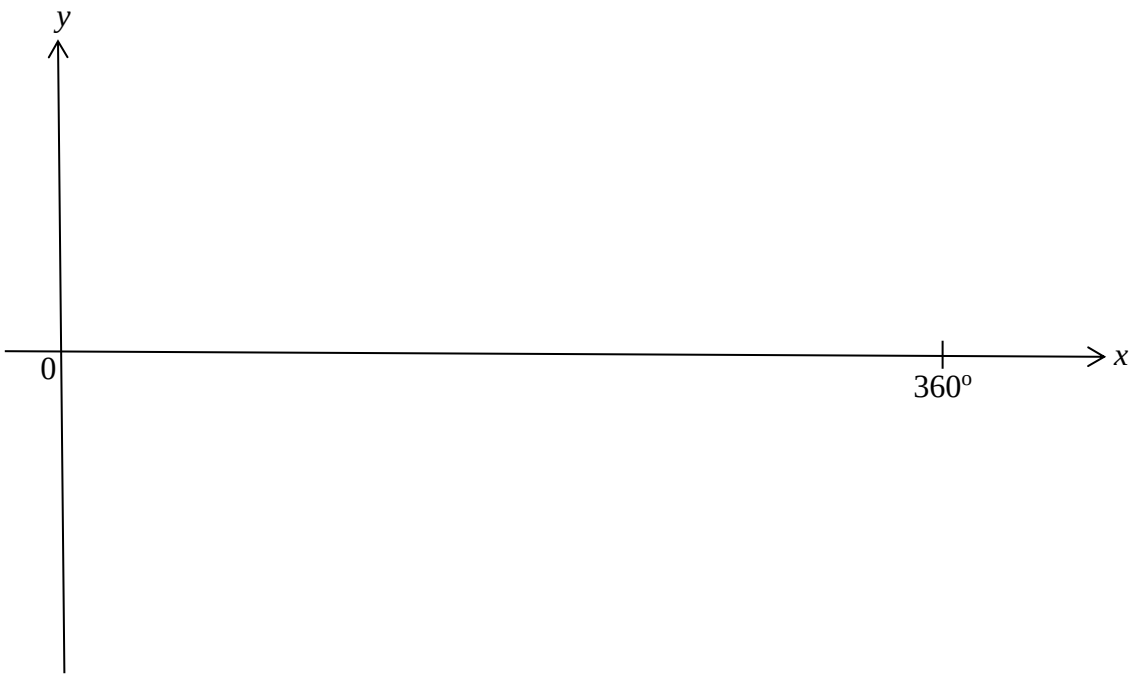
(a) State the least and greatest values of $\mathrm{f}(x)$. [2]

(b) State the period of $\mathrm{f}(x)$. [1]

(ii) The graph of $\mathrm{g}(x) = \tan ax$, where a is a constant, has a period of 480°. Find the value of a. [1]

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13 **(iii)** On the same axes, sketch the graphs of f(x) and g(x) for $0° \le x \le 360°$. [4]



(iv) State the number of solutions of the equation $3\sin\left(\frac{x}{2}\right) = \tan ax - 1$ for $0° \le x \le 360°$. [1]

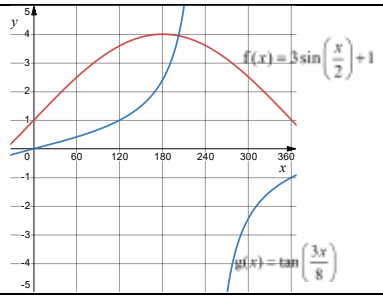
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FMS(S) Sec 4 Exp Prelim Examination 2023
Additional Mathematics Paper 1

Answer Key

1(i)	$\frac{4}{\sqrt{7}}$	**1(ii)**	$\frac{\sqrt{3}}{2}+\frac{3}{4}$ or $\frac{2\sqrt{3}+3}{4}$
2	$-4\sqrt{5} < a < 4\sqrt{5}$ or $-\sqrt{80} < a < \sqrt{80}$	**3**	$A=1,\ B=-5$
4	$-\frac{8}{15}$ cm/s or -0.533 cm/s (to 3 s.f.)	**5(ii)**	1601 years (to nearest whole number)
5(iii)	25.0 grams (to 3 s.f.)	**7(i)**	$r = 4.8$ Hence, there is no constant term because r must be a positive integer/whole number.
8(i)	(1, 7)	**8(ii)**	$h=\frac{5}{2}$ or 2.5
9(i)	$x=\frac{1}{2}$	**9(ii)**	$\frac{dy}{dx}=1-\frac{4}{(1-2x)^2}$ or $\frac{(2x+1)(2x-3)}{(1-2x)^2}$ or $1-\frac{4x+2}{(1-2x)^2}+\frac{2}{1-2x}$
9(iii)	$\left(\frac{3}{2},\frac{7}{2}\right)$ and $\left(-\frac{1}{2},-\frac{1}{2}\right)$	**9(iv)**	y has a minimum point at $x=\frac{3}{2}$ and a maximum point at $x=-\frac{1}{2}$.
10(i)	D (3, 1)	**10(ii)**	$y=-\frac{3}{2}x+\frac{7}{2}$ or $2y+3x=7$
10(iii)	$C\left(\frac{41}{13},-\frac{16}{13}\right)$, 14 units^2	**11(a)**	$b^2-4ac=-207$. Since the discriminant is negative, there is no solution for the equation.
11(b)(ii)	$\frac{\sqrt{14}}{6}-\frac{1}{6}$	**12(ii)**	The giraffe changes direction/ moves in the opposite direction at t = 4. So the total distance travelled by the giraffe in the interval t = 0 to t = 7 is not obtained by finding the value of s when t = 7.
12(iii)	452 cm	**13(i)(a)**	Greatest value = 4 Least value = -2
13(i)(b)	720° or 4π	**13(ii)**	$\frac{3}{8}$
13(iii)		**13(iv)**	1 solution

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