Name:	Class:	Class Register Number:

中正中学

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PRELIMINARY EXAMINATION 2021

SECONDARY 4

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **Tuesday 14 September 2021**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
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The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Question Number	Marks Obtained
1	
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Total Marks	

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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Identities

$$\sin^2 A + \cos^2 A = 1$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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1 Given that $p(x)=(x+1)^3+5(x-1)^2+ax+b$ has a factor of $x-1$ and leaves a remainder of 21 when divided by $x-2$, find the value of a and of b. [4]

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2 (i) Given that $f(x) = x^3 + 3x^2 - 4$, show that $x+2$ is a factor of $f(x)$, and hence factorise $f(x)$ completely. [3]

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(ii) Hence, express $\frac{2x^3+6x^2+7}{x^3+3x^2-4}$ in partial fractions. [5]

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3 **(a)** Given that $\frac{\sqrt[3]{2^2}\times 3^{\frac{1}{5}}}{6^3}\times\left(\frac{2}{3}\right)^{-2} = 2^a \times 3^b$, find the fractions, a and b. [3]

(b) Given that $3^{x+2}\times 5^{x-1} = 9^x \times 5^{2x}$, evaluate 15^x without using a calculator. [3]

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(c) Given that n is a positive integer, explain why the largest prime factor of $2(5^{n+1})-4(5^n)+5^{n+2}$ is 31. [3]

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4 **(a)** Given that $\int_1^5 f(x)\,dx = 5$, find the value of

(i) $\int_5^1 f(x)\,dx$, [1]

(ii) $\int_1^{1.5} f(x)\,dx + \int_{1.5}^5 \left[f(x) - \frac{3}{2}x \right] dx$. [3]

(b) $y = g(x)$ is the equation of a non-horizontal straight line. [1]

Given that $\int_0^5 g(x)\,dx = 0$ and $g(2.5) = 0$, explain what the result implies about the line $y = g(x)$.

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5 **(i)** Given that $y = \frac{16x}{1+x^2}$, find the range of values of x for which y is increasing. [4]

(ii) Another curve has equation $y = \frac{16x}{1+x^2} + d$, where d is a constant. [1]

Explain why the range of values of x for which y is increasing for this curve is the same as the answer in **(i)**.

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6 A circle, C_1, has an equation $x^2 - 6x + y^2 + 10y = 66$.

(i) Are the x-axis and y-axis tangents to C_1? Explain your answer. [3]

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(ii) Does the point $(2, -4)$ lie inside, on or outside of C_1? Show your working clearly. [2]

A second circle, C_2, is the reflection of C_1 on the y-axis.

(iii) Find the equation of C_2. [2]

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7 The point $(a, 2)$ is a stationary point on the graph $y = x^3 - x^2 - x + b$, where a and b are positive constants.

(i) Show that $a = 1$. [2]

(ii) Find the value of b. [1]

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(iii) Find the coordinates of the other stationary point. [1]

(iv) Determine the nature of each of the stationary points. [3]

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8 **(a)** The average person's blood pressure is modelled by $y = k\sin(5\pi t) + 100$, where y millimetres of mercury (mmHg) represents the blood pressure at time t, measured in minutes and k is a positive constant.

The maximum blood pressure of an average person is 120 mmHg.

(i) Find the value of k. Hence, state the minimum blood pressure of an average person. [2]

(ii) State the period of y and the number of cycles for $0 \le t \le 0.6$. [2]

(iii) Sketch the graph of $y = k\sin(5\pi t) + 100$ for $0 \le t \le 0.6$. [2]

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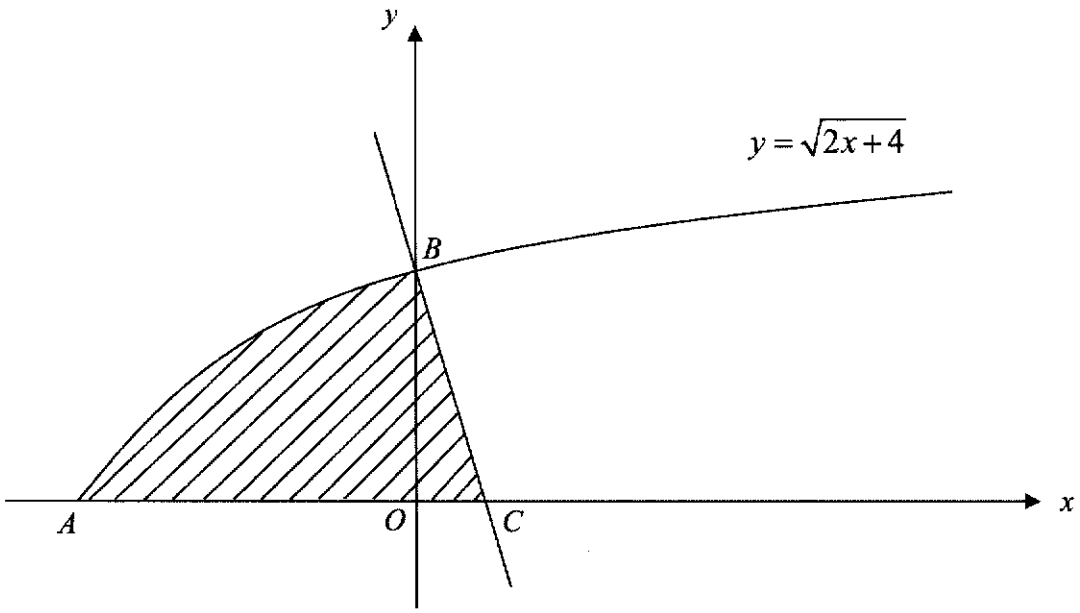
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(b) Solve the equation $2\cos 2x - 3\sec x + 2 = 0$ for $-\pi \leq x \leq \pi$. [5]

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9



The diagram shows part of the curve $y = \sqrt{2x+4}$, meeting the x-axis at the point A and the y-axis at the point B. The normal to the curve at B meets the x-axis at the point C.

(i) Write down the coordinates of B. [1]

(ii) Find the coordinates of C. [4]

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(iii) Find the area of the shaded region. [4]

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10 A particle travels in a straight line so that, t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = t^2 - 2kt - 6k$, where k is a constant.
The minimum velocity of the particle occurs when $t = 2$.

(i) Show that $k = 2$. [2]

(ii) Find an expression, in terms of t, for the displacement of the particle. Hence, show that the particle will return to O only once during its journey. [5]

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(iii) Find the total distance travelled by the particle for the first 7 seconds. [4]

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11 **(a)** A, B and C are the angles of a triangle. [2]
Show that $\sin C = \sin A \cos B + \sin B \cos A$.

(b) Given that P and Q are angles such that $\sin P = -\frac{4}{5}$ and $\cos Q = \frac{12}{13}$, where both P and Q are in the same quadrant, find the exact value of

(i) $\tan 2Q$, [3]

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(ii) $\operatorname{cosec}(P-Q)$, [3]

(iii) $\sin\frac{P}{2}$. [2]

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12 The cost per submarine, \$$y$ in millions, of assembling x submarines can be modelled by $y = \frac{5}{2}x^2 - 20x + 100$, where $x \leq 8$.

(i) Explain the meaning of the constant term 100 in this model. [1]

(ii) Express $y = \frac{5}{2}x^2 - 20x + 100$ in the form of $y = a(x-h)^2 + k$. [2]

(iii) Explain the significance of $x = h$ in **(ii)**. [1]

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PRELIMINARY EXAMINATION 2021

SECONDARY 4

ADDITIONAL MATHEMATICS 4049/02

Paper 2

Friday 17 September 2021

2 hours 15 minutes

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No additional materials are required.

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1 (a) Solve the simultaneous equations

$$y = x^2 - 6,$$
$$y = 2x - 3.$$ [3]

(b) Prove that $\dfrac{3\cos\theta + \cos 2\theta - 1}{\cos^2\theta + 2\cos\theta} = 2 - \sec\theta$. [4]

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2 The expression $8\sin\theta - 3\cos\theta$ is defined for $0^0 \le \theta \le 180^0$.

Using $R\sin(\theta - \alpha)$, where $R > 0$ and $0^0 < \alpha < 90^0$,

(i) solve the equation $8\sin\theta - 3\cos\theta = 5$, [5]

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(ii) state the smallest value of $\frac{1}{(8\sin\theta - 3\cos\theta)^2}$ and find the corresponding value of θ, for $0^O \leq \theta \leq 180^O$. [2]

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3 **(a)** Given that $4x^2 - \frac{x}{h} - 3 = 0$, where $h \neq 0$, show that there are no real values of h for which the equation has equal real roots. [3]

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(b) Find the range of values of the positive constant k for which the line $y = kx$ lies completely above the curve $y = -16x^2 + (2k-1)x - 1$. [4]

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4 (i) By considering the general term in the binomial expansion of $\left(\frac{2}{x^2}+x^3\right)^{16}$, explain why every term is dependent on x. [3]

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(ii) Find the term independent of x in the expansion of $\left(\frac{2}{x^2}+x^3\right)^{16}\left(4-3x^2\right)$. [3]

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5 An object moving in a certain medium, with speed v m/s, experiences a resistance to motion of R newtons. R and v are related by the equation $R = av^2 + bv$, where a and b are constants.

The table shows the experimental values of the variables v and R.

v	5	10	15	20	25
R	17	44	81	128	185

(i) On the grid on page 11, plot $\frac{R}{v}$ against v and draw a straight line graph. [3]

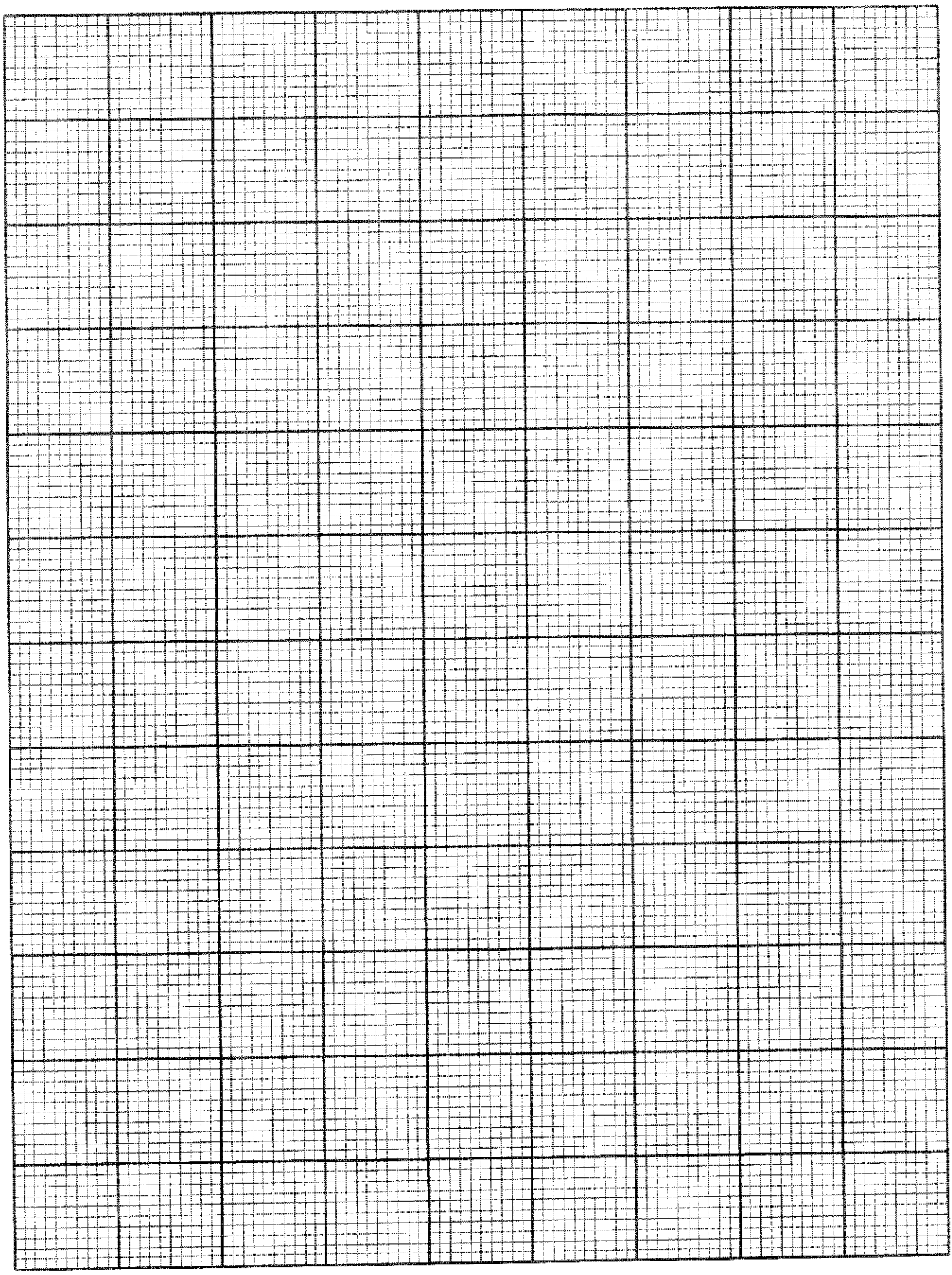
(ii) Use your graph to estimate the value of a and of b. [3]

(iii) In a different medium, R is directly proportional to v and $R = 30$ when $v = 5$.

Draw a suitable line on your graph to illustrate the second situation and use it to determine the value of v for which the resistance is the same in both mediums. [3]

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6 (a) A cone has a volume of $\left(24+5\sqrt{3}\right)\pi$ cm^3 and a base radius of $(2+\sqrt{3})$ cm. Find its height in the form of $\left(p+q\sqrt{3}\right)$, where p and q are integers. [4]

(b) Solve the equation $e^x\left(3e^x-2\right)=5$. [3]

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(c) Sam just bought a brand new car. The value, V dollars, of the car depreciates over time.
It is given that $V = 179200e^{kt} + 8800$, where k is a constant and t is the time in years since it was bought.

(i) Find the initial value of the car. [1]

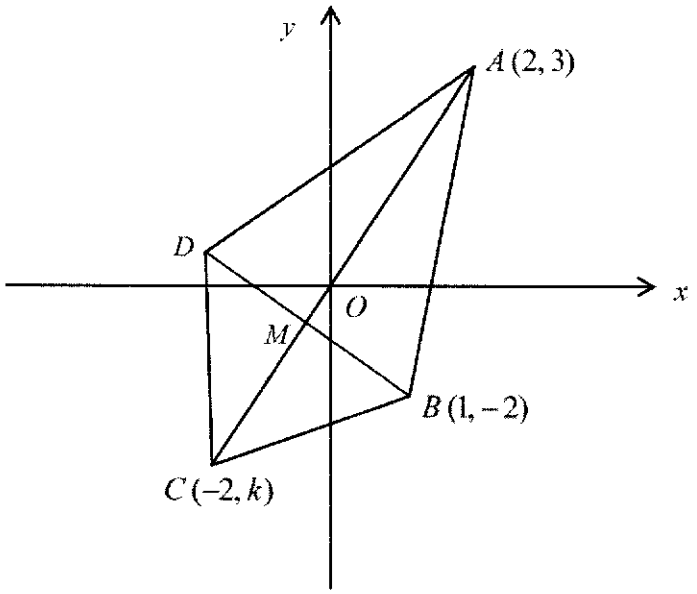
(ii) Calculate the value of k if, after 5 years, the value of the car is halved. [3]

(iii) After having driven the car for a long time, Sam decided to sell his car.
A second-hand car dealer offers to buy the car from him for $8000.
Explain, with clear mathematical reasoning, why Sam should not accept the offer. [2]

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7 The diagram shows a kite $ABCD$ in which the line AC passes through the origin O and has a gradient of 1.5.



(i) Find the value of k. [2]

(ii) Find the equation of the line BD. [3]

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(iii) The line AC and line BD meets at point M.

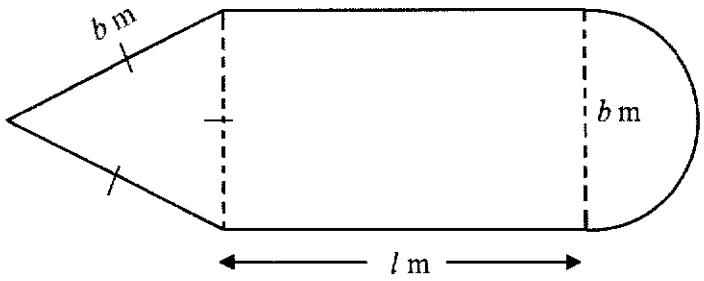
Show that the coordinates of M is $\left(-\frac{8}{13}, -\frac{12}{13}\right)$. [3]

(iv) The coordinates of D are (a,b). Prove that $13\left(a+\frac{8}{13}\right)^2+13\left(b+\frac{12}{13}\right)^2=49$. [2]

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8 A farmer just bought a piece of land to grow his crops. This piece of land consists of an equilateral triangle of sides b m, a rectangle with sides b m and l m as well as a semi-circle of diameter b m as shown in the diagram below.



(i) Given that the perimeter of the land is 600 m, express l in terms of b. [2]

(ii) Given that the area of the land is A m^2, show that $A = 300b + \left(\frac{2\sqrt{3} - 8 - \pi}{8}\right)b^2$. [3]

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(iii) Given that b can vary, find the value of b which gives a stationary value of A. [3]

(iv) Find the nature of this stationary value and explain why the farmer will be happy with this information. [3]

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9 **(i)** Find $\frac{d}{dx}(x\cos 2x)$. [2]

(ii) Find $\frac{d}{dx}(x^2 \sin 2x)$. [2]

(iii) Using the results to parts **(i)** and **(ii)**, find $\int x^2 \cos 2x \, dx$. [4]

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10 A curve $y = f(x)$ is such that $f''(x) = 12e^{3x} + 7$ and the point $A\left(0, -\frac{2}{3}\right)$ lies on the curve. The gradient of the curve at A is 9. Find the equation of the curve. [5]

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11 When the height of liquid in a container is h m, the volume of liquid is V m^3 where

$$V = 0.06\left[(4h+1)^3 - 7\right] \text{ and } h \geq 0.25$$

(i) Find an expression for $\frac{dV}{dh}$. [2]

The liquid leaks out of the container at a constant rate of 0.98 m^3/s.

(ii) Find the value of $\frac{dh}{dt}$ when $V = 1.2$. [4]

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(iii) Briefly explain the significance of your answer for **(ii)**. [1]

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1 Given that $p(x) = (x+1)^3 + 5(x-1)^2 + ax + b$ has a factor $x-1$ and it leaves a remainder of 21 when divided by $x-2$, find the value of a and of b. [4]

$$p(1) = 0$$
$$(1+1)^3 + 5(1-1)^2 + a + b = 0$$
$$8 + a + b = 0$$
$$b = -a - 8 \quad ----(1)$$

$$p(2) = 21$$
$$(2+1)^3 + 5(2-1)^2 + 2a + b = 21$$
$$27 + 5 + 2a + b = 21$$
$$b = -2a - 11 \quad ----(2)$$

Subst. (1) into (2),
$-a - 8 = -2a - 11$
$a = -3$

Subst. $a = 12$ into (1),
$b = -(-3) - 8$
$b = -5$

$\therefore a = -3, b = -5$

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2 **(i)** Given that $f(x) = x^3 + 3x^2 - 4$, show that $x+2$ is a factor of $f(x)$, and hence factorise $f(x)$ completely. [3]

Let $f(x) = x^3 + 3x^2 - 4$

By inspection,

$$\begin{aligned} f(-2) &= (-2)^3 + 3(-2)^2 - 4 \\ &= -8 + 12 - 4 \\ &= 0 \end{aligned}$$

Since remainder $= 0$, by factor theorem, $(x+2)$ is a factor of $f(x)$. [Shown]

$$\begin{array}{r} x^2 + x - 2 \\ x+2 \overline{)\, x^3 + 3x^2 + 0x - 4} \\ -\underline{\left(x^3 + 2x^2\right)} \\ \quad x^2 + 0x \\ \quad -\underline{\left(x^2 + 2x\right)} \\ \qquad -2x - 4 \\ \qquad -\underline{\left(-2x - 4\right)} \\ \qquad\quad 0 \end{array}$$

$$\begin{aligned} f(x) &= (x+2)(x^2 + x - 2) \\ &= (x-1)(x+2)^2 \end{aligned}$$

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(ii) Hence, express $\dfrac{2x^3+6x^2+7}{x^3+3x^2-4}$ in partial fractions. [5]

$$x^3+3x^2+0x-4\overline{)2x^3+6x^2+0x+7}$$

$$\begin{array}{r} 2 \\ \\ -\underline{\left(2x^3+6x^2+0x-8\right)} \\ 15 \end{array}$$

$$\frac{2x^3+6x^2+7}{x^3+3x^2-4}=\frac{2x^3+6x^2+7}{(x-1)(x+2)^2}$$

$$=2+\frac{15}{(x-1)(x+2)^2}$$

Let $\dfrac{15}{(x-1)(x+2)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$, where A, B and C are constants.

$$15=A(x+2)^2+B(x-1)(x+2)+C(x-1)$$

When $x=1$, $\quad 15=A(3)^2$

$$A=\frac{5}{3}$$

When $x=-2$, $\quad 15=C(-2-1)$

$$C=-5$$

When $x=0$, $\quad 15=\frac{5}{3}(2)^2+B(-1)(2)-5(-1)$

$$15=\frac{20}{3}-2B+5$$

$$B=-\frac{5}{3}$$

$$\therefore \frac{2x^3+6x^2+7}{x^3+3x^2-4}=2+\frac{5}{3(x-1)}-\frac{5}{3(x+2)}-\frac{5}{(x+2)^2}$$

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3 **(a)** Given that $\frac{\sqrt[3]{2^2}\times 3^{\frac{1}{5}}}{6^3}\times\left(\frac{2}{3}\right)^{-2} = 2^a \times 3^b$, find the fractions, a and b. [3]

$$\frac{\sqrt[3]{2^2}\times 3^{\frac{1}{5}}}{6^3}\times\left(\frac{2}{3}\right)^{-2} = \frac{2^{\frac{2}{3}}\times 3^{\frac{1}{5}}}{2^3\times 3^3}\times\left(\frac{3^2}{2^2}\right)$$

$$= 2^{\frac{2}{3}-3-2}\times 3^{\frac{1}{5}-3+2}$$

$$= 2^{-\frac{13}{3}}\times 3^{-\frac{4}{5}}$$

$$\therefore a = -\frac{13}{3}, b = -\frac{4}{5}$$

(b) Given that $3^{x+2}\times 5^{x-1} = 9^x \times 5^{2x}$, evaluate 15^x without using a calculator. [3]

$$3^{x+2}\times 5^{x-1} = 9^x \times 5^{2x}$$

$$9\left(3^x\right)\times\frac{5^x}{5} = 3^{2x}\times 5^{2x}$$

$$\frac{9}{5} = \frac{(3^{2x})(5^{2x})}{(3^x)(5^x)}$$

$$\frac{(3^{2x})(5^{2x})}{(3^x)(5^x)} = \frac{9}{5}$$

$$(3^x)(5^x) = \frac{9}{5}$$

$$15^x = \frac{9}{5}$$

(c) Given that n is a positive integer, explain why the largest prime factor of $2(5^{n+1})-4(5^n)+5^{n+2}$ is 31. [3]

$$2(5^{n+1})-4(5^n)+5^{n+2} = 2(5)(5^n)-4(5^n)+5^2(5^n)$$

$$= 10(5^n)-4(5^n)+25(5^n)$$

$$= 5^n\left(10-4+25\right)$$

$$= 5^n\left(31\right)$$

Since n is a positive integer, 5^n is a multiple of 5. There are 2 prime factors, 5 and 31. Thus, the largest prime factor of $2(5^{n+1})-4(5^n)+5^{n+2}$ is 31.

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4 **(a)** Given that $\int_1^5 f(x)\,dx = 5$, find the value of

(i) $\int_5^1 f(x)\,dx$, [1]

$$\begin{aligned}\int_5^1 f(x)\,dx &= -\int_1^5 f(x)\,dx \\ &= -\int_1^5 f(x)\,dx \\ &= -5\end{aligned}$$

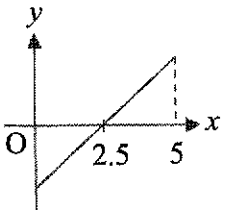
(ii) $\int_1^{1.5} f(x)\,dx + \int_{1.5}^5 \left[f(x) - \frac{3}{2}x\right] dx$. [3]

$$\begin{aligned}\int_1^{1.5} f(x)\,dx + \int_{1.5}^5 \left[f(x) - \frac{3}{2}x\right] dx &= \int_1^{1.5} f(x)\,dx + \int_{1.5}^5 f(x)\,dx - \int_{1.5}^5 \frac{3}{2}x\,dx \\ &= \int_1^5 f(x)\,dx - \left[\frac{3x^2}{2(2)}\right]_{1.5}^5 \\ &= 5 - \left(\frac{3(5)^2}{4} - \frac{3(1.5)^2}{4}\right) \\ &= 5 - \left(17\frac{1}{16}\right) \\ &= -12\frac{1}{16} \text{ or } -12.0625\end{aligned}$$

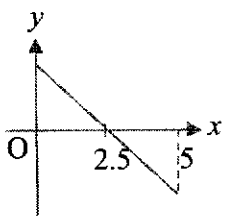
(b) $y = g(x)$ is the equation of a non-horizontal straight line. [1]

Given that $\int_0^5 g(x)\,dx = 0$ and $g(2.5) = 0$, explain what the result implies about the line $y = g(x)$.

The line $y = g(x)$ intersects the x-axis at $x = 2.5$. The two area between line and the x-axis in the interval $0 \le x \le 5$ lies above the x-axis and below the x-axis are equal.



OR



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5 **(i)** Given that $y = \dfrac{16x}{1+x^2}$, find the range of values of x for which y is increasing. [4]

$$y = \frac{16x}{1+x^2}$$

$$\frac{dy}{dx} = \frac{(1+x^2)(16)-(16x)(2x)}{\left(1+x^2\right)^2}$$

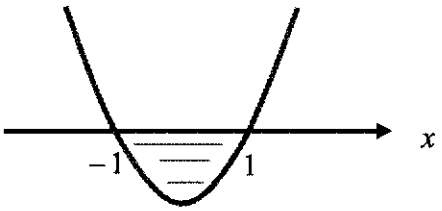
$$\frac{dy}{dx} = \frac{16+16x^2-32x^2}{\left(1+x^2\right)^2}$$

$$\frac{dy}{dx} = \frac{16-16x^2}{\left(1+x^2\right)^2}$$

For y to be increasing, $\dfrac{dy}{dx} > 0$.

Since $\left(1+x^2\right)^2 > 0$ and $\dfrac{16-16x^2}{\left(1+x^2\right)^2} > 0$,

$$16-16x^2 > 0$$
$$-16x^2 > -16$$
$$x^2 - 1 < 0$$
$$(x+1)(x-1) < 0$$



$$\therefore -1 < x < 1$$

(ii) Another curve has equation $y = \dfrac{16x}{1+x^2} + d$, where d is a constant. [1]

Explain why the range of values of x for which y is increasing for this curve is the same as the answer in **(i)**.

d is a constant. The value of d will only result in the translation of the respective graphs vertically upwards or downwards, therefore, it has the same gradient of the curve, and does not affect the range of values of x. Thus, it has the same answer as (i).

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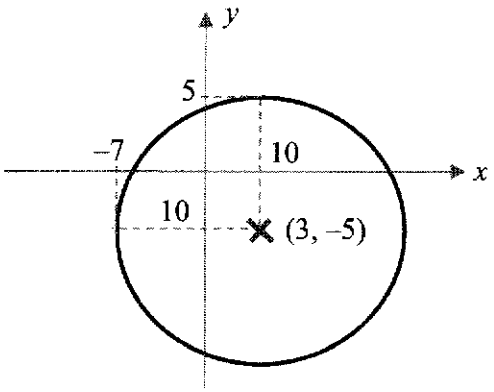
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6 A circle, C_1, has an equation $x^2-6x+y^2+10y=66$.

(i) Are the x-axis and y-axis tangents to C_1? Explain your answer. [3]

$$x^2-6x+y^2+10y=66$$
$$(x-3)^2-9+(y+5)^2-25=66$$
$$(x-3)^2+(y+5)^2=66+9+25$$
$$(x-3)^2+(y+5)^2=100$$
$$(x-3)^2+(y+5)^2=10^2$$

Centre of Circle = $(3,-5)$,
Radius of Circle = 10 units



EITHER

The x-coordinate of the point on circumference west (right) of the centre of circle = $3-10$
$= -7$

Since the x-coordinate of the point $\neq 0$, y-axis is not a tangent to C_1.

The y-coordinate of the point on circumference north (top) of the centre of circle = $-5+10$
$= 5$

Since the y-coordinate of the point $\neq 0$, x-axis is not a tangent to C_1.

OR

The centre of the circle is 5 units from the x-axis and 3 units from the y-axis. As the radius of the circle (10 units) is larger than the distance of the centre from both axes ($10 > 3$ and $10 > 5$), the circle will intersect both axes twice. Therefore, the x-axis and y-axis are not tangents to C_1.

Alternative Solution

$$x^2-6x+y^2+10y=66$$
$$x^2-6x+y^2+10y-66=0$$

Let $x=0$, $y^2+10y-66=0$

$$y=\frac{-10\pm\sqrt{10^2-4(1)(-66)}}{2(1)}$$
$$=\frac{-10\pm\sqrt{364}}{2}$$
$$y=4.539... \quad \text{or} \quad y=-14.539..$$

∴The circle intersects the y-axis at 2 points and the y-axis is not a tangent to C_1.

Let $y=0$, $x^2-6x-66=0$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-66)}}{2(1)}$$
$$=\frac{6\pm\sqrt{300}}{2}$$
$$x=11.660... \quad \text{or} \quad x=-5.660..$$

∴The circle intersects the x-axis at 2 points and the x-axis is not a tangent to C_1.

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(ii) Does the point $(2,-4)$ lie inside, on or outside of C_1? Show your working clearly. [2]

Distance between $(2,-4)$ and centre $= \sqrt{(3-2)^2+(-5-(-4))^2}$

$= \sqrt{2}$ units

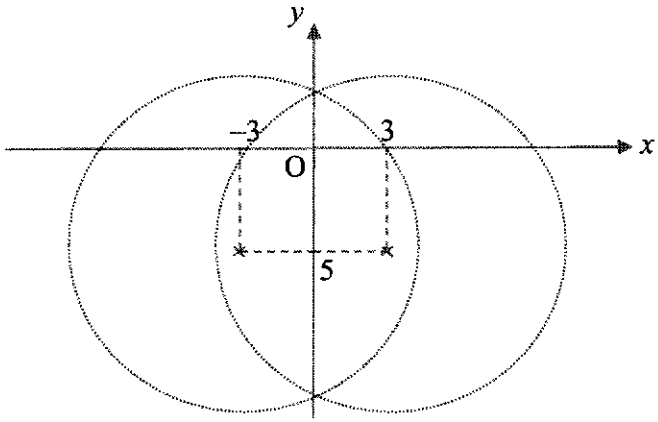
Since distance between $(2,-4)$ and centre is smaller than the radius, i.e. $\sqrt{2} < 10$, therefore the point $(2,-4)$ lies inside the circle.

A second circle, C_2, is the reflection of C_1 on the y-axis.

(iii) Find the equation of C_2. [2]

New centre of circle $= (-3,-5)$

Equation of new circle: $(x+3)^2+(y+5)^2 = 100$



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7 The point $(a, 2)$ is a stationary point on the graph $y = x^3 - x^2 - x + b$, where a and b are positive constants.

(i) Show that $a = 1$. [2]

$y = x^3 - x^2 - x + b$

$\frac{dy}{dx} = 3x^2 - 2x - 1$

when $x = a$, $\frac{dy}{dx} = 0$

$3a^2 - 2a - 1 = 0$

$(3a + 1)(a - 1) = 0$

$3a + 1 = 0$ or $a - 1 = 0$

$a = -\frac{1}{3}$ (rejected $a > 0$) or $a = 1$ (shown)

(ii) Find the value of b. [1]

$y = x^3 - x^2 - x + b$

when $x = 1$, $y = 2$

$2 = (1)^3 - (1)^2 - (1) + b$

$b = 3$

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(iii) Find the coordinates of the other stationary point. [1]

when $x = -\frac{1}{3}$, $y = \left(-\frac{1}{3}\right)^3 - \left(-\frac{1}{3}\right)^2 - \left(-\frac{1}{3}\right) + 3$

$$y = 3\frac{5}{27}$$

Coordinates of the other stationary point = $\left(-\frac{1}{3}, 3\frac{5}{27}\right)$

(iv) Determine the nature of each of the stationary points. [3]

$$\frac{dy}{dx} = 3x^2 - 2x - 1$$

$$\frac{d^2y}{dx^2} = 6x - 2$$

when $x = 1$, $\frac{d^2y}{dx^2} = 6(1) - 2$

$$\frac{d^2y}{dx^2} = 4 > 0$$

So $(1, 2)$ is a minimum point.

when $x = -\frac{1}{3}$, $\frac{d^2y}{dx^2} = 6\left(-\frac{1}{3}\right) - 2$

$$\frac{d^2y}{dx^2} = -4 < 0$$

So $\left(-\frac{1}{3}, 3\frac{5}{27}\right)$ is a maximum point.

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8 (a) The average person's blood pressure is modelled by $y = k\sin(5\pi t) + 100$, where y millimetres of mercury (mmHg) represents the blood pressure at time t, measured in minutes and k is a constant.

The maximum blood pressure of an average person is 120 mmHg.

(i) Find the value of k. Hence, state the minimum blood pressure of an average person. [2]

$\max.\ \text{of}\ \sin(5\pi t) = 1$

$120 = k(1) + 100$

$k = 120 - 100$

$k = 20$

Since $\min.\ \text{of}\ \sin(5\pi t) = -1$,

Min. Blood Pressure $= 20(-1) + 100$

$= 80$ mmHg

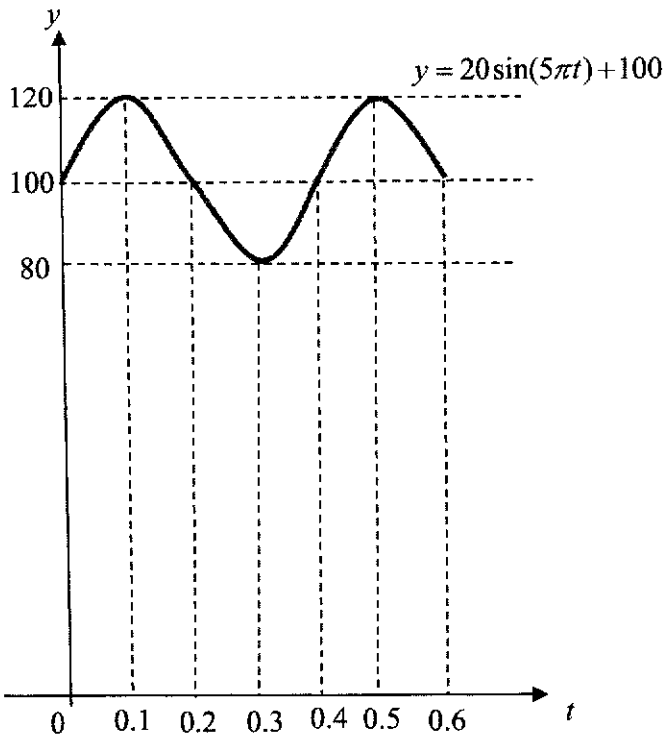
(ii) State the period of y and the number of cycles for $0 \le t \le 0.6$. [2]

$$\text{Period} = \frac{2\pi}{5\pi}$$

$= 0.4$ minutes

$$\text{Number of cycles for } 0 \le t \le 0.6 = \frac{0.6}{0.4} = 1.5$$

(iii) Sketch the graph of $y = k\sin(5\pi t) + 100$ for $0 \le t \le 0.6$. [2]



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(b) Solve the equation $2\cos 2x - 3\sec x + 2 = 0$ for $-\pi \le x \le \pi$. [5]

$$2\cos 2x - 3\sec x + 2 = 0$$

$$2\left(2\cos^2 x - 1\right) - \frac{3}{\cos x} + 2 = 0$$

$$4\cos^2 x - 2 - \frac{3}{\cos x} + 2 = 0$$

$$4\cos^2 x = \frac{3}{\cos x}$$

$$\cos^3 x = \frac{3}{4}$$

$$\cos x = \sqrt[3]{\frac{3}{4}}$$

x is in 1st or 4th quad

$$\text{basic } \angle = \cos^{-1}\left(\sqrt[3]{\frac{3}{4}}\right)$$

$$\text{basic } \angle = 0.43097...$$

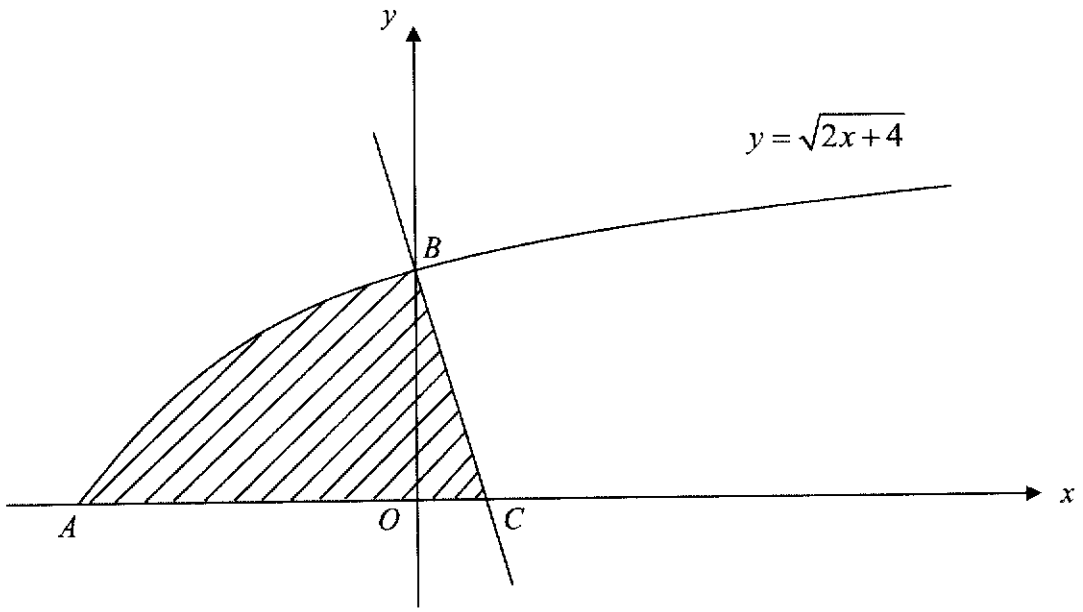
$$x = 0.43097... \quad or \quad -0.43097...$$

$$x = 0.431 \quad or \quad -0.431 \text{ (3sf)}$$

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9



The diagram shows part of the curve $y=\sqrt{2x+4}$, meeting the x-axis at the point A and the y-axis at the point B. The normal to the curve at B meets the x-axis at the point C.

(i) Write down the coordinates of B. [1]

When $x=0$, $y=\sqrt{0+4}$

$=2$

$\therefore B(0,2)$

(ii) Find the coordinates of C. [4]

$\frac{dy}{dx}=\frac{1}{2}\left[(2x+4)^{-\frac{1}{2}}\right]\times 2$

$=(2x+4)^{-\frac{1}{2}}$

At B, $\frac{dy}{dx}=(0+4)^{-\frac{1}{2}}$

$=\frac{1}{2}$

Gradient of normal to the curve at $B=-2$

Equation of normal to the curve (ie. line BC) at B: $y=-2x+2$

When $y=0$,

$0=-2x+2$

$x=1$

$\therefore C(1,0)$

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(iii) Find the area of the shaded region. [4]

When $y = 0$,

$0 = \sqrt{2x+4}$

$2x = -4$

$\therefore A(-2, 0)$

Area of the shaded region $= \int_{-2}^{0} \sqrt{2x+4}\, dx + \frac{1}{2}(1)(2)$

$$= \left[\frac{(2x+4)^{\frac{3}{2}}}{\frac{3}{2}(2)} \right]_{-2}^{0} + 1$$

$$= \left[\frac{(0+4)^{\frac{3}{2}}}{3} - \frac{(-4+4)^{\frac{3}{2}}}{3} \right] + 1$$

$= 3\frac{2}{3}$ or 3.67(3s.f.) units^2

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10 A particle travels in a straight line so that, t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = t^2 - 2kt - 6k$, where k is a constant.
The minimum velocity of the particle occurs when $t = 2$.

(i) Show that $k = 2$. [2]

$v = t^2 - 2kt - 6k$

$\frac{dv}{dt} = 2t - 2k$

At minimum velocity, $t = 2$, $\frac{dv}{dt} = 0$

$2(2) - 2k = 0$

$2k = 4$

$k = \frac{4}{2}$

$k = 2$ (shown)

(ii) Find an expression, in terms of t, for the displacement of the particle. Hence, show that the particle will return to O only once during its journey. [5]

$v = t^2 - 2kt - 6k$

$= t^2 - 4t - 12$

$s = \int t^2 - 4t - 12 \, dt$

$= \frac{1}{3}t^3 - 2t^2 - 12t + C$, where C is a constant

When $t = 0, s = 0$

$\frac{1}{3}(0)^3 - 2(0)^2 - 12(0) + C = 0$

$\therefore C = 0$

So $s = \frac{1}{3}t^3 - 2t^2 - 12t$

When $s = 0$,

$\frac{1}{3}t^3 - 2t^2 - 12t = 0$

$\frac{1}{3}t\left(t^2 - 6t - 36\right) = 0$

$\frac{1}{3}t = 0$ or $t^2 - 6t - 36 = 0$

$t = 0$ or $t = \frac{-(-6) \pm \sqrt{(6)^2 - 4(1)(-36)}}{2(1)}$

$t = 0$ or $t = \frac{6 \pm \sqrt{180}}{2}$

$t = 0$ (NA), $t = 9.71$ or $t = -3.71$ (rejected $t > 0$)

Therefore, the particle will return to O only once at $t = 9.71$ s during its journey.

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(iii) Find the total distance travelled by the particle for the first 7 seconds. [4]

$$v = t^2 - 4t - 12$$

For turning point, $v = 0$

$$t^2 - 4t - 12 = 0$$
$$(t-6)(t+2) = 0$$
$$t = 6 \text{ or } t = -2$$

Distance travelled from 0s to 6s $= \left|\frac{1}{3}(6)^3 - 2(6)^2 - 12(6)\right|$
$= 72$ m

Distance travelled from 6s to 7s $= \left|\frac{1}{3}(6)^3 - 2(6)^2 - 12(6)\right| - \left|\frac{1}{3}(7)^3 - 2(7)^2 - 12(7)\right|$
$= 72 - 67\frac{2}{3}$
$= 4\frac{1}{3}$ m

Total distance travelled by the particle for the first 7 seconds $= 72 + 4\frac{1}{3}$
$= \frac{229}{3}$
$= 76\frac{1}{3}$ or 76.3 (3sf) m

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11 **(a)** A, B and C are the angles of a triangle. [2]
Show that $\sin C = \sin A\cos B + \sin B\cos A$.

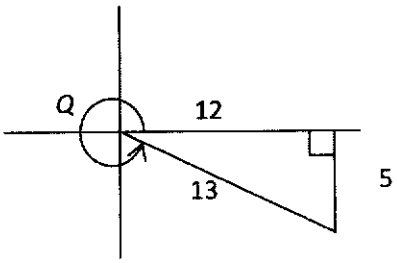
$$A + B + C = 180°$$
$$A + B = 180° - C$$
$$\sin(A+B) = \sin(180° - C)$$
$$\sin A\cos B + \sin B\cos A = \sin C \text{ (shown)}$$

Alternative Soln

$$\sin C = \sin\left(180^0 - C\right)$$
$$= \sin(A+B)$$
$$= \sin A\cos B + \sin B\cos A\,(Shown)$$

(b) Given that P and Q are angles such that $\sin P = -\frac{4}{5}$ and $\cos Q = \frac{12}{13}$, where both P and Q are in the same quadrant, find the exact value of

(i) $\tan 2Q$, [3]

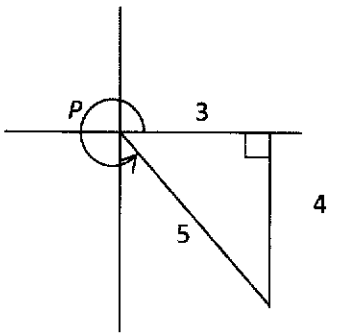


$$\tan 2Q = \frac{2\tan Q}{1-\tan^2 Q}$$
$$= \frac{2\left(-\frac{5}{12}\right)}{1-\left(-\frac{5}{12}\right)^2}$$
$$= -\frac{720}{714}$$
$$-\frac{120}{119}$$

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(ii) $\operatorname{cosec}(P-Q)$, [3]



$$\operatorname{cosec}(P-Q) = \frac{1}{\sin(P-Q)}$$

$$= \frac{1}{\sin P\cos Q - \sin Q\cos P}$$

$$= \frac{1}{\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right)}$$

$$= -\frac{65}{33}$$

(iii) $\sin\frac{P}{2}$. [2]

$$\cos 2\left(\frac{P}{2}\right) = 1 - 2\sin^2\left(\frac{P}{2}\right)$$

$$2\sin^2\frac{P}{2} = 1 - \cos P$$

$$\sin\frac{P}{2} = \pm\sqrt{\frac{1}{2}(1-\cos P)}$$

$$\sin\frac{P}{2} = \pm\sqrt{\frac{1}{2}\left(1-\frac{3}{5}\right)}$$

$$\sin\frac{P}{2} = \pm\sqrt{\frac{1}{5}}$$

$$\sin\frac{P}{2} = \pm\frac{\sqrt{5}}{5}$$

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12 The cost per submarine, \$$y$ in millions, of assembling x submarines can be modelled by $y = \frac{5}{2}x^2 - 20x + 100$, where $x \leq 8$.

(i) Explain the meaning of the constant term 100 in this model. [1]

There is a fixed price of \$100 **million** incurred, even when **no** submarines were assembled.

(ii) Express $y = \frac{5}{2}x^2 - 20x + 100$ in the form of $y = a(x-h)^2 + k$. [2]

$$y = \frac{5}{2}x^2 - 20x + 100$$
$$y = \frac{5}{2}\left(x^2 - 8x + 40\right)$$
$$y = \frac{5}{2}\left(x^2 - 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 40\right)$$
$$y = \frac{5}{2}\left[(x-4)^2 + 24\right]$$
$$y = \frac{5}{2}(x-4)^2 + 60$$

(iii) Explain the significance of $x = h$ in **(ii)**. [1]

$x = 4$ occurs at the minimum point of the curve. Hence, the cost will be lowest when we assemble 4 submarines.

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PRELIMINARY EXAMINATION 2021

SECONDARY 4

ADDITIONAL MATHEMATICS 4049/02

Paper 2 **Friday 17 September 2021**

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	

This document consists of **21** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \quad \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \quad \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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1 (a) Solve the simultaneous equations

$$y = x^2 - 6,$$
$$y = 2x - 3.$$ [3]

$y = x^2 - 6 \quad ---(1)$

$y = 2x - 3 \quad ---(2)$

Subst. eqn (2) into (1),

$x^2 - 6 = 2x - 3$

$x^2 - 2x - 6 + 3 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$

Subst. into eqn (2),

$y = 2(-1) - 3$ or $y = 2(3) - 3$

$y = -5$ $\quad$ $y = 3$

$x = -1,$ $\quad$ or $x = 3,$

$y = -5$ $\quad$ $y = 3$

(b) Prove that $\dfrac{3\cos\theta + \cos 2\theta - 1}{\cos^2\theta + 2\cos\theta} = 2 - \sec\theta.$ [4]

LHS

$$= \frac{3\cos\theta + \cos 2\theta - 1}{\cos^2\theta + 2\cos\theta}$$

$$= \frac{3\cos\theta + (2\cos^2\theta - 1) - 1}{\cos\theta(\cos\theta + 2)}$$

$$= \frac{2\cos^2\theta + 3\cos\theta - 2}{\cos\theta(\cos\theta + 2)}$$

$$= \frac{(2\cos\theta - 1)(\cos\theta + 2)}{\cos\theta(\cos\theta + 2)}$$

$$= \frac{(2\cos\theta - 1)}{\cos\theta}$$

$$= \frac{2\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}$$

$$= 2 - \sec\theta$$

= RHS [Proven]

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2 The expression $8\sin\theta - 3\cos\theta$ is defined for $0^o \le \theta \le 180^o$.

Using $R\sin(\theta - \alpha)$, where $R > 0$ and $0^o < \alpha < 90^o$,

(i) solve the equation $8\sin\theta - 3\cos\theta = 5$. [5]

Let $8\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$

$= R\sin\theta\cos\alpha - R\sin\theta\cos\alpha$

Comparing coefficients,

$R\cos\alpha = 8 ---(1)$

$R\sin\alpha = 3 ---(2)$

$(2) \div (1)$, $\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{3}{8}$

$\tan\alpha = \dfrac{3}{8}$

$\alpha = \tan^{-1}\left(\dfrac{3}{8}\right)$

$\alpha = 20.556...^o$

$= 20.6^o\ (1d.p.)$

and

$(2)^2 + (1)^2$, $R^2 = 8^2 + 3^2$

$R = \sqrt{8^2 + 3^2}$

$= \sqrt{73}$ *or* $= 8.54(3.s.f)$

$\therefore\ 8\sin\theta - 3\cos\theta = \sqrt{73}\sin(\theta - 20.556...^o)$

Since $8\sin\theta - 3\cos\theta = 5$

$\sqrt{73}\sin(\theta - 20.556...^o) = 5$

$\sin(\theta - 20.556...^o) = \dfrac{5}{\sqrt{73}}$

$(\theta - 20.556...^o) = \sin^{-1}\left(\dfrac{5}{\sqrt{73}}\right)$

θ lies in 1st or 2nd quad since $0^o < \theta < 180^o$

Basic angle $= 35.8175...^o$

$(\theta - 20.556...^o) = 35.8175...^o$ or $180^o - 35.8175...^o$

$\theta = 35.8175...^o + 20.556...^o$ or $180^o - 35.8175...^o + 20.556...^o$

$= 56.3735...^o$ or $164.7385...^o$

$= 56.4^o\ (1dp)$ or $164.7^o\ (1dp)$

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(ii) state the smallest value of $\dfrac{1}{(8\sin\theta - 3\cos\theta)^2}$ and find the corresponding value of θ, for $0^O \le \theta \le 180^O$. [2]

Smallest value of $\dfrac{1}{(8\sin\theta - 3\cos\theta)^2}$

$= \dfrac{1}{\text{Largest value of } (8\sin\theta - 3\cos\theta)^2}$

$= \dfrac{1}{\left(\sqrt{73}\right)^2}$

$= \dfrac{1}{73}$ --- **[B1]**

occurs when $\sin\left(\theta - 20.556...^O\right) = 1$

$\left(\theta - 20.556...^O\right) = \sin^{-1}(1)$

$\left(\theta - 20.556...^O\right) = 90^O$

$\theta = 90^O + 20.556...^O$

$\theta = 110.556...^O$

$= 110.6^O \text{ (1dp)}$

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3 **(a)** Given that $4x^2 - \frac{x}{h} - 3 = 0$, where $h \neq 0$, show that there are no real values of h for which the equation has equal real roots. [3]

$$4x^2 - \frac{x}{h} - 3 = 0$$
$$4hx^2 - x - 3h = 0$$

For the equation to have equal real roots,

Discriminant = 0

$$(-1)^2 - 4(4h)(-3h) = 0$$
$$1 + 48h^2 = 0$$
$$48h^2 = -1$$
$$h^2 = -\frac{1}{48} \text{ (No real solutions)}$$

OR

$$\left(-\frac{1}{h}\right)^2 - 4(4)(-3) = 0$$
$$\frac{1}{h^2} + 48 = 0$$
$$\frac{1}{h^2} = -48$$
$$h^2 = -\frac{1}{48}$$

$\therefore$ Since $h^2 \geq 0$, there are no real values of h for which $h^2 = -\frac{1}{48}$. [Shown]

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(b) Find the range of values of the positive constant k for which the line $y = kx$ lies completely above the curve $y = -16x^2 + (2k-1)x - 1$. [4]

$y = -16x^2 + (2k-1)x - 1 \quad ---(1)$

$y = kx \quad ---(2)$

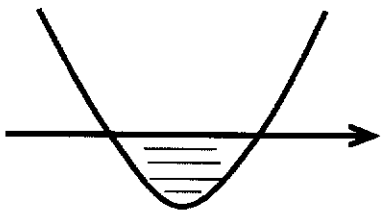
Substitute (2) into (1),

$$-16x^2 + (2k-1)x - 1 = kx$$
$$-16x^2 + 2kx - x - kx - 1 = 0$$
$$-16x^2 + kx - x - 1 = 0$$
$$16x^2 - kx + x + 1 = 0$$
$$16x^2 + (1-k)x + 1 = 0$$

Since the line lies completely above the curve,

Discriminant < 0

$$(1-k)^2 - 4(16)(1) < 0$$
$$1 - 2k + k^2 - 64 < 0$$
$$k^2 - 2k - 63 < 0$$
$$(k-9)(k+7) < 0$$



$$-7 < k < 9$$

Since $k > 0$,

$\therefore 0 < k < 9$

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4 (i) By considering the general term in the binomial expansion of $\left(\frac{2}{x^2}+x^3\right)^{16}$, explain why every term is dependent on x. [3]

General term, T_{r+1}

$$=\binom{16}{r}\left(\frac{2}{x^2}\right)^{(16-r)}\left(x^3\right)^r$$

$$=\binom{16}{r}(2)^{16-r}\left(x^{-2}\right)^{16-r}x^{3r}$$

$$=\binom{16}{r}(2)^{16-r}x^{-32+2r}x^{3r}$$

$$=\binom{16}{r}(2)^{16-r}x^{5r-32}$$

If there is a **term independent** of x in this expansion, it will be a **constant**.

Consider $x^{5r-32}=x^0$

$$5r-32=0$$

$$r=\frac{32}{5}$$

Since r is **not** a whole number or a non-negative integer, there does not exist a term independent of x (ie a constant term), hence every term is dependent of x in this binomial expansion.

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(ii) Find the term independent of x in the expansion of $\left(\frac{2}{x^2}+x^3\right)^{16}\left(4-3x^2\right)$. [3]

Since from part (i), we have shown that the **first expansion does not have any constant term**, by selective distribution, observe that the only way to obtain a **constant term** from the combined expansion is to get an $\frac{a}{x^2}$ term in the first expansion multiplied with the $\left(-3x^2\right)$ term in the second factor.

Consider $x^{5r-32}=x^{-2}$

$$5r-32=-2$$
$$5r=30$$
$$r=6$$

$$T_{6+1}=\binom{16}{6}(2)^{16-6}x^{5(6)-32}$$

$$=8008(1024)(x)^{-2}$$

$$=\frac{8\,200\,192}{x^2}$$

Hence by selective expansion,

term independent of x in $\left(\frac{2}{x^2}+x^3\right)^{16}\left(4-3x^2\right)$

$$=\left(\frac{8\,200\,192}{x^2}\right)\times\left(-3x^2\right)$$

$$=-24\,600\,576$$

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5 An object moving in a certain medium, with speed v m/s, experiences a resistance to motion of R newtons. R and v are related by the equation $R = av^2 + bv$, where a and b are constants.

The table shows the experimental values of the variables v and R.

v	5	10	15	20	25
R	17	44	81	128	185

(i) On the grid on page 11, plot $\frac{R}{v}$ against v and draw a straight line graph. [3]

Refer to grid page.

v	5	10	15	20	25
$\frac{R}{v}$	3.4	4.4	5.4	6.4	7.4

(ii) Use your graph to estimate the value of a and of b. [3]

$R = av^2 + bv$

$\frac{R}{v} = av + b$

From the graph,

a = gradient of graph

$= \frac{7-3}{23-3}$

$= 0.2$

b = vertical intercept

$= 2.4$

(iii) In a different medium, R is directly proportional to v and $R = 30$ when $v = 5$.
Draw a suitable line on your graph to illustrate the second situation and use it to determine the value of v for which the resistance is the same in both mediums. [3]

$R = kv$, where k is a constant.

Since $v = 5$ and $R = 30$,

$30 = k(5)$

$k = 6$

$\therefore R = 6v$

Draw line $\frac{R}{v} = 6$.

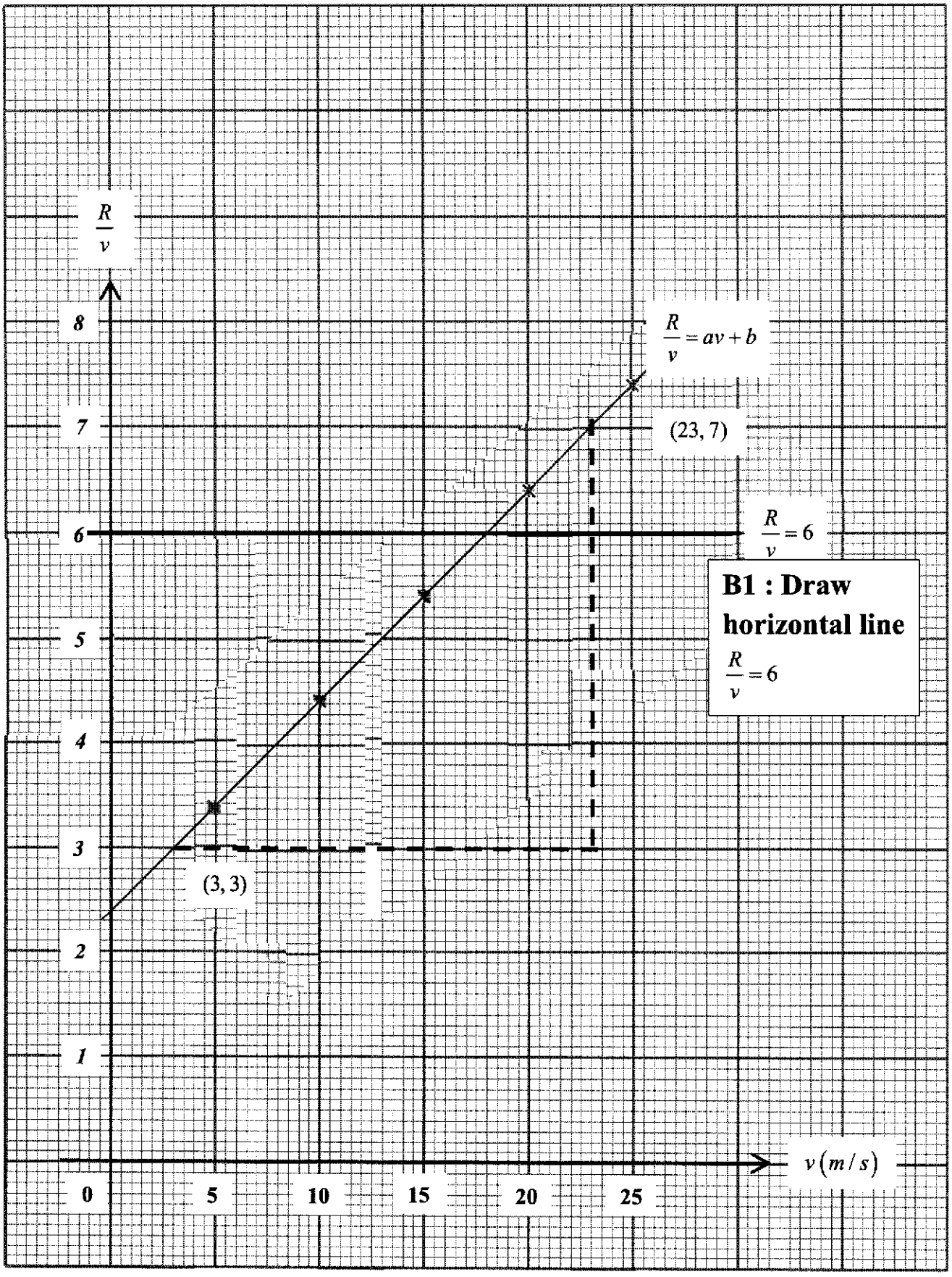
From the point of intersection,

$v = 18\ m/s$

Accept 17.5 to 18.5

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6 (a) A cone has a volume of $\left(24+5\sqrt{3}\right)\pi$ cm^3 and a base radius of $(2+\sqrt{3})$ cm. Find its height in the form of $\left(p+q\sqrt{3}\right)$, where p and q are integers. [4]

$$\text{Volume of cone} = \frac{1}{3}\times(\text{base area})\times\text{height}$$

$$\left(24+5\sqrt{3}\right)\pi = \frac{1}{3}\times\left[\pi\left(2+\sqrt{3}\right)^2\right]\times\text{height}$$

$$\text{height} = \frac{3\left(24+5\sqrt{3}\right)\pi}{\pi\left(2+\sqrt{3}\right)^2}$$

$$= \frac{3\left(24+5\sqrt{3}\right)}{7+4\sqrt{3}}$$

$$= \frac{3\left(24+5\sqrt{3}\right)}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{3\left(168-96\sqrt{3}+35\sqrt{3}-60\right)}{49-16(3)}$$

$$= \frac{3\left(108-61\sqrt{3}\right)}{1}$$

$$= \left(324-183\sqrt{3}\right)\text{ cm}$$

(b) Solve the equation $e^x\left(3e^x-2\right)=5$. [3]

$$e^x\left(3e^x-2\right)=5$$

Let $u = e^x$.

$$u(3u-2)=5$$
$$3u^2-2u-5=0$$
$$(3u-5)(u+1)=0$$

$$u=\frac{5}{3} \quad \text{or} \quad u=-1$$
$$e^x=\frac{5}{3} \quad \text{or} \quad e^x=-1 \quad (\text{no solution, } e^x>0)$$

$$x=\ln\left(\frac{5}{3}\right)$$
$$x=0.511\,(3\text{ s.f})$$

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(c) Sam just bought a brand new car. The value, V dollars, of the car depreciates over time.

It is given that $V = 179200e^{kt} + 8800$, where k is a constant and t is the time in years since it was bought.

(i) Find the initial value of the car. [1]

$V = 179200e^{kt} + 8800$

When $t = 0$,

$$V_o = 179\,200\left(e^{o}\right) + 8800$$
$$= 188\,000$$

$\therefore$ Initial value of the car = \$188 000.

(ii) Calculate the value of k if, after 5 years, the value of the car is halved. [3]

When $t = 5$, $V = \dfrac{188\,000}{2}$

$$179\,200e^{5k} + 8\,800 = 94\,000$$
$$179\,200e^{5k} = 85\,200$$
$$e^{5k} = \frac{213}{448}$$
$$5k = \ln\left(\frac{213}{448}\right)$$
$$k = -0.1487...$$
$$k = -0.149\,(3\text{ s.f.})$$

(iii) After having driven the car for a long time, Sam decided to sell his car.
A second-hand car dealer offers to buy the car from him for \$8000.
Explain, with clear mathematical reasoning, why Sam should not accept the offer.

[2]

$V = 179\,200e^{-0.1487t} + 8800$

Since $179\,200e^{-0.1487t} > 0$ **or** As t become a very large number, $e^{-0.1487t}$ approaches 0

$\therefore V > 8800$ $V \to 8800$

Since the value of the car is always more than \$ 8800, Sam **should not accept** the offer as he will have incurred a loss.

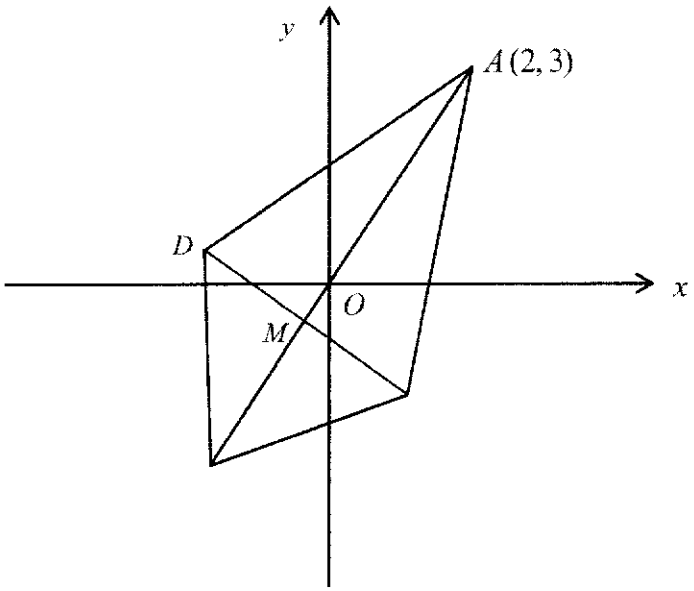
Or

The offer of \$8000 is less than the value of the car which is \$ 8800 so he will have suffered a loss if he accepts the offer.

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7 The diagram shows a kite $ABCD$ in which the line AC passes through the origin O and has a gradient of 1.5.



(i) Find the value of k. [2]

$$\frac{3-k}{2-(-2)} = 1.5$$
$$3-k = 6$$
$$k = -3$$

(ii) Find the equation of the line BD. [3]

Gradient of $BD = \dfrac{-1}{1.5}$

$$= -\frac{2}{3}$$

Using $B(1, -2)$,

Equation of line BD is $y-(-2) = -\dfrac{2}{3}(x-1)$

$$y = -\frac{2}{3}x + \frac{2}{3} - 2$$
$$y = -\frac{2}{3}x - \frac{4}{3}$$

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(iii) The line AC and line BD meets at point M.

Show that the coordinates of M is $\left(-\frac{8}{13}, -\frac{12}{13}\right)$. [3]

$y = \frac{3}{2}x \quad ---(1)$

$y = -\frac{2}{3}x - \frac{4}{3} \quad ---(2)$

(1) = (2) :

$\frac{3}{2}x = -\frac{2}{3}x - \frac{4}{3}$

$\frac{3}{2}x + \frac{2}{3}x = -\frac{4}{3}$

$\frac{13}{6}x = -\frac{4}{3}$

$39x = -24$

$x = -\frac{8}{13}$

Subst. into (1),

$y = 1.5\left(-\frac{8}{13}\right)$

$y = -\frac{12}{13}$

$\therefore M = \left(-\frac{8}{13}, -\frac{12}{13}\right)$ [Shown]

(iv) The coordinates of D are (a,b). Prove that $13\left(a+\frac{8}{13}\right)^2 + 13\left(b+\frac{12}{13}\right)^2 = 49$. [2]

Since $ABCD$ is a kite, M =midpoint of BD

$DM = BM$

$\sqrt{\left(a+\frac{8}{13}\right)^2 + \left(b+\frac{12}{13}\right)^2} = \sqrt{\left(-\frac{8}{13}-1\right)^2 + \left(-\frac{12}{13}+2\right)^2}$

$\sqrt{\left(a+\frac{8}{13}\right)^2 + \left(b+\frac{12}{13}\right)^2} = \sqrt{\frac{49}{13}}$

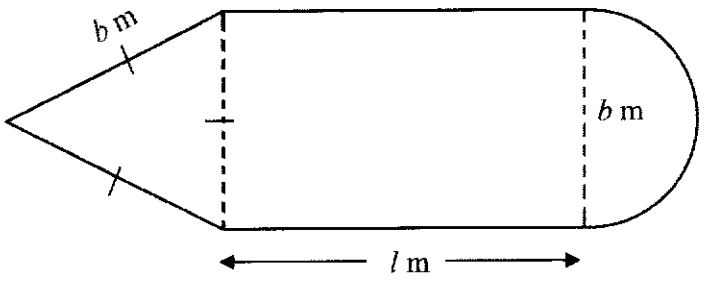
$\left(a+\frac{8}{13}\right)^2 + \left(b+\frac{12}{13}\right)^2 = \frac{49}{13}$

$13\left(a+\frac{8}{13}\right)^2 + 13\left(b+\frac{12}{13}\right)^2 = 49$ *[Shown]*

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8 A farmer just bought a piece of land to grow his crops. This piece of land consists of an equilateral triangle of sides b m, a rectangle with sides b m and l m as well as a semi-circle of diameter b m as shown in the diagram below.



(i) Given that the perimeter of the land is 600 m, express l in terms of b. [2]

(i) Perimeter of land = 600 m

$$2b + 2l + \frac{1}{2}\pi b = 600$$

$$4b + 4l + \pi b = 1200$$

$$4l = 1200 - 4b - \pi b$$

$$l = 300 - b - \frac{\pi}{4}b$$

(ii) Given that the area of the land is A m^2, show that $A = 300b + \left(\frac{2\sqrt{3} - 8 - \pi}{8}\right)b^2$. [3]

$$A = \frac{1}{2}\left(b^2\right)\sin 60^o + bl + \frac{1}{2}\pi\left(\frac{b}{2}\right)^2$$

$$= \frac{b^2}{2}\left(\frac{\sqrt{3}}{2}\right) + b\left(300 - b - \frac{\pi}{4}b\right) + \frac{\pi}{8}b^2$$

$$= \frac{\sqrt{3}}{4}b^2 + 300b - b^2 - \frac{\pi}{4}b^2 + \frac{\pi}{8}b^2$$

$$= 300b + \left(\frac{\sqrt{3}}{4} - 1 - \frac{\pi}{4} + \frac{\pi}{8}\right)b^2$$

$$= 300b + \left(\frac{2\sqrt{3} - 8 - 2\pi + \pi}{8}\right)b^2$$

$$= 300b + \left(\frac{2\sqrt{3} - 8 - \pi}{8}\right)b^2 \quad \text{[Shown]}$$

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(iii) Given that b can vary, find the value of b which gives a stationary value of A. [3]

(iii) $A = 300b + \left(\frac{2\sqrt{3} - 8 - \pi}{8}\right)b^2$

$$\frac{dA}{db} = 300 + \left(\frac{2\sqrt{3} - 8 - \pi}{8}\right).2b$$

$$= 300 + \left(\frac{2\sqrt{3} - 8 - \pi}{4}\right)b$$

When A is stationary, $\frac{dA}{db} = 0$.

$$300 + \left(\frac{2\sqrt{3} - 8 - \pi}{4}\right)b = 0$$

$$\left(\frac{2\sqrt{3} - 8 - \pi}{4}\right)b = -300$$

$$\left(2\sqrt{3} - 8 - \pi\right)b = -1200$$

$$b = \frac{-1200}{\left(2\sqrt{3} - 8 - \pi\right)}$$

$$b = 156.301...$$

$$b = 156\,m\,(3\,\text{s.f.})$$

(iv) Find the nature of this stationary value and explain why the farmer will be happy with this information. [3]

(iv) $\frac{dA}{db} = 300 + \left(\frac{2\sqrt{3} - 8 - \pi}{4}\right)b$

$$\therefore \frac{d^2A}{db^2} = \left(\frac{2\sqrt{3} - 8 - \pi}{4}\right)$$

$$= -7.677... < 0$$

$\therefore A$ has a **maximum** value at $b = 156$ m.

The farmer will be happy as he will be able to **maximize his area** of land to grow his crops for **possibly maximum profits**.

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9 **(i)** Find $\frac{d}{dx}(x\cos 2x)$. [2]

$$\frac{d}{dx}(x\cos 2x) = \cos 2x - 2x\sin 2x$$

(ii) Find $\frac{d}{dx}\left(x^2 \sin 2x\right)$. [2]

$$\frac{d}{dx}\left(x^2 \sin 2x\right) = 2x\sin 2x + 2x^2\cos 2x$$

(iii) Using the results to parts **(i)** and **(ii)**, find $\int x^2 \cos 2x \, dx$. [4]

$$\int 2x\sin 2x + 2x^2\cos 2x \, dx = x^2 \sin 2x + c_1$$

$$\int 2x\sin 2x \, dx + \int 2x^2\cos 2x \, dx = x^2 \sin 2x + c_1$$

$$\int 2x^2\cos 2x \, dx = x^2 \sin 2x - \int 2x\sin 2x \, dx + c_1$$

$$\int 2x^2\cos 2x \, dx = x^2 \sin 2x + \left(x\cos 2x - \int \cos 2x \, dx\right) + c_2$$

$$\int 2x^2\cos 2x \, dx = x^2 \sin 2x + x\cos 2x - \frac{1}{2}\sin 2x + c_3$$

$$\int x^2\cos 2x \, dx = \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + c,$$

where c_1, c_2, c_3 and c are constants

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10 A curve $y = f(x)$ is such that $f''(x) = 12e^{3x} + 7$ and the point $A\left(0, -\frac{2}{3}\right)$ lies on the curve. The gradient of the curve at A is 9. Find the equation of the curve. [5]

$f'(x) = \int (12e^{3x} + 7)\, dx$

$f'(x) = \frac{12e^{3x}}{3} + 7x + C_1$

Given $f'(x) = 9$ at $A\left(0, -\frac{2}{3}\right)$,

$\frac{12e^{3(0)}}{3} + 7(0) + C_1 = 9$

$C_1 = 5$

$f'(x) = 4e^{3x} + 7x + 5$

$\therefore f(x) = \int (4e^{3x} + 7x + 5)\, dx$

$f(x) = \frac{4}{3}e^{3x} + \frac{7}{2}x^2 + 5x + C$

At $A\left(0, -\frac{2}{3}\right)$,

$-\frac{2}{3} = \frac{4}{3}e^{3(0)} + \frac{7}{2}(0)^2 + 5(0) + C$

$C = -2$

$\therefore$ Equation of the curve is : $y = \frac{4}{3}e^{3x} + \frac{7}{2}x^2 + 5x - 2$

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11 When the height of liquid in a container is h m, the volume of liquid is V m^3, where

$$V = 0.06\left[(4h+1)^3 - 7\right] \text{ and } h \geq 0.25.$$

(i) Find an expression for $\frac{dV}{dh}$. [2]

$$V = 0.06\left[(4h+1)^3 - 7\right]$$
$$\frac{dV}{dh} = 0.06(3)(4h+1)^2(4)$$
$$= 0.72(4h+1)^2 \text{ or } \frac{18}{25}(4h+1)^2$$

The liquid leaks out of the container at a constant rate of 0.98 m^3/s.

(ii) Find the value of $\frac{dh}{dt}$ when $V = 1.2$. [4]

When $V = 1.2$,

$$1.2 = 0.06\left[(4h+1)^3 - 7\right]$$
$$1.2 = 0.06\left[(4h+1)^3 - 7\right]$$
$$20 = (4h+1)^3 - 7$$
$$(4h+1)^3 = 27$$
$$4h+1 = 3$$
$$h = \frac{1}{2}$$

When $h = \frac{1}{2}$,

$$\frac{dV}{dh} = 0.72\left[4\left(\frac{1}{2}\right)+1\right]^2$$
$$= 6.48 \text{ m}^3/\text{m}$$

Given $\frac{dV}{dt} = -0.98$ m^3/s,

$$\frac{dV}{dt} = \frac{dh}{dt} \times \frac{dV}{dh}$$
$$-0.98 = \frac{dh}{dt} \times 6.48$$
$$\frac{dh}{dt} = -\frac{0.98}{6.48}$$
$$= -0.151 \text{ m/s (3 s.f.)} \text{ or } -\frac{49}{324} m/s$$

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(iii) Briefly explain the significance of your answer for **(ii)**. [1]

When the volume of the liquid in the container = 1.2 m^3, the **height** of the liquid in the container is **decreasing** at a rate of 0.151 m/s.

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