Name:	Class:	Class Register Number:

CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2023

SECONDARY 4

ADDITIONAL MATHEMATICS 4049/02

Paper 2 **Tuesday 29 August 2023**

Candidates answer on the Question Paper. **2 hours 15 minutes**

MARKS SCHEME

This document consists of **20** printed pages and **2** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 **(a)** The line $4x = 3y + 2$ intersects the curve $x^2 - xy + 5 = 0$ at the points A and B. Find the midpoint of AB. [5]

$$4x = 3y + 2 \quad ...(1)$$
$$x^2 - xy + 5 = 0 \quad ...(2)$$

From (1):
$$3y = 4x - 2$$
$$y = \frac{4x - 2}{3} \quad ...(3)$$

Sub. (3) into (2):
$$x^2 - x\left(\frac{4x - 2}{3}\right) + 5 = 0$$ M1 f.t. – substitution
$$3x^2 - 4x^2 + 2x + 15 = 0$$
$$-x^2 + 2x + 15 = 0$$
$$x^2 - 2x - 15 = 0$$
$$(x - 5)(x + 3) = 0$$ M1 f.t. – solving quadratic
$$x = 5 \text{ or } -3 \text{ sub. into (3):}$$
$$y = 6 \text{ or } -\frac{14}{3}$$ M1 – either correct x or y values

$\therefore A\left(-3, -\frac{14}{3}\right)$ **and** ***B*(5, 6)**

$$\text{Midpoint of } AB = \left(\frac{-3 + 5}{2}, \frac{-\frac{14}{3} + 6}{2}\right)$$ M1 f.t. – midpoint formula
$$= \left(1, \frac{2}{3}\right)$$ A1

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(b) Find the least value of the integer h for which hx^2+5x+h is positive for all real values of x. [3]

For $hx^2+5x+h>0$,

discriminant <0

$25-4(h)h<0$

$25-4h^2<0$

$(5-2h)(5+2h)<0$

$h<-\frac{5}{2}$ or $h>\frac{5}{2}$

B1 – discriminant

M1 f.t. – factorising quadratic

Since $h>0$, $h>\frac{5}{2}$.

∴ Least integer value of h = 3.

A1

(c) Given that the line $y=3x+p$ is tangent to the curve $y=x^2+5x+q$, where p and q are integers, prove that p and q are consecutive numbers. [4]

$y=3x+p$ ... (1)

$y=x^2+5x+q$ ... (2)

Sub. (1) into (2):

$3x+p=x^2+5x+q$

M1 – substitution

$x^2+5x+q-3x-p=0$

$x^2+2x+q-p=0$

M1 – forming quadratic

$a=1, b=2, c=k-c$

Line is tangent to curve $\rightarrow b^2-4ac=0$

$2^2-4(1)(q-p)=0$

$4-4q+4p=0$

$4q=4+4p$

$q=1+p$

M1 f.t. – any use of discriminant

A1 – with explanation

Since $q=1+p$, q will always be the next number after p.
Hence, p and q are consecutive numbers (proved).

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Alternative:

$y = x^2 + 5x + q$

$\frac{dy}{dx} = 2x + 5$

Since line is tangent to curve and gradient of line = 3,

$2x + 5 = 3$

$2x = -2$

$x = -1$

M1 – equate $\frac{dy}{dx}$ to 3

M1 f.t. – finding x

$y = 3x + p$ ... (1)

$y = x^2 + 5x + q$ ... (2)

Sub. (1) into (2) and $x = -1$:

$3x + p = x^2 + 5x + q$

$3(-1) + p = (-1)^2 + 5(-1) + q$

$-3 + p = -4 + q$

$q - p = 1$

M1 – substitution

Since the difference between q and p is 1, q will always be the next number after p.
Hence, p and q are consecutive numbers (proved).

A1 – with explanation

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2 **(a)** By considering the general term in the binomial expansion of $\left(x^3-\frac{2}{x}\right)^7$, explain why there are only odd powers of x in this expansion. [3]

$$T_{r+1}=\binom{7}{r}\left(x^3\right)^{7-r}\left(-\frac{2}{x}\right)^r$$
$$=\binom{7}{r}\left(x^{21-3r}\right)(-2)^r\left(x\right)^{-r}$$
$$=\binom{7}{r}(-2)^r\,x^{21-4r}$$

B1 – general formula

Power of $x = 21 - 4r$

M1 f.t. – finding powers of x

Since **$4r$ is an even number** for all non-negative integer values of r and 21 is an odd number,
then **$21 - 4r$ is always an odd number**.

Therefore, there are only odd powers of x in this expansion.

A1 – conclusion

(b) Find the term independent of x in the expansion of $\left(x^3-\frac{2}{x}\right)^7\left(\frac{5}{x}-2x^2\right)$. [3]

Consider $21-4r=1$,
$4r=20$
$r=5$

M1 f.t. – identifying x term

$$\left(x^3-\frac{2}{x}\right)^7\left(\frac{5}{x}-2x^2\right)=\left[\ldots+\binom{7}{5}(-2)^5\,x^{21-4(5)}+\ldots\right]\left(\frac{5}{x}-2x^2\right)$$
$$=\left(\ldots-672x+\ldots\right)\left(\frac{5}{x}-2x^2\right)$$

M1 f.t. – expansion

Term independent of $x = -672\times 5$
$= -3360$

A1

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3 The expression $7\sin\theta + 3\cos\theta$ is defined for $0° \le \theta \le 360°$.

(a) Using $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, solve the equation

$$7\sin\theta = 5 - 3\cos\theta .$$ [5]

$$7\sin\theta + 3\cos\theta = R\sin(\theta + \alpha)$$
$$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$$

Comparing,
$7 = R\cos\alpha$ and $3 = R\sin\alpha$

$$R = \sqrt{7^2 + 3^2}$$
$$= \sqrt{58}$$

M1 – attempt to find R

$$\tan\alpha = \frac{3}{7}$$
$$\alpha = \tan^{-1}\left(\frac{3}{7}\right)$$
$$= 23.1985...°$$

M1 – attempt to find α
[only accept $\tan^{-1}\left(\frac{3}{7}\right)$ or $\tan^{-1}\left(\frac{7}{3}\right)$]

$$\therefore 7\sin\theta + 3\cos\theta = \sqrt{58}\sin(\theta + 23.1985...°)$$

$$7\sin\theta = 5 - 3\cos\theta$$
$$7\sin\theta + 3\cos\theta = 5$$
$$\sqrt{58}\sin(\theta + 23.1985...°) = 5$$

M1 f.t. – substitute R-form

$$\sin(\theta + 23.1985...°) = \frac{5}{\sqrt{58}}$$
$$\text{basic angle} = \sin^{-1}\left(\frac{5}{\sqrt{58}}\right)$$
$$= 41.0359...°$$

M1 f.t. – basic angle

$$\theta + 23.1985...° = 41.0359...° \quad \text{or} \quad 180° - 41.0359...°$$
$$\theta = 17.8° \quad \text{or} \quad 115.8° \text{ (1 dec. pl.)}$$

A1

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(b) State the largest and smallest values of $\left(7\sin\theta+3\cos\theta\right)^2-12$ and find the corresponding values of θ. [4]

largest value of $\left(7\sin\theta+3\cos\theta\right)^2-12=\left(\sqrt{58}\right)^2-12$ or $\left(-\sqrt{58}\right)^2-12$

$=46$ B1

occurs when $\theta+23.1985...°=90°$ or $270°$

$\theta=66.8°$ or $246.8°$ (1 dec. pl.) B1

smallest value of $\left(7\sin\theta+3\cos\theta\right)^2-12=\left(0\right)^2-12$

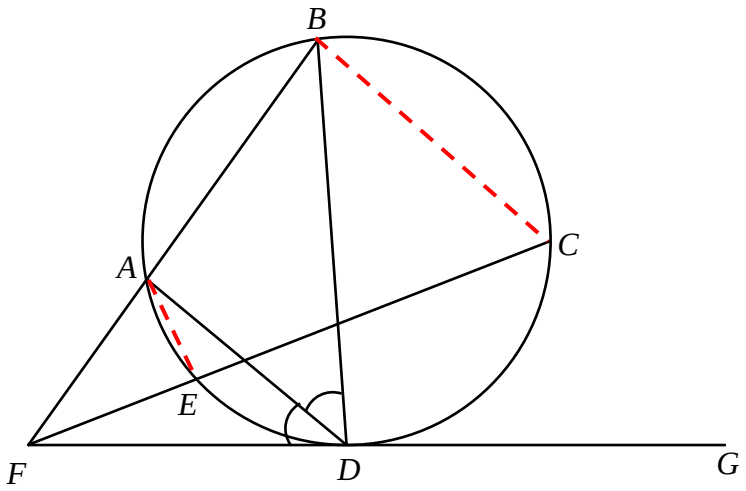
$=-12$ B1

occurs when $\theta+23.1985...°=180°$ or $360°$

$\theta=156.8°$ or $336.8°$ (1 dec. pl.) B1

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4



The diagram shows a circle passing through the points A, B, C, D and E. The straight line FDG is tangent to the circle at D while FAB and FEC are secant lines.
Given that angle FDA = angle ADB,

(a) show that triangle ABD is an isosceles triangle, [2]

$\angle ABD = \angle FDA$ (**alternate segment theorem**) B1
$= \angle ADB$

Since $\angle ABD = \angle ADB$, they form **base angles of isosceles triangle**. B1
Thus, triangle ABD is an isosceles triangle. (shown)

(b) prove that $AF \times BF = EF \times CF$. [3]

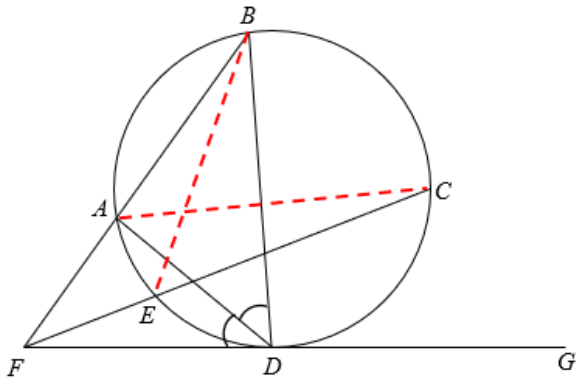
$\angle FAE = \angle FCB$ (**exterior ∠ of cyclic quadrilateral**) B1

$\angle AFE = \angle CFB$ (**common ∠**) B1

Thus, triangle AFE is similar to triangle CFB.

$\frac{AF}{CF} = \frac{EF}{BF}$ (**ratio of corresponding sides are equal**) B1
$AF \times BF = EF \times CF$ (proved)

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Alternative:

$\angle FBE = \angle FCA$ (**$\angle$s in same segment**) B1

$\angle EFB = \angle AFC$ (**common $\angle$**) B1

Thus, triangle EFB is similar to triangle AFC.

$\frac{AF}{EF} = \frac{CF}{BF}$ (**ratio of corresponding sides are equal**) B1

$AF \times BF = EF \times CF$ (proved)

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5 An object is heated in an oven until it reaches a temperature of X °C. It is then allowed to cool under room temperature. Its temperature, T °C, can be modelled by $T = 18 + 62e^{-kt}$, where t is the time in minutes since the object starts cooling.

(a) Find the value of X. [1]

When $t = 0$, $T = X$.

$$\begin{aligned} X &= 18 + 62e^{-k(0)} \\ &= 18 + 62 \\ &= 80 \end{aligned}$$

B1

When $t = 10$, the temperature of the object is 65 °C.

(b) Find the temperature of the object an hour later, giving your answer to one decimal place. [5]

When $t = 10$, $T = 65$.

$$65 = 18 + 62e^{-k(10)}$$

B1 – sub. 10 and 65 correctly

$$\frac{47}{62} = e^{-10k}$$

$$\ln\frac{47}{62} = \ln e^{-10k}$$

M1 – Take ln on both sides

$$\ln\frac{47}{62} = -10k$$

$$k = -\frac{1}{10}\ln\frac{47}{62}$$

$$= 0.0276986...$$

M1 – finding k

When $t = 60$,

$$T = 18 + 62e^{-\left(-\frac{1}{10}\ln\frac{47}{62}\right)(60)}$$

$$= 29.765...$$

$$= 29.8 \text{ °C (1 dec. pl.)}$$

When $t = 70$,

$$T = 18 + 62e^{-\left(-\frac{1}{10}\ln\frac{47}{62}\right)(70)}$$

$$= 26.9193...$$

$$= 26.9 \text{ °C (1 dec. pl.)}$$

M1 f.t.– sub. $t = 60$ or 70

A1

(c) What does the model suggest about the room temperature? Explain your answer. [2]

Since $e^{-kt} > 0$, then $62e^{-kt} > 0$.
Thus, **as t becomes larger**, **$62e^{-kt}$ approaches zero**.
Therefore, T will **approach 18 °C** as t becomes larger.
Hence, the room temperature is **18 °C.**

B1 – explanation

B1 – temperature

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6 The equation of a curve is $y = \ln\left(\frac{2x-1}{3x-1}\right)$, where $x > \frac{1}{2}$.

(a) Find $\frac{dy}{dx}$, expressing it as a single fraction. [3]

$$y = \ln\left(\frac{2x-1}{3x-1}\right)$$
$$= \ln(2x-1) - \ln(3x-1)$$

B1 – quotient law of logarithms

$$\frac{dy}{dx} = \frac{2}{2x-1} - \frac{3}{3x+1}$$
$$= \frac{2(3x-1)-3(2x-1)}{(2x-1)(3x-1)}$$
$$= \frac{1}{(2x-1)(3x+1)}$$

M1 f.t. – differentiate either term

A1 – single fraction

Alternative:

$$\frac{dy}{dx} = \frac{\frac{2(3x-1)-3(2x-1)}{(3x-1)^2}}{\frac{2x-1}{3x-1}}$$
$$= \frac{\frac{1}{(3x-1)^2}}{\frac{2x-1}{3x-1}}$$
$$= \frac{1}{(2x-1)(3x+1)}$$

B1 – differentiate ln in the form $\frac{f'(x)}{f(x)}$

M1 – apply quotient rule to numerator.

A1 – single fraction

(b) Explain why the curve will be almost parallel to the x-axis as x becomes very large. [2]

Since $(2x-1)(3x+1)$ becomes a very large number as x becomes very large, $\frac{dy}{dx}$ approaches **0**, the curve will **almost horizontal**, thus it will be almost parallel to the x-axis.

M1

A1

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(c) Find the value of x at the instant when the rate of change of x is twice the rate of change of y. [3]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{(2x-1)(3x-1)} \times 2\left(\frac{dy}{dt}\right)$$

M1 f.t. – substitution

$$1 = \frac{2}{(2x-1)(3x-1)}$$

B1 – substitute $\frac{dx}{dt} = 2\frac{dy}{dt}$

$$(2x-1)(3x-1) = 2$$

$$6x^2 - 5x + 1 - 2 = 0$$

$$6x^2 - 5x - 1 = 0$$

$$(x-1)(6x+1) = 0$$

$$x = 1 \text{ or } -\frac{1}{6} \quad (\text{rej.} \because x > \frac{1}{2})$$

A1

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7 **(a)** Show that $\tan A + \cot A = 2\text{cosec } 2A$. [3]

Alternative:

$$\text{LHS} = \tan A + \cot A$$
$$= \tan A + \frac{1}{\tan A}$$
$$= \frac{\tan^2 A + 1}{\tan A}$$
$$= \frac{\sec^2 A}{\tan A}$$
$$= \frac{\frac{1}{\cos^2 A}}{\frac{\sin A}{\cos A}}$$
$$= \frac{1}{\sin A \cos A}$$
$$= \frac{2}{2\sin A \cos A}$$
$$= 2\text{cosec } 2A \text{ (shown)}$$

$$\text{LHS} = \tan A + \cot A$$
$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$
$$= \frac{1}{\sin A \cos A}$$
$$= \frac{2}{2\sin A \cos A}$$
$$= \frac{2}{\sin 2A}$$
$$= 2\text{cosec } 2A \text{ (shown)}$$

M1 – $\frac{\sin A}{\cos A}$ or $\frac{1}{\tan A}$

M1 – for $\sin^2 A + \cos^2 A = 1$ or for $\tan^2 A + 1 = \sec^2 A$

A1 – award only if last 3 steps are shown

(b) Hence, solve the equation $\frac{1}{\tan A + \cot A} = \frac{1}{4}$ for $0 \le A \le 2\pi$. [3]

$$\frac{1}{\tan A + \cot A} = \frac{1}{4}$$
$$\tan A + \cot A = 4$$
$$2\text{cosec} 2A = 4$$
$$\text{cosec} 2A = 2$$
$$\sin 2A = \frac{1}{2}$$
$$\text{basic } \angle = \sin^{-1}\left(\frac{1}{2}\right)$$
$$= \frac{\pi}{6}$$
$$2A = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$
$$A = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

B1 – substitution of **(a)**

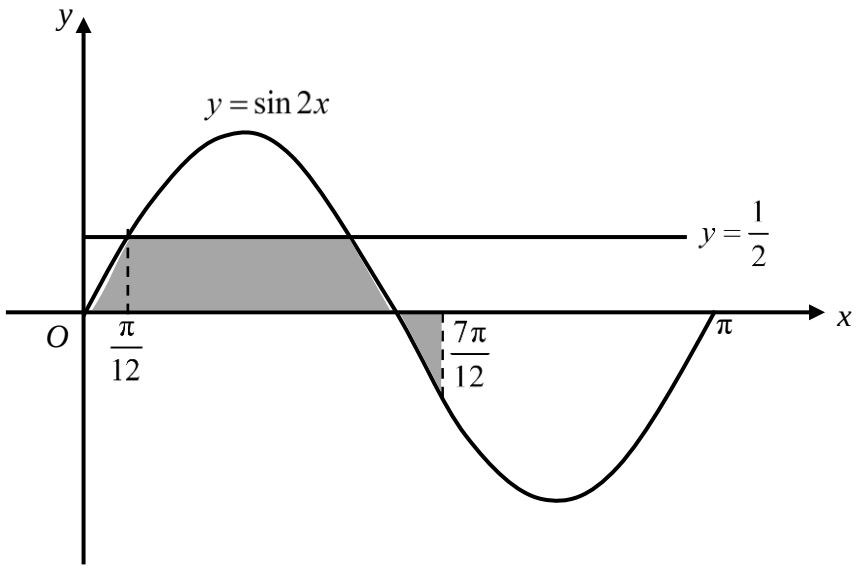
M1 – for finding $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

A1

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(c) The diagram shows, for $0 \le x \le \pi$, the curve $y = \sin 2x$ and the line $y = \frac{1}{2}$.

Showing all your working, find the area of the shaded region. [5]



Area bounded by curve, x-axis and lines $x = 0$ and $x = \frac{\pi}{12}$

$= \int_0^{\frac{\pi}{12}} \sin 2x \, dx$

$= \left[\frac{-\cos 2x}{2} \right]_0^{\frac{\pi}{12}}$

M1 – integrate $\sin 2x$

$= \left[-\frac{\cos \frac{\pi}{6}}{2} \right] - \left[-\frac{\cos 0}{2} \right]$

$= \left(-\frac{\sqrt{3}}{4} + \frac{1}{2} \right)$ units^2 or 0.066987... units^2

DM1 – evaluate with either limits 0 to $\frac{\pi}{12}$ or $\frac{5\pi}{12}$ to $\frac{\pi}{2}$ or $\frac{\pi}{2}$ to $\frac{7\pi}{12}$

Area of rectangle $= \left(\frac{5\pi}{12} - \frac{\pi}{12} \right) \times \frac{1}{2}$

$= \frac{\pi}{6}$ units^2

M1 – area of rectangle

Area of shaded region $= 3 \times \left(-\frac{\sqrt{3}}{4} + \frac{1}{2} \right) + \frac{\pi}{6}$

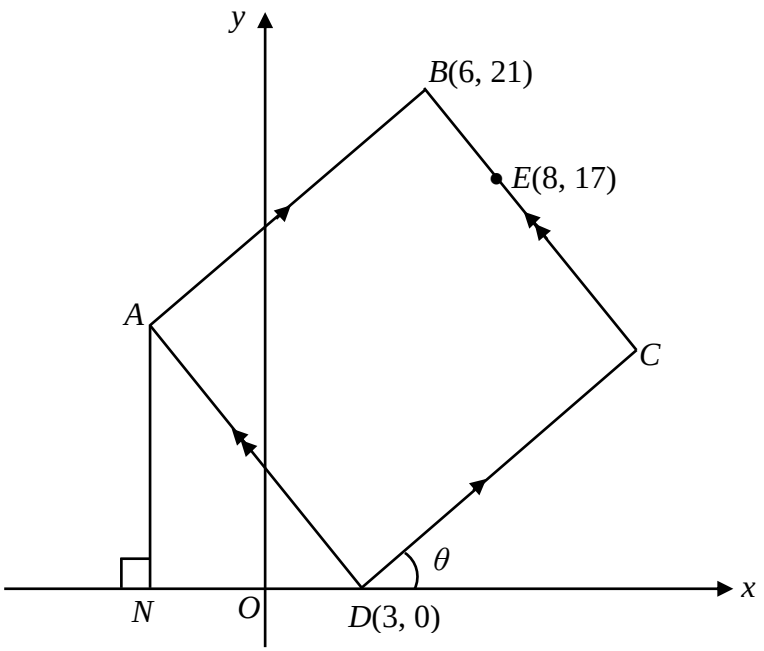
$= 0.725$ units^2 (3 sig.fig)

M1 f.t. – area under curve × 3

A1

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8



The diagram shows a parallelogram with vertices A, $B(6, 21)$, C and $D(3, 0)$. The point $E(8, 17)$ lies on BC. The line CD makes an angle θ with the positive x-axis such that $\tan\theta = 1$. A line is drawn, parallel to the y-axis, from A to meet the x-axis at N.

(a) Show that the coordinates of A are $(-3, 12)$. [5]

$m_{AB} = m_{CD}$
$= 1$

B1 – gradient = tan θ

Eq. of AB: $y - 21 = (1)(x - 6)$
$y = x + 15$ ...(1)

$m_{AD} = m_{BE}$
$= \dfrac{17 - 21}{8 - 6}$
$= -2$

B1 – gradient formula

Eq. of AD: $y - 0 = -2(x - 3)$
$y = -2x + 6$ ...(2)

M1 f.t. – finding equation of AD or AB

Sub. (1) into (2):
$x + 15 = -2x + 6$
$3x = -9$
$x = -3$ sub. into (1):
$y = 12$

M1 f.t. – substitution

∴ $A(-3, 12)$ (shown)

A1 (A.G.)

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Alternative:

Eq. of CD: $y-0=(1)(x-3)$

$y=x-3$ ...(1)

$m_{BC}=m_{BE}$

$=\dfrac{17-21}{8-6}$

$=-2$

B1 – gradient formula

Eq. of BC: $y-21=-2(x-6)$

$y=-2x+33$ ...(2)

M1 f.t. – finding equation of BC

Sub. (1) into (2):

$x-3=-2x+33$

$3x=36$

$x=12$ sub. into (1):

$y=9$

$\therefore$ ***C*(12, 9)**

M1 f.t. – substitution

Midpoint of AC = Midpoint of BD

$\left(\dfrac{x+12}{2}, \dfrac{y+9}{2}\right)=\left(\dfrac{3+6}{2}, \dfrac{0+21}{2}\right)$

$\left(\dfrac{x+12}{2}, \dfrac{y+9}{2}\right)=\left(\dfrac{9}{2}, \dfrac{21}{2}\right)$

M1 f.t. – equating midpoints

Comparing,

$\dfrac{x+12}{2}=\dfrac{9}{2}$ and $\dfrac{y+9}{2}=\dfrac{21}{2}$

$x+12=9$ and $y+9=21$

$x=-3$ $y=12$

A1 (A.G.)

$\therefore$ ***A*(−3, 12) (shown)**

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(b) Hence, find the area of parallelogram *ABCD*. [2]

area of parallelogram $ABCD = 2 \times$ area of triangle ABD

$$= 2 \times \frac{1}{2}\begin{vmatrix} -3 & 3 & 6 & -3 \\ 12 & 0 & 21 & 12 \end{vmatrix}$$

M1 – allow clockwise direction

$$= 2 \times \frac{1}{2}\left[(63+72)-(-63+36)\right]$$

$$= 135 + 27$$

$$= 162 \text{ units}^2$$

A1

Alternative:

If students have found the coordinates of C in part (a),

$$\text{area of parallelogram } ABCD = \frac{1}{2}\begin{vmatrix} 3 & 12 & 6 & -3 & 3 \\ 0 & 9 & 21 & 12 & 0 \end{vmatrix}$$

M1 f.t. – allow clockwise direction

$$= \frac{1}{2}\left[(27+252+72)-(36-63+54)\right]$$

$$= \frac{1}{2}(351-27)$$

$$= 162 \text{ units}^2$$

A1

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(c) A point F with y-coordinate of 5 lies on the line CD.
Explain why $AEFN$ is a parallelogram. [5]

Eq. of CD: $y-0=(1)(x-3)$

$y=x-3$ ...(3)

B1

Let coordinates of F be $(x, 5)$.

Sub. $(8, y)$ into (3):

$y=8-3$

$=5$

M1 f.t. – substitution

$\therefore$ $F(8, 5)$

Coordinates of $N = (-3, 0)$

length of $AN=\sqrt{(-3+3)^2+(12-0)^2}$

$=12$ units

length of $EF=\sqrt{(8-8)^2+(17-5)^2}$

$=12$ units

M1 f.t. – finding either lengths

Since F lies directly below E, the line EF is a vertical line.
Thus, the lines EF and AN are parallel.

A1 – vertical lines are parallel

And that the lengths of AN and EF are equal (i.e., 12 units),
they form a pair of parallel and equal opposite sides.
Thus, $AEFN$ is a parallelogram.

A1 – conclusion

Alternative 1:

gradient of $AE=\dfrac{17-12}{8-(-3)}=\dfrac{5}{11}$ gradient of $FN=\dfrac{5-0}{8-(-3)}=\dfrac{5}{11}$

M1 f.t. – finding either gradients

Since gradients of lines AE and FN are the same, the lines AE and FN are parallel.

Since F lies directly below E, the line EF is a vertical line.
Thus, the lines EF and AN are parallel.

A1 – vertical lines are parallel

Since there are 2 pairs of parallel lines, $AEFN$ is a parallelogram.

A1 – conclusion

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Alternative 2:

Midpoint of $AF = \left(\frac{-3+8}{2}, \frac{12+5}{2}\right)$ $= \left(\frac{5}{2}, \frac{17}{2}\right)$	M1 f.t. – midpoint of AF
Midpoint of $NE = \left(\frac{-3+8}{2}, \frac{0+17}{2}\right)$ $= \left(\frac{5}{2}, \frac{17}{2}\right)$	M1 f.t. – midpoint of AE
Since the diagonals intersect at the same point, *AEFN* is a parallelogram.	A1 – conclusion

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9 **(a)** Differentiate $x \ln x$ with respect to x. [2]

$$\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x) = x\left(\frac{1}{x}\right) + (1)\ln x$$
$$= 1 + \ln x$$

B1, B1 – for each term

(b) A curve $y = \mathrm{f}(x)$ is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 24x^2 + \frac{16}{x}$, where $x > 0$. The line $y = 24x - 40$ is parallel to the tangent of the curve at $P(1, -16)$.

By using the result found in part **(a)**, find the equation of the curve. [6]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \int 24x^2 + \frac{16}{x}\,\mathrm{d}x$$
$$= \frac{24x^3}{3} + 16\ln x + c$$
$$= 8x^3 + 16\ln x + c, \text{ where } c \text{ is a constant.}$$

B1, B1 – for each integral

When $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 24$.
$$24 = 8(1)^3 + 16\ln(1) + c$$
$$c = 16$$

M1 f.t. – finding c

$$\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = 8x^3 + 16\ln x + 16$$

$$y = \int 8x^3 + 16\ln x + 16\,\mathrm{d}x \quad \textbf{or} \quad y = \int 8x^3 + 16(1 + \ln x)\,\mathrm{d}x$$
$$= \frac{8x^4}{4} + 16\left(x\ln x - \int 1\,\mathrm{d}x\right) + 16x + c_1$$
$$= \frac{8x^4}{4} + 16(x\ln x - x + c_2) + 16x + c_1$$
$$= 2x^4 + 16x\ln x - 16x + 16x + c_3$$
$$2x^4 + 16x\ln x + c_3, \text{ where } c_1, c_2 \text{ and } c_3 \text{ are constants.}$$

M1 f.t. – reverse

When $x = 1$, $y = -16$.
$$-16 = 2(1)^4 + 16(1)\ln(1) + c_3$$
$$-16 = 2 + c_3$$
$$c_3 = -18$$

M1 f.t. – finding c_3

$$\therefore y = 2x^4 + 16x\ln x - 18$$

A1

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Continuation of working space for question **9(b)**.

(c) Explain why the condition $x > 0$ is necessary. [1]

ln x is defined for $x > 0$.

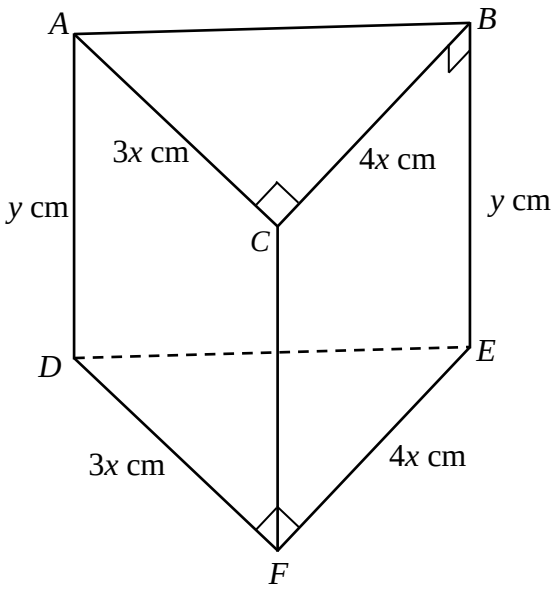
or

ln x is undefined for $x < 0$.

B1

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10



The diagram shows a solid prism with right-angled triangular ends that are perpendicular to the parallel sides *AD*, *BE* and *CF*, which are each y cm in length. The right-angled triangular ends have sides *AC* and *DF*, which are $3x$ cm, and sides *BC* and *EF*, which are $4x$ cm.

Given that the volume of the prism is 1200 cm^3,

(a) find an expression for y in terms of x, [2]

$$\text{Vol. of prism} = \frac{1}{2}(3x)(4x)y$$ — B1 – volume formula

$$1200 = 6x^2y$$

$$y = \frac{200}{x^2}$$ — B1

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(b) show that the total surface area of the prism, S cm^2, is given by $S = 12x^2 + \frac{2400}{x}$. [3]

$AB = \sqrt{(3x)^2 + (4x)^2}$
$= \sqrt{25x^2}$
$= 5x$ cm

B1 – finding length of AB

Total surface area $= 2 \times \frac{1}{2}(3x)(4x) + 3xy + 4xy + 5xy$
$= 12x^2 + 12xy$
$= 12x^2 + 12x\left(\frac{200}{x^2}\right)$
$= 12x^2 + \frac{2400}{x}$ (shown)

M1 – at least 3 correct surfaces

A1 (A.G.)

(c) Given that x can vary, find the value of x for which the total surface area of the prism is a stationary value. [3]

$\frac{dS}{dx} = 24x - \frac{2400}{x^2}$

B1 – differentiation

For S to be stationary, $\frac{dS}{dx} = 0$

$24x - \frac{2400}{x^2} = 0$
$24x = \frac{2400}{x^2}$
$x^3 = 100$
$x = \sqrt[3]{100}$
$= 4.64$ (3 sig. fig.)

M1 f.t.

A1

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(d) Explain why this value of x gives the smallest surface area possible. [2]

$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 24 + \frac{4800}{x^3}$

M1 f.t. – differentiation or 1st derivative test

When $x = \sqrt[3]{100}$,

$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 24 + \frac{4800}{\left(\sqrt[3]{100}\right)^3}$

$= 72 \quad > 0 \quad \because$ minimum

Since $\frac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0$, the surface area is the smallest when $x = 4.64$.

A1 – conclude '> 0' (must show '72')

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