Anglo-Chinese School
(Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 1
Preliminary Examination 2023

1		Height = $\frac{45\sqrt{5}-38\sqrt{7}}{6-\sqrt{35}}\times\frac{6+\sqrt{35}}{6+\sqrt{35}}$			
		$=\frac{270\sqrt{5}+45\sqrt{175}-228\sqrt{7}-38\sqrt{245}}{1}$			
		$=270\sqrt{5}+225\sqrt{7}-228\sqrt{7}-266\sqrt{5}$			
		$=4\sqrt{5}-3\sqrt{7}$			
2	(a)	$\mathrm{f}(x)=x^3-(2h+1)x^2-(k-2h)x+k$			
		$\mathrm{f}(1)=(1)^3-(2h+1)(1)^2-(k-2h)(1)+k$			
		$1-2h-1-k+2h+k=0$ (shown)			
		By factor theorem, $x-1$ is a factor of f(x)			
	(b)	$x^3-(2h+1)x^2-(k-2h)x+k=(x-1)(x^2-2hx-k)$			
		$\mathrm{f}(x)=(x-1)(x^2-2hx-k)$			
		Since f(x) has three real roots when f(x) = 0, $x^2-2hx-k=0$ has two real roots.			
		$b^2-4ac\geq 0$			
		$(-2h)^2-4(1)(-k)\geq 0$			
		$4h^2+4k\geq 0$			
		$h^2+k\geq 0$			
3	(a)	$T=30°\mathrm{C}$			
	(b)	$1000=2030-2000e^{-k(1.5)}$			
		$2000e^{-k(1.5)}=1030$			
		$k=0.442392$			
		$1300=2030-2000e^{-0.442392t}$			
		$2000e^{-0.442392t}=730$			
		$\ln e^{-0.442392t}=\ln\frac{730}{2000}$			
		$t=2.28h$			
	(c)	$y=2030$			

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4	(a)	$\mathrm{f}''(x)=\dfrac{18}{(1-2x)^3}$			
		$\mathrm{f}'(x)=\dfrac{18(1-2x)^{-2}}{-2\times-2}+c_1$			
		$=\dfrac{9(1-2x)^{-2}}{2}+c_1$			
		$2\mathrm{f}'(-1)=1$			
		$2\left(\dfrac{9}{2(1-2(-1))^2}+c_1\right)=1$			
		$\dfrac{1}{2}+c_1=\dfrac{1}{2}$			
		$c_1=0$			
		$\mathrm{f}'(x)=\dfrac{9}{2(1-2x)^2}$			
		$\mathrm{f}(x)=\dfrac{9}{4(1-2x)}+c_2$			
		Sub (1,0)			
		$0=\dfrac{9}{4(1-2(1))}+c_2$			
		$c_2=\dfrac{9}{4}$			
		$y=\dfrac{9}{4(1-2x)}+\dfrac{9}{4}$			
5	(a)	$\mathrm{f}'(x)=\dfrac{(p-x)\left[2(x+2)(1)\right]-(x+2)^2(-1)}{(p-x)^2}$			
		$=\dfrac{2(p-x)(x+2)+(x+2)^2}{(p-x)^2}$			
		$=\dfrac{(x+2)(2p+2-x)}{(p-x)^2}$			
	(b)	For $\mathrm{f}(x)$ is an increasing function, $\mathrm{f}'(x)>0$			
		$(x+2)(2p+2-x)>0$			

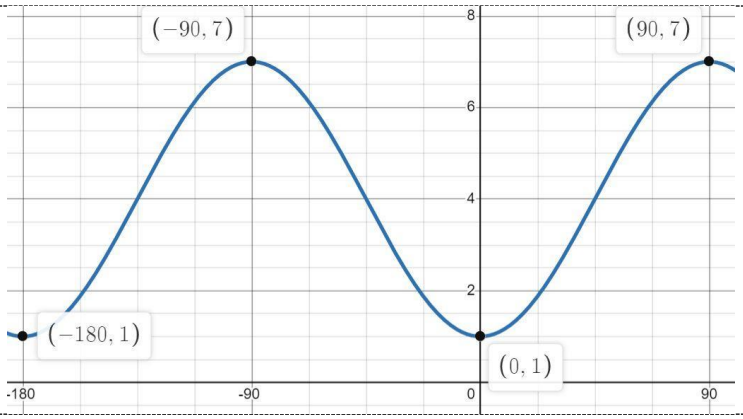
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		$-x^2+2px+4p+4>0$			
		$x^2-2px-4p-4<0$ ---(1)			
		$(x+2)(x-8)<0$			
		$x^2-6x-16<0$ ----(2)			
		Comparing (1) & (2)			
		$-2p=-6$			
		$p=3$			
		Alternative method			
		Comparing this with $(x+2)(2p+2-x)>0$ in (a), $(x+2)(x-8)<0$ $(x+2)(8-x)>0$ $2p+2=8$ $p=3$			
	(c)	$\frac{(x+2)^2}{3-x}=0 \Rightarrow x=-2$			
		$f'(x)=\frac{(-2+2)(2(4)+2+2)}{(3+2)^2}$ $=0$ (gradient of tangent)			
		Gradient of normal is undefined (vertical line) $\therefore$ Eqn. of normal $x=-2$			
6	(a)	$5\sin^2 A-3\cos^2 A=7\sin 2A$			
		$5\sin^2 A-3\cos^2 A=7(2\sin A\cos A)$			
		$5\sin^2 A-14\sin A\cos A-3\cos^2 A=0$			
		$(5\sin x+\cos x)(\sin x-3\cos x)=0$			
		$\frac{\sin A}{\cos A}=-\frac{1}{5}$ or $\frac{\sin A}{\cos A}=3$			
		$\tan A=-\frac{1}{5}$ (reject, A is acute) or $\tan A=3$ (shown)			
	(b)	$\cos(60^\circ+A)=\cos 60^\circ\cos A-\sin 60^\circ\sin A$			
		$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{10}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{10}}\right)$			

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		$= \frac{1-3\sqrt{3}}{2\sqrt{10}}$			
	(c)	From (b), $\cos(60^\circ + A)$ is a negative ratio. So $60^\circ + A$ lies in the 2nd or 4th quadrant. Both 60° and A are both acute, $60^\circ + A$ cannot exceed 180°. Thus $60^\circ + A$ lies in the 2nd quadrant and is obtuse.			
7	(a)	$y = (3x-1)\left(\sqrt{6x+1}\right)$			
		$\frac{dy}{dx} = 3\left(\sqrt{6x+1}\right) + \frac{1}{2}(3x-1)(6x+1)^{-\frac{1}{2}}(6)$			
		$= \frac{18x+3+9x-3}{\sqrt{6x+1}}$			
		$= \frac{27x}{\sqrt{6x+1}}$			
	(b)	$\int_0^4 \frac{9x-3}{\sqrt{6x+1}}\,dx$			
		$= \frac{1}{3}\int_0^4 \frac{27x}{\sqrt{6x+1}}\,dx - \int_0^4 \frac{3}{\sqrt{6x+1}}\,dx$			
		$= \frac{1}{3}\left[(3x-1)\sqrt{6x+1}\right]_0^4 - 3\left[\frac{(6x+1)^{\frac{1}{2}}}{6\times\frac{1}{2}}\right]_0^4$			
		$= \frac{1}{3}\left(11\sqrt{25}+\sqrt{1}\right) - \left(\sqrt{25}-\sqrt{1}\right)$			
		$= \frac{44}{3}$ or 16.7 (3s.f)			
8	(a)	$T_3 = \binom{n}{2}(2x)^{n-2}\left(-\frac{3}{x}\right)^2$			
		$= \binom{n}{2}(2)^{n-2}(x)^{n-2}(-3)^2\left(x^{-1}\right)^2$			
		$\frac{n(n-1)}{2}(2)^{n-2}(-3)^2 = \frac{270}{8}\left(2^n\right)$			
		$\frac{n(n-1)}{2}(2)^{-2}(-3)^2 = \frac{270}{8}$			

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		$\frac{n(n-1)}{2}=15$			
		$n^2-n-30=0$			
		$(n-6)(n+5)=0$			
		$n=6$ or $n=-5$(reject)			
8	(b)	$\left(1+x^2\right)\left(2x-\frac{3}{x}\right)^6=\left(1+x^2\right)\left(-576x^4+2160x^2+...\right)$			
		Coefficient of $x^4=-576+2160=1584$			
9	(a)	$a=-3$			
		$b=4$			
	(b)				
	(c)	$1-\frac{1}{2}\cos 2x=0$			
		$6-\frac{6}{2}\cos 2x=0$			
		$4-3\cos 2x=-2$			
		$y=-2$			
		No solution			
10	(a)	$49^x-7^{x+1}=18$			
		$7^{2x}-7^x\times 7=18$			
		Let $y=7^x$			
		$y^2-7y-18=0$			

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		$(y-9)(y+2)=0$			
		$y=-2$ or $y=9$			
		(reject)			
		$7^x=9$			
		$x=\log_7 9$			
		$a=7,\ b=9$			
	(b)	$-3x^2+qx-8<0$			
		$b^2-4ac<0$			
		$q^2-4(-3)(-8)<0$			

		$q^2-96<0$			
		$(q-\sqrt{96})(q+\sqrt{96})<0$	M1		
		$-\sqrt{96}<q<\sqrt{96}$			
		$-9.798<q<9.798$			
		Largest value of the integer q = 9	A1		
					[8]
11	(a)	$DA = GC + x$			
		$x=26\cos\theta$			
		$GC=13\sin\theta$			
		$AD = 13\sin\theta+26\cos\theta$			
		$T = 11+13+26+26\cos\theta+13\sin\theta$			
		$T = 50+26\cos\theta+13\sin\theta$			
	(b)	$\sqrt{26^2+13^2}$			
		$=\sqrt{845}$ or $13\sqrt{5}$			
		$\tan^{-1}\frac{13}{26}$			
		$T = 50+13\sqrt{5}\cos(\theta-26.6°)$			
	(c)	Max $T = 50+13\sqrt{5}$			
		$=79.1$ (3s.f)			
		This is not possible as the maximum distance of T is 79.1 m.			

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12	(a)	$a + 4$			
	(b)	centre = $(a, a+4)$			
		$\sqrt{(a-8)^2 + (a+4-13)^2} = a$			
		$(a-8)^2 + (a-9)^2 = a$			
		$a^2 - 16a + 64 + a^2 - 18a + 81 = a^2$			
		$a^2 - 34a + 145 = 0$			
		$a = 29$ (rejected) or $a = 5$			
		centre = $(5, 9)$			
		$(x-5)^2 + (y-9)^2 = 25$			
	(c)	Gradient of radius = $\frac{13-9}{8-5} = \frac{4}{3}$			
		Gradient of line L = $-\frac{3}{4}$			
		Equation of line L: $y = -\frac{3}{4}x + c$			
		$13 = -\frac{3}{4}(8) + c$			
		$c = 19$			
		$y = -\frac{3}{4}x + 19$			
13	(a)	$\frac{4x^3 + 2x^2 - 5}{x^2(2x-1)} = 2 + \frac{4x^2 - 5}{x^2(2x-1)}$			
		$\frac{4x^2 - 5}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$			
		$4x^2 - 5 = A(x)(2x-1) + B(2x-1) + Cx^2$			
		Let $x = 0$			
		$-5 = B(-1)$			
		$B = 5$			
		Let $x = \frac{1}{2}$			

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		$-4=\frac{1}{4}C$			
		$C=-16$			
		Let $x=1$			
		$-1=A+5-16$			
		$-1=A+5-16$			
		$A=10$			
		$\frac{4x^3+2x^2-5}{x^2(2x-1)}=2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}$			
	(b)	$\int\frac{4x^3+2x^2-5}{x^2(2x-1)}\,\mathrm{d}x$			
		$=\int 2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}\,\mathrm{d}x$			
		$=2x+10\ln x-\frac{5}{x}-8\ln(2x-1)+c$			

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