Name:	Class	Index Number

Jurong West Secondary School

Preliminary Examinations 2020

80

ADDITIONAL MATHEMATICS 4047/01

Secondary Four Express/ Five Normal Academic **27 August 2020**

Paper 1 **1000 - 1200**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of **17** printed pages.

Setter: Ms Sharron Chiam

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Downloaded from SingaKids.com

1 A cone with base radius $\left(5+2\sqrt{3}\right)$ cm and a slant height l cm has a curved surface area of $\left(51-3\sqrt{3}\right)\pi$ cm^2. Without using a calculator, obtain an expression for l in the form of $\left(a+b\sqrt{3}\right)$, where a and b are integers. [4]

2 The acute angles A and B are such that $\tan\left(A+B\right)=8$ and $\tan A=\frac{1}{5}$. Without using a calculator, find the exact value of $\cos B$. [5]

Downloaded from SingaKids.com

3 **(i)** Find the range of values of m for which the curve $y=(4+m)x^2-4x+m+1$ has a maximum point. [1]

(ii) Find the range of values of m for which the curve $y=(4+m)x^2-4x+m+1$ is always negative for all real values of x.

[3]

(iii) Hence state the range of values of m for which the curve has a maximum point and is always negative for all values of x. [1]

Downloaded from SingaKids.com

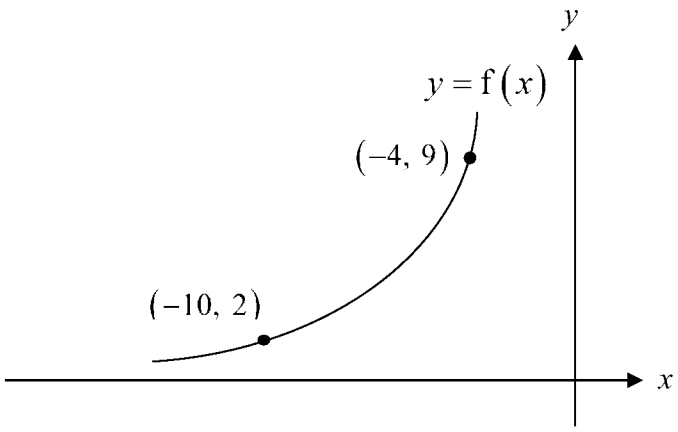
4 **(a)** It is given that $\int \mathrm{f}(x)\,\mathrm{d}x = 3x^2 \ln x + kx^2 + c$ where k and c are constants.

(i) If $\int_1^e \mathrm{f}(x)\,\mathrm{d}x = e^2 + 2$, show that $k = -2$. [3]

(ii) Find $\mathrm{f}(x)$. [2]

Downloaded from SingaKids.com

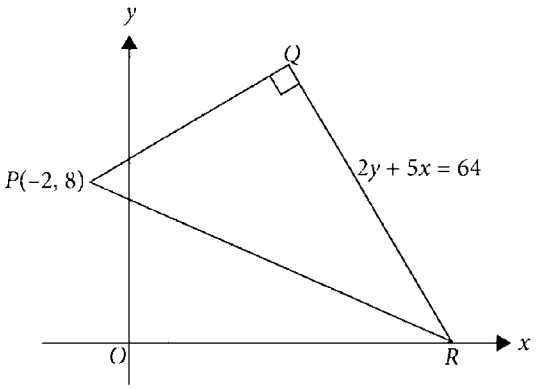
(b) The figure shows part of the curve $y = \mathrm{f}(x)$. $(-10, 2)$ and $(-4, 9)$ are two points on the curve.



Given that $\int_{-10}^{-4} y \, \mathrm{d}x = 30$, find $\int_{2}^{9} x \, \mathrm{d}y$. [2]

Downloaded from SingaKids.com

5



The diagram shows a triangle PQR in which the point P is $(-2, 8)$, the point R lies on the x-axis and angle PQR is $90°$. The equation of QR is $2y + 5x = 64$.

(i) Find the coordinates of Q. [5]

Downloaded from SingaKids.com

(ii) Given that M is the midpoint of PR and that $PQRS$ is a rectangle, find the coordinates of M and of S. [3]

Downloaded from SingaKids.com

6 **(a)** Solve the equation $\lg(20+5x)-\lg(10-x)=1$. [3]

(b) Given that $\log_a p = x$ and $\log_a q = y$, express in terms of x and y,

(i) $\log_{pq} a$, [2]

(ii) $\log_p aq$. [3]

Downloaded from SingaKids.com

7 **(i)** Express $4x^2 - 12x + 5$ in the form of $a(x-b)^2 - c$. [2]

(ii) Sketch the graph of $y = |4x^2 - 12x + 5|$, indicating the intercepts of both axes and the coordinates of the turning point. [3]

(iii) Determine the set of values of m for which the equation $|4x^2 - 12x + 5| = m$ has 4 solutions. [1]

:

(iv) Solve $|4x^2 - 12x + 5| = 5$. [2]

Downloaded from SingaKids.com

8 A curve is such that $\frac{d^2y}{dx^2} = 2 + \frac{16}{x^3}$ and $(2, 7)$ is a minimum point on the curve.

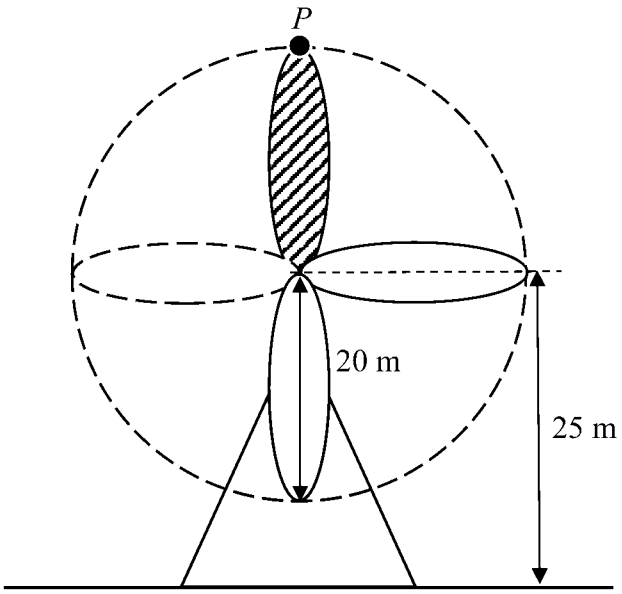
(i) Find the equation of the curve. [5]

Downloaded from SingaKids.com

(ii) Find the value of x for which the curve has a maximum gradient, and find this maximum gradient. [3]
(You are not required to show that gradient is maximum.)

Downloaded from SingaKids.com

9 The diagram shows a windmill with blades 20 m in length. The centre of their circular motion is a point 25 m above the ground. One of the blades has been painted with stripes and the tip of the stripped blade is currently at point P, 45 m above the ground. When in operation, the windmill takes 6 seconds to complete one revolution.



(i) The height above the ground, y m, of the tip of the stripped blade is modelled by the equation $y = a\cos kt + b$, where t in the time in seconds after leaving point P.
Find the value of a and of b. [2]

(ii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [1]

Downloaded from SingaKids.com

(iii) Sketch the graph of $y = a\cos kt + b$ for $0 \le t \le 6$. [2]

(iv) Find the length of time for which the tip of the stripped blade is at most 15 m above the ground. [3]

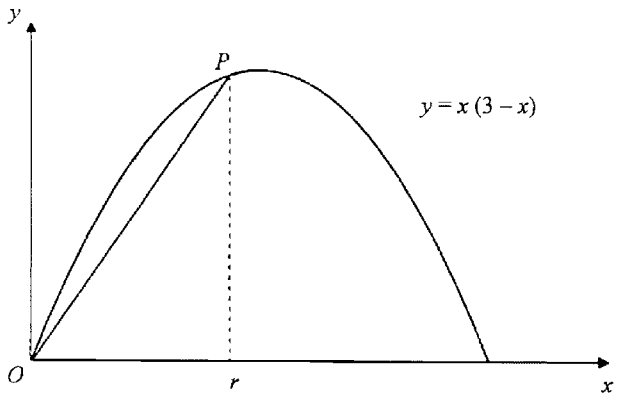
Downloaded from SingaKids.com

10 **(i)** Express $\dfrac{x+1}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \dfrac{x+1}{x(2x-1)^2}\,\mathrm{d}x$. [4]

Downloaded from SingaKids.com

11



The diagram shows the trajectory of an athlete during a long jump which can be represented by the equation $y = x(3-x)$ where x and y are the horizontal distance and vertical height of the jump respectively. O is the point where the athlete takes-off from the ground. When $x = r$ m, the athlete is at point P.

(i) Show that the distance, s m, between O and P is given by $s = \sqrt{r^4 - 6r^3 + 10r^2}$. [3]

(ii) Show that $\frac{ds}{dr} = \frac{r(2r^2 - 9r + 10)}{\sqrt{r^4 - 6r^3 + 10r^2}}$. [2]

Downloaded from SingaKids.com

(iii) Given that r can vary, find the values of r for which s is stationary. [3]

(iv) By using the first derivative test, determine whether the smaller of these values of r will give a maximum or a minimum value of s. [2]

End of Paper

Downloaded from SingaKids.com

Jurong West Secondary School

Preliminary Examinations 2020

100

ADDITIONAL MATHEMATICS 4047/02

Secondary Four Express / Five Normal (Academic) **31 Aug 2020**

Paper 2 **0800 - 1030**

2 hour 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of **18** printed pages.

Setter: Mr Seoh Soon Peng

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Downloaded from SingaKids.com

1 Solve the simultaneous equations. [5]

$$4^{y} = 2^{8-2x}$$
$$3^{3(x-2)} = 27^{y}$$

Downloaded from SingaKids.com

2 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Show that $(\alpha - 2)(\beta - 2) = 4$. [3]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha - 2}$ and $\frac{\beta}{\beta - 2}$. [4]

Downloaded from SingaKids.com

3 **(i)** Prove that $\dfrac{\cot^2\theta - \tan^2\theta}{\cos ec^2\theta} = 1 - \tan^2\theta$. [4]

(ii) Hence, solve the equation $\cot^2\theta - \tan^2\theta = -5\cos ec^2\theta$ for $0° < \theta < 360°$. [3]

Downloaded from SingaKids.com

4 **(i)** Write down the first three terms in the expansion, in ascending power of x, of $(1+kx)^6$, where k is a positive constant. r [2]

(ii) Write down the first three terms in the expansion, in ascending power of x, of $(2-x)^n$, where n is a positive constant. [2]

(iii) The first three terms in the expansion, in ascending power of x, of $(1+kx)^6(2-x)^n$ are $1024+25600x+ax^2$, where a is a positive constant. Find the value of k, of n and of a. [5]

Downloaded from SingaKids.com

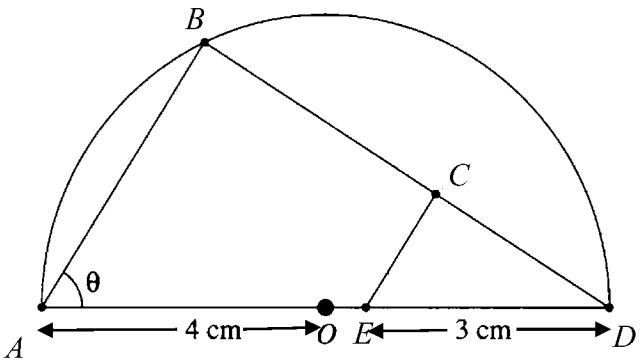
5 A circle, C_1, passes through the points $A(-4,5)$ and $B(-4,-3)$. The line $y = 6$ is a tangent to C_1. It is given that x-coordinate for centre of C_1 is $x > -7$.

(i) Find the equation of C_1. [5]

(ii) Find the equation of the tangent to the circle at $(3,-4)$. [3]

Downloaded from SingaKids.com

6



The diagram shows a triangle ABD. Points A, B and D lies on the circumference of the circle with centre, O, and diameter AD. AB is parallel to EC. Angle $BAD = \theta$, $ED = 3$ cm and $AO = 4$ cm.

(i) Show that $AB + BC + CE = 5\sin\theta + 11\cos\theta$. [3]

Downloaded from SingaKids.com

(ii) Express $5\sin\theta + 11\cos\theta$ in the form of $R\sin(\theta + \alpha)$. [4]

(iii) Find the maximum value of $AB + BC + CE$ and the corresponding value of θ. [3]

(iv) Find the value of θ of which $AB + BC + CE = 9$ cm. [2]

Downloaded from SingaKids.com

7 **(a)** Given that $\mathrm{f}(x) = e^{5x}(2x-7)$, where $x > 3.3$.

Show that $\mathrm{f}(x)$ is an increasing function. [3]

(b) A particle moves along the curve $y = \dfrac{\ln x}{x}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.4 units per second. Find the rate of change of the x-coordinate of the particle when $x = 2$. [4]

Downloaded from SingaKids.com

(c) Given that $y = \sin x + p\cos^2 x$, where p is a constant, and that

[5]

Downloaded from SingaKids.com

8 **(a)** Solve the equation $2x^3 + 3x^2 - 13x + 8 = 0$. [4]

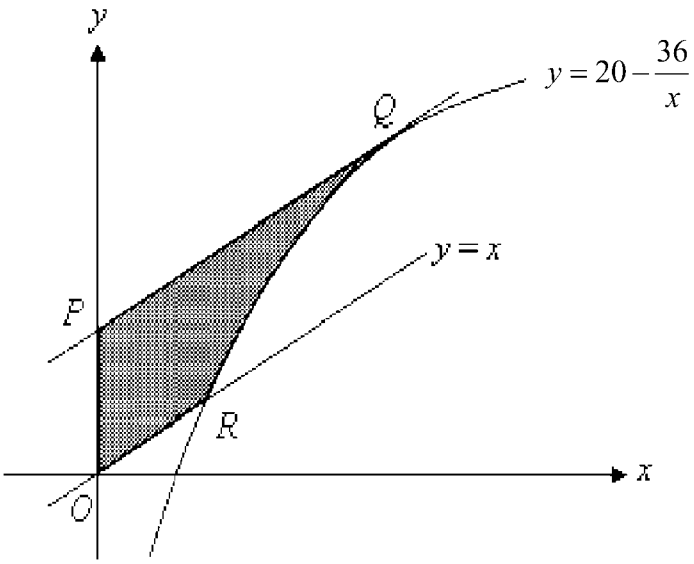
(b) The term containing the highest power of x in a polynomial f(x) is $6x^3$. One of the roots of the equation f(x) = 0 is k and that $2x^2 + 3x + 4$ is a quadratic factor of f(x). f(x) leaves a remainder of 108 when divided by $x - 1$.

(i) Show that $k = -3$. [4]

Downloaded from SingaKids.com

(ii) Find the number of real roots of the equation f(x) = 0, justifying your answer. [2]

9



The diagram shows part of the curve $y = 20 - \frac{36}{x}$ and line $y = x$.

OR and PQ are parallel lines. Find

(i) the coordinates of R, [3]

Downloaded from SingaKids.com

(ii) the coordinates of Q, [3]

(iii) the area of the shaded region. [3]

Downloaded from SingaKids.com

10 A particle starts from rest, travels in a straight line so that t is the time in seconds after passing through a fixed point O. Its velocity, v m/s, is given by $v = 2t^2 - 15t + 18$.

(i) Find the acceleration(s) of the particle when it comes to instantaneous rest. [3]

Downloaded from SingaKids.com

(ii) Find the minimum velocity of the particle. [3]

(iii) Calculate the average speed in the first 6 seconds. [5]

Downloaded from SingaKids.com

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{x^b}{10^a}$, where a and b are constants. Measured values of x and y are given in the following table.

x	2	3	4	5
y	0.08	0.30	0.64	1.25

(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable line on your graph, solve the equation $y = x^{-2}$. [3]

--End of Paper--

Downloaded from SingaKids.com

Downloaded from SingaKids.com

Name:	Class	Index Number

Jurong West Secondary School

Preliminary Examinations 2020

80

ADDITIONAL MATHEMATICS 4047/01

Secondary Four Express/ Five Normal Academic **27 August 2020**

Paper 1 **1000 - 1200**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of **17** printed pages.

Setter: Ms Sharron Chiam

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Downloaded from SingaKids.com

1 A cone with base radius $\left(5+2\sqrt{3}\right)$ cm and a slant height l cm has a curved surface area of $\left(51-3\sqrt{3}\right)\pi$ cm^2. Without using a calculator, obtain an expression for l in the form of $\left(a+b\sqrt{3}\right)$, where a and b are integers. [4]

Solution:

Curved surface area of cone $= \pi rl$

$\pi\left(5+2\sqrt{3}\right)l = \left(51-3\sqrt{3}\right)\pi$

$l = \dfrac{\left(51-3\sqrt{3}\right)\pi}{\pi\left(5+2\sqrt{3}\right)}$ [M1] – Correct use of formula

$= \dfrac{51-3\sqrt{3}}{5+2\sqrt{3}} \times \dfrac{5-2\sqrt{3}}{5-2\sqrt{3}}$ [M1] – Correct use of conjugate surds

$= \dfrac{255-102\sqrt{3}-15\sqrt{3}+6(3)}{25-4(3)}$ [M1]

$= \dfrac{273-117\sqrt{3}}{13}$

$= \left(21-9\sqrt{3}\right)$ cm [A1]

2 The acute angles A and B are such that $\tan\left(A+B\right)=8$ and $\tan A = \dfrac{1}{5}$. Without using a calculator, find the exact value of $\cos B$. [5]

Solution:

$\tan\left(A+B\right)=8$

$\dfrac{\tan A+\tan B}{1-\tan A\tan B}=8$

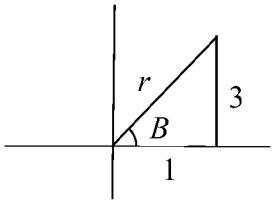
$\dfrac{\frac{1}{5}+\tan B}{1-\frac{1}{5}\tan B}=8$ [M1] – Correct use of double angle formula

$\dfrac{1}{5}+\tan B = 8\left(1-\dfrac{1}{5}\tan B\right)$ [M1]

$\dfrac{1}{5}+\tan B = 8-\dfrac{8}{5}\tan B$

$\dfrac{13}{5}\tan B = \dfrac{39}{5}$

$\tan B = 3$ [M1]



By Pythagoras' Theorem,

$r = \sqrt{1^2+3^2}$

Downloaded from SingaKids.com

$r = \sqrt{10}$ [M1]

$\cos B = \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$

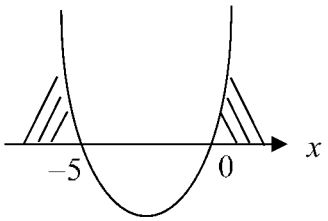
$\cos B = \frac{\sqrt{10}}{10}$ [A1]

3 **(i)** Find the range of values of m for which the curve $y = (4+m)x^2 - 4x + m + 1$ has a maximum point. [1]

Solution:
For curve to have a maximum point,
$4 + m < 0$
$m < -4$ [B1]

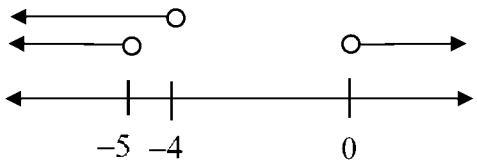
(ii) Find the range of values of m for which the curve $y = (4+m)x^2 - 4x + m + 1$ is always negative for all real values of x. [3]

Solution:
For curve to be always negative,
$b^2 - 4ac < 0$
$(-4)^2 - 4(4+m)(m+1) < 0$ } [M1] – Either seen for correct use of discriminant
$16 - 4(m^2 + 5m + 4) < 0$
$16 - 4m^2 - 20m - 16 < 0$
$-4m^2 - 20m < 0$ [M1]
$m^2 + 5m > 0$
$m(m+5) > 0$
$m < -5$ or $m > 0$ [A1]



(iii) Hence state the range of values of m for which the curve has a maximum point and is always negative for all values of x. [1]

Solution:
$m < -5$ [B1]



Downloaded from SingaKids.com

4 **(a)** It is given that $\int \mathrm{f}(x)\,\mathrm{d}x = 3x^2 \ln x + kx^2 + c$ where k and c are constants.

(i) If $\int_1^e \mathrm{f}(x)\,\mathrm{d}x = e^2 + 2$, show that $k = -2$. [3]

Solution:

$\int_1^e \mathrm{f}(x)\,\mathrm{d}x = e^2 + 2$

$\left[3x^2 \ln x + kx^2 + c\right]_1^e = e^2 + 2$

$\left[3e^2 \ln e + ke^2 + c - (3\ln 1 + k + c)\right] = e^2 + 2$ [M1] – Use of definite integrals

$3e^2 + ke^2 + c - (k + c) = e^2 + 2$

$3e^2 + ke^2 - k = e^2 + 2$ [M1]

$3e^2 - e^2 - 2 = k - ke^2$

$2e^2 - 2 = k - ke^2$

$2(e^2 - 1) = -k(e^2 - 1)$

$\therefore k = -2$ [A1]

(ii) Find $\mathrm{f}(x)$. [2]

Solution:

$\int \mathrm{f}(x)\,\mathrm{d}x = 3x^2 \ln x - 2x^2 + c$

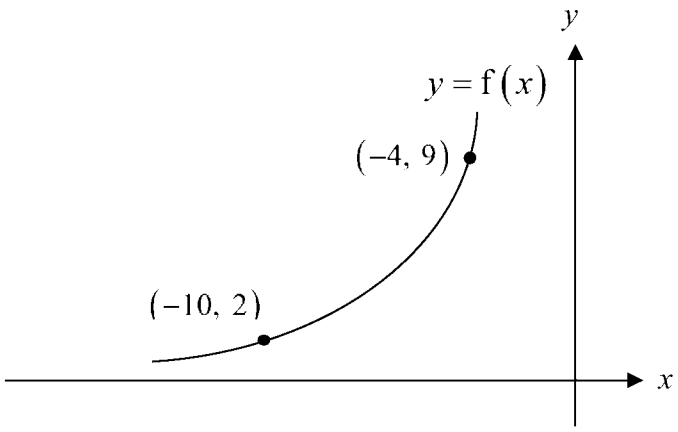
$\mathrm{f}(x) = 6x \ln x + 3x^2\left(\frac{1}{x}\right) - 4x$ [M1] – Correct use of product rule

$= 6x \ln x - x$

$= x(6 \ln x - 1)$ } [A1] – Either seen

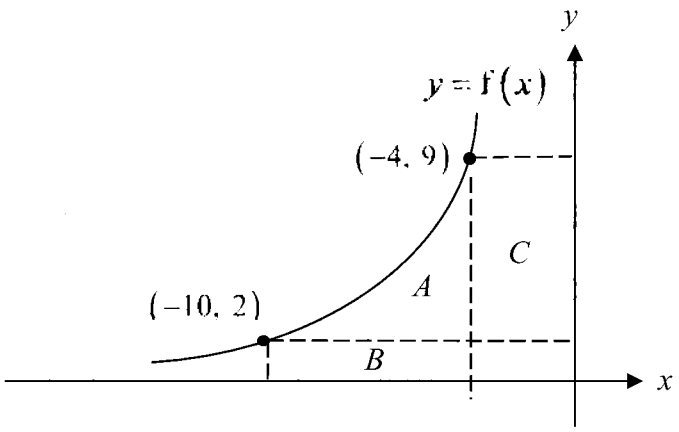
Downloaded from SingaKids.com

(b) The figure shows part of the curve $y = \mathrm{f}(x)$. $(-10, 2)$ and $(-4, 9)$ are two points on the curve.



Given that $\int_{-10}^{-4} y \, \mathrm{d}x = 30$, find $\int_{2}^{9} x \, \mathrm{d}y$. [2]

Solution:



Area of $A + B = 30$

Area of $B = 6(2)$
$= 12$

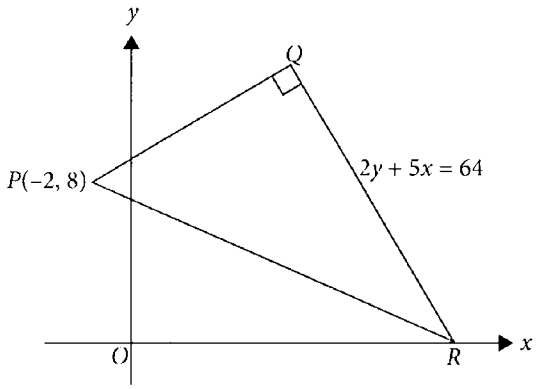
Area of $A = 30 - 12$ [M1]
$= 18$

Area of $C = 7(4)$
$= 28$

$\int_{2}^{9} x \, \mathrm{d}y = \text{Area of } A + C$
$= 28 + 18$
$= 46$ [A1]

Downloaded from SingaKids.com

5



The diagram shows a triangle PQR in which the point P is $(-2, 8)$, the point R lies on the x-axis and angle PQR is $90°$. The equation of QR is $2y + 5x = 64$.

(i) Find the coordinates of Q. [5]

Solution:

$2y + 5x = 64$

$2y = -5x + 64$

$y = -\frac{5}{2}x + 32$

Gradient of $QR = -\frac{5}{2}$

Gradient of $PQ = \frac{2}{5}$ [B1]

Equation of PQ:

$8 = \frac{2}{5}(-2) + c$ [M1] – Attempt to find c

$8 = \frac{2}{5}(-2) + c$

$c = \frac{44}{5}$

$\therefore$ Equation of PQ is $y = \frac{2}{5}x + \frac{44}{5}$. [A1]

$y = -\frac{5}{2}x + 32$ ---- (1)

$y = \frac{2}{5}x + \frac{44}{5}$ ---- (2)

(1) = (2)

$-\frac{5}{2}x + 32 = \frac{2}{5}x + \frac{44}{5}$ [M1]

$-\frac{29}{10}x = \frac{-116}{5}$

$x = 8$

Downloaded from SingaKids.com

Subst. $x = 8$ into (1)

$$y = -\frac{5}{2}(8) + 32$$
$$= 12$$

$\therefore\ Q(8, 12)$ [A1]

(ii) Given that M is the midpoint of PR and that $PQRS$ is a rectangle, find the coordinates of M and of S. [3]

Solution:

When $y = 0$,

$$2(0) + 5x = 64$$
$$x = 12.8$$ [M1]

$\therefore\ R(12.8, 0)$

Coordinates of $M = \left(\frac{-2 + 12.8}{2}, \frac{8 + 0}{2}\right)$
$= (5.4, 4)$ [A1]

Let $S(x, y)$

$$\left(\frac{x+8}{2}, \frac{y+12}{2}\right) = (5.4, 4)$$

$\frac{x+8}{2} = 5.4$ and $\frac{y+12}{2} = 4$

$x + 8 = 10.8$ $y + 12 = 8$

$x = 2.8$ $y = -4$

$\therefore\ S(2.8, -4)$ [A1]

Downloaded from SingaKids.com

6 **(a)** Solve the equation $\lg(20+5x)-\lg(10-x)=1$. [3]

Solution:

$\lg(20+5x)-\lg(10-x)=1$

$\lg\left(\frac{20+5x}{10-x}\right)=1$ [M1]

$\frac{20+5x}{10-x}=10$ [M1]

$20+5x=100-10x$

$15x=80$

$x=5\frac{1}{3}$ [A1]

(b) Given that $\log_a p=x$ and $\log_a q=y$, express in terms of x and y,

(i) $\log_{pq} a$, [2]

Solution:

$\log_{pq} a=\frac{\log_a a}{\log_a pq}$ [M1]

$=\frac{1}{\log_a p+\log_a q}$

$=\frac{1}{x+y}$ [A1]

(ii) $\log_p aq$. [3]

Solution:

$\log_p aq=\log_p a+\log_p q$ [M1]

$=\frac{\log_a a}{\log_a p}+\frac{\log_a q}{\log_a p}$ [M1]

$=\frac{1+y}{x}$ [A1]

Or

$\log_p aq=\frac{\log_a aq}{\log_a p}$ [M1]

$=\frac{\log_a a+\log_a q}{\log_a p}$ [M1]

$=\frac{1+y}{x}$ [A1]

Downloaded from SingaKids.com

7 **(i)** Express $4x^2-12x+5$ in the form of $a(x-b)^2-c$. [2]

Solution:

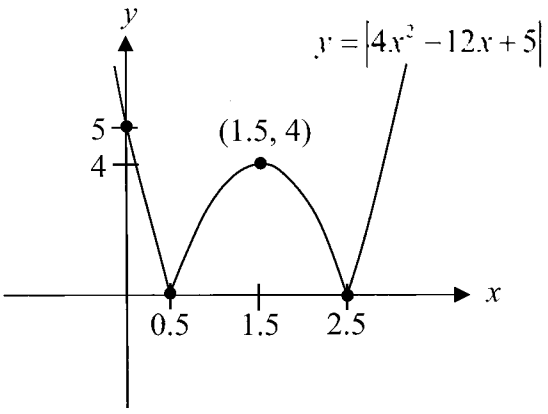
$$4x^2-12x+5=4\left(x^2-3x+\frac{5}{4}\right)$$
$$=4\left[x^2-3x+\left(-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2+\frac{5}{4}\right]$$
$$=4\left[\left(x-\frac{3}{2}\right)^2-1\right]$$
$$=4\left(x-\frac{3}{2}\right)^2-4$$

[B1] – For $4\left(x-\frac{3}{2}\right)^2$ seen

[B1] – For -4 seen

(ii) Sketch the graph of $y=\left|4x^2-12x+5\right|$, indicating the intercepts of both axes and the coordinates of the turning point. [3]

Solution:



[B1] – Correct shape
[B1] – Turning point at (1.5, 4)
[B1] – Correct intercepts

(iii) Determine the set of values of m for which the equation $\left|4x^2-12x+5\right|=m$ has 4 solutions. [1]

Solution:

$0<m<4$ [B1]

(iv) Solve $\left|4x^2-12x+5\right|=5$. [2]

Solution:

$\left|4x^2-12x+5\right|=5$

$4x^2-12x+5=5$ or $4x^2-12x+5=-5$ [M1]

$4x^2-12x=0$ $4x^2-12x+10=0$ (N.A)

$4x(x-3)=0$ $b^2-4ac=(-12)^2-4(4)(10)$

Downloaded from SingaKids.com

$x = 0$ or $x = 3$ $= -16$ [A1]

8 A curve is such that $\frac{d^2y}{dx^2} = 2 + \frac{16}{x^3}$ and $(2, 7)$ is a minimum point on the curve.

(i) Find the equation of the curve. [5]

Solution

$$\frac{d^2y}{dx^2} = 2 + \frac{16}{x^3}$$

$$\frac{dy}{dx} = \int 2 + 16x^{-3}\, dx$$

$$= 2x + \frac{16x^{-2}}{-2} + c$$ [M1] – Correct differentiation

$$= 2x - \frac{8}{x^2} + c$$

At $(2, 7)$, $\frac{dy}{dx} = 0$

$$2(2) - \frac{8}{(2)^2} + c = 0$$ [M1] – Attempt to find c

$$4 - 2 + c = 0$$

$$c = -2$$

$$\frac{dy}{dx} = 2x - \frac{8}{x^2} - 2$$

$$y = \int 2x - 8x^{-2} - 2\, dx$$

$$= \frac{2x^2}{2} - \frac{8x^{-1}}{(-1)} - 2x + c$$ [M1] – Correct differentiation

$$= x^2 + \frac{8}{x} - 2x + c$$

At $(2, 7)$,

$$7 = (2)^2 + \frac{8}{2} - 2(2) + c$$ [M1] – Attempt to find c

$$7 = 4 + 4 - 4 + c$$

$$c = 3$$

$$\therefore y = x^2 + \frac{8}{x} - 2x + 3$$ [A1]

Downloaded from SingaKids.com

(ii) Find the value of x for which the curve has a maximum gradient, and find this maximum gradient. [3]
(You are not required to show that gradient is maximum.)

Solution:
For maximum gradient,

$\frac{d^2y}{dx^2} = 0$
$2 + \frac{16}{x^3} = 0$ } [M1] – Either seen

$2x^3 + 16 = 0$
$2x^3 = -16$
$x^3 = -8$
$x = -2$ [A1]

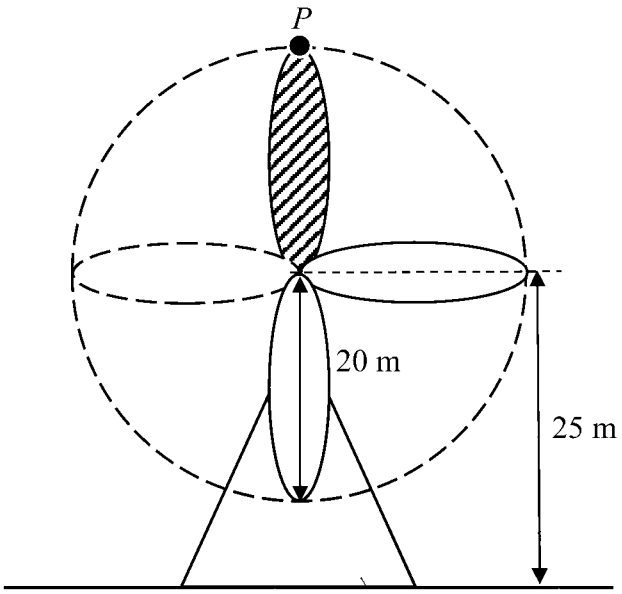
When $x = -2$,

$$\frac{dy}{dx} = 2(-2) - \frac{8}{(-2)^2} - 2$$
$$= -8$$ [A1]

$\therefore$ Maximum gradient $= -8$

Downloaded from SingaKids.com

9 The diagram shows a windmill with blades 20 m in length. The centre of their circular motion is a point 25 m above the ground. One of the blades has been painted with stripes and the tip of the stripped blade is currently at point P, 45 m above the ground. When in operation, the windmill takes 6 seconds to complete one revolution.



(i) The height above the ground, y m, of the tip of the stripped blade is modelled by the equation $y = a\cos kt + b$, where t in the time in seconds after leaving point P.
Find the value of a and of b. [2]

Solution:

$$\text{Amplitude} = \frac{45-5}{2}$$
$$= 20$$

$\therefore\ a = 20$ [B1]

$b = 25$ [B1]

(ii) Show that the value of k is $\frac{\pi}{3}$ radians per second. [1]

Solution:

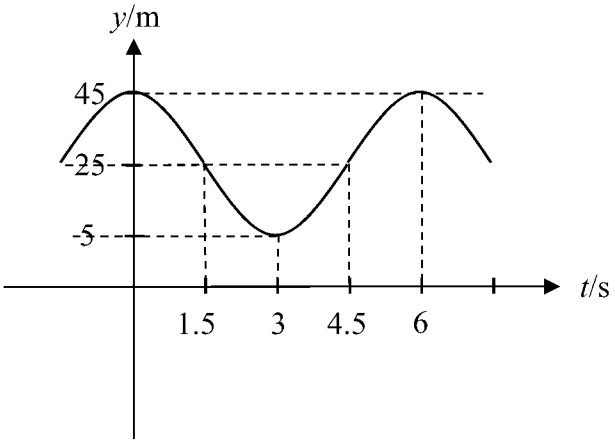
$$\frac{2\pi}{k} = 6$$

$k = \frac{\pi}{3}$ (shown) [B1]

Downloaded from SingaKids.com

(iii) Sketch the graph of $y = a\cos kt + b$ for $0 \le t \le 6$. [2]

Solution:



[B2, 1, 0]
1 cosine, correct amplitude and positioned correctly

(iv) Find the length of time for which the tip of the stripped blade is at most 15 m above the ground. [3]

Solution:

$20\cos\frac{\pi}{3}t + 25 = 15$ [M1]

$20\cos\frac{\pi}{3}t = -10$

$\cos\frac{\pi}{3}t = -0.5$

Basic angle $= \cos^{-1}(0.5)$

$= \frac{\pi}{3}$ [M1]

$\frac{\pi}{3}t = \pi - \frac{\pi}{3}, \qquad \pi + \frac{\pi}{3}$

$t = 2, \quad 4$

$\therefore$ Length of time $= 2$ s [A1]

Downloaded from SingaKids.com

10 **(i)** Express $\dfrac{x+1}{x(2x-1)^2}$ in partial fractions. [5]

Solution:

$$\frac{x+1}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{(2x-1)^2}$$

$$x+1 = A(2x-1)^2 + Bx(2x-1) + Cx$$

[M1] – Either seen

Let $x = 0$

$A = 1$ [M1] – For evaluating A and C

Let $x = \dfrac{1}{2}$

$$\frac{3}{2} = \frac{1}{2}C$$

$$C = 3$$

Let $x = 1$

$2 = 1(1)^2 + B(1) + 3$ [M1] – For evaluating B

$B + 4 = 2$

$B = -2$

$$\therefore \frac{x+1}{x(2x-1)^2} = \frac{1}{x} - \frac{2}{2x-1} + \frac{3}{(2x-1)^2}$$ [A2] – Minus 1 for each error

(ii) Hence find $\int \dfrac{x+1}{x(2x-1)^2}\,\mathrm{d}x$. [4]

Solution:

$$\int \frac{x+1}{x(2x-1)^2}\,\mathrm{d}x = \int \frac{1}{x} - \frac{2}{2x-1} + \frac{3}{(2x-1)^2}\,\mathrm{d}x$$ [M1]

$$= \int \frac{1}{x} - \frac{2}{2x-1} + 3(2x-1)^{-2}\,\mathrm{d}x$$

$$= \ln x - \ln(2x-1) + \frac{3(2x-1)^{-1}}{(-1)(2)} + c$$

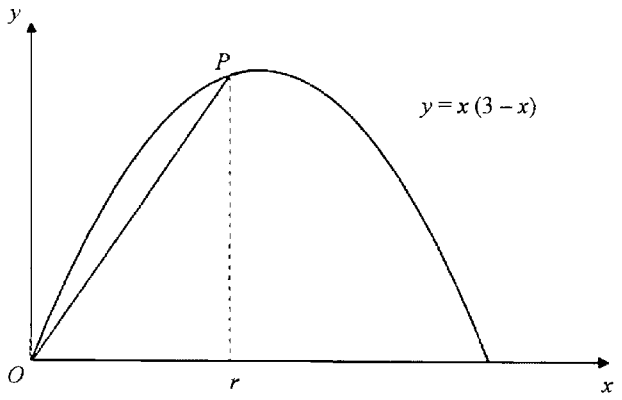
$$= \ln x - \ln(2x-1) - \frac{3}{2(2x-1)} + c$$

$$= \ln \frac{x}{2x-1} - \frac{3}{2(2x-1)} + c$$

[A3] – Either seen
Minus 1 for each error including + c

Downloaded from SingaKids.com

11



The diagram shows the trajectory of an athlete during a long jump which can be represented by the equation $y = x(3-x)$ where x and y are the horizontal distance and vertical height of the jump respectively. O is the point where the athlete takes-off from the ground. When $x = r$ m, the athlete is at point P.

(i) Show that the distance, s m, between O and P is given by $s = \sqrt{r^4 - 6r^3 + 10r^2}$. [3]

Solution:

When $x = r$,

$y = r(3-r)$ [B1]

$s^2 = r^2 + \left[r(3-r)\right]^2$ [M1] – Use of Pythagoras' Theorem

$s^2 = r^2 + \left(3r - r^2\right)^2$

$s^2 = r^2 + 9r^2 - 6r^3 + r^4$

$s^2 = 10r^2 - 6r^3 + r^4$

$s = \sqrt{r^4 - 6r^3 + 10r^2}$ (shown) [A1]

(ii) Show that $\dfrac{ds}{dr} = \dfrac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}}$. [2]

Solution:

$s = \sqrt{r^4 - 6r^3 + 10r^2}$

$s = \left(r^4 - 6r^3 + 10r^2\right)^{\frac{1}{2}}$

$\dfrac{ds}{dr} = \dfrac{1}{2}\left(r^4 - 6r^3 + 10r^2\right)^{-\frac{1}{2}}\left(4r^3 - 18r^2 + 20r\right)$ [M1]

$= \left(r^4 - 6r^3 + 10r^2\right)^{-\frac{1}{2}}\left(2r^3 - 9r^2 + 10r\right)$

$= \dfrac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}}$ [A1]

Downloaded from SingaKids.com

(iii) Given that r can vary, find the values of r for which s is stationary. [3]

Solution:

For stationary value of s, $\frac{ds}{dr} = 0$.

$$\frac{r\left(2r^2 - 9r + 10\right)}{\sqrt{r^4 - 6r^3 + 10r^2}} = 0$$ [M1] – Knowledge of stationary value

$$r\left(2r^2 - 9r + 10\right) = 0$$

$$r\left(2r - 5\right)\left(r - 2\right) = 0$$

$r = 0$ (N.A) or $r = 2$ or $r = 2.5$ [A2] – Minus 1 for each error

(iv) By using the first derivative test, determine whether the smaller of these values of r will give a maximum or a minimum value of s. [2]

Solution:

r	< 2	2	> 2
$\frac{ds}{dr}$	> 0	$= 0$	< 0
Sketch of tangent			
Outline of graph			

[M1]

$\therefore$ s is maximum when $r = 2$. [B1]

End of Paper

Downloaded from SingaKids.com

Name:

Answer Scheme

Index Number

Jurong West Secondary School

Preliminary Examinations 2020

100

ADDITIONAL MATHEMATICS 4047/02

Secondary Four Express / Five Normal (Academic) **31 Aug 2020**

Paper 2 **0800 - 1030**

2 hour 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of **18** printed pages.

Setter: Mr Seoh Soon Peng

Downloaded from SingaKids.com

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Downloaded from SingaKids.com

1 Solve the simultaneous equations. [5]

$$4^{y} = 2^{8-2x}$$
$$3^{3(x-2)} = 27^{y}$$

$2^{2y} = 2^{8-2x}$ [M1] Same base 2 or 3 seen

$2y = 8 - 2x$ [M1] comparing powers $2y = 8 - 2x$ or $y = 4 - x$ seen

$y = 4 - x$ --(1)

$3^{3x-6} = 3^{3y}$

$3x - 6 = 3y$

$x - 2 = y$ --(2)

Sub (2) into (1)

$x - 2 = 4 - x$ [M1]

$x = 3$ [A1]

When $x = 3$, sub into (1)

$y = 1$ [A1]

Downloaded from SingaKids.com

2 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Show that $(\alpha - 2)(\beta - 2) = 4$. [3]

$\alpha + \beta = 2$

$\alpha\beta = 4$ [B1] either $\alpha + \beta$ or $\alpha\beta$ seen

LHS

$= (\alpha - 2)(\beta - 2)$

$= \alpha\beta - 2\alpha - 2\beta + 4$ [M1] Correct expansion

$= \alpha\beta - 2(\alpha + 2) + 4$

$= 4 - 2(2) + 4$

$= 4(shown)$ [A1]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha - 2}$ and $\frac{\beta}{\beta - 2}$. [4]

SOR:

$\frac{\alpha}{\alpha - 2} + \frac{\beta}{\beta - 2}$

$= \frac{\alpha(\beta - 2) + \beta(\alpha - 2)}{(\alpha - 2)(\beta - 2)}$ [M1] Express as algebraic expression with common denominator

$= \frac{2\alpha\beta - 2(\alpha + \beta)}{(\alpha - 2)(\beta - 2)}$

$= \frac{2(4) - 2(2)}{4}$

$= 1$ [A1]

POR:

$(\frac{\alpha}{\alpha - 2})(\frac{\beta}{\beta - 2})$

$= \frac{\alpha\beta}{(\alpha - 2)(\beta - 2)}$

$= \frac{4}{4}$

$= 1$ [B1]

$x^2 - x + 1 = 0$ [A1]

Downloaded from SingaKids.com

3 **(i)** Prove that $\dfrac{\cot^2\theta - \tan^2\theta}{\cos ec^2\theta} = 1 - \tan^2\theta$. [4]

LHS

$[\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}][\sin^2\theta]$ [M1] Attempt to change any terms into $\sin\theta$ or $\cos\theta$

$= [\dfrac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta}][\sin^2\theta]$

$= \dfrac{(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)}{\cos^2\theta}$ [M1] use of $a^2 - b^2 = (a+b)(a-b)$

$= \dfrac{(\cos^2\theta - \sin^2\theta)}{\cos^2\theta}$ [M1] $\cos^2\theta + \sin^2\theta = 1$

$= 1 - \tan^2\theta$ [A1]

= RHS (Shown)

(ii) Hence, solve the equation $\cot^2\theta - \tan^2\theta = -5\cos ec^2\theta$ for $0° < \theta < 360°$. [3]

$\dfrac{\cot^2\theta - \tan^2\theta}{\cos ec^2\theta} = -5$

$1 - \tan^2\theta = -5$ [M1]

$\tan^2\theta = 6$

$\tan\theta = \pm\sqrt{6}$ [M1]

$\alpha = 67.792$

$\theta = 67.8°, 112.2°, 247.8°, 292.2°$ [A1]

Downloaded from SingaKids.com

4 **(i)** Write down the first three terms in the expansion, in ascending power of x, of $(1+kx)^6$, where k is a positive constant. [2]

$(1+kx)^6 = 1+6kx+15k^2x^2+\ldots$ [B2, 1, 0] minus 1 for each error

(ii) Write down the first three terms in the expansion, in ascending power of x, of $(2-x)^n$, where n is a positive constant. [2]

$(2-x)^n = 2^n + \binom{n}{1}(2^{n-1})(-x) + \binom{n}{2}(2^{n-2})(-x)^2 + \ldots$

$= 2^n - n(2^{n-1})(x) + (\frac{n(n-1)}{2})(2^{n-2})(x^2) + \ldots$ [B2, 1, 0] minus 1 for each error

(iii) The first three terms in the expansion, in ascending power of x, of $(1+kx)^6(2-x)^n$ are $1024+25600x+ax^2$, where a is a positive constant. Find the value of k, of n and of a. [5]

$(1+kx)^6(2-x)^n = (1+6kx+15k^2x^2+\ldots)(2^n - n(2^{n-1})(x) + (\frac{n(n-1)}{2})(2^{n-2})(x^2)+\ldots)$

$= (2^n - n2^{n-1}x + n(n-1)2^{n-3}x^2 + 2^n(6)kx - 6nk2^{n-1}x^2 + 2^n(15)k^2x^2 + \ldots)$[M1] Attempt to expand/ Attempt to show comparing coefficients

$2^n = 1024$ [M2, 1 , 0] Minus 1 for each incorrect comparison

$2^n = 2^{10}$

$n = 10$

When $n = 10$

$-n2^{n-1} + 2^n(6)k = 25600$

$k = 5$

When $n = 10$, $k = 5$

$n(n-1)2^{n-3} - 6nk2^{n-1} + 2^n(15)k^2 = a$

$a = 241920$ [A2, 1, 0] Minus 1 for each error

Downloaded from SingaKids.com

5 A circle, C_1, passes through the points $A(-4,5)$ and $B(-4,-3)$. The line $y = 6$ is a tangent to C_1. It is given that x-coordinate for centre of C_1 is $x > -7$.

(i) Find the equation of C_1. [5]

$y = \dfrac{5+(-3)}{2} = 1$ [B1]

Radius = 6 – 1 = 5 [B1]

$(5-1)^2 + (-4-x)^2 = 5^2$ [M1] Length formula

$(-4-x)^2 = 9$

$-4-x = \pm 3$

$x = -1$ or $x = -7$ (rejected) [A1]

$(-1, 1)$

$(x+1)^2 + (y-1)^2 = 25$ [A1]

(ii) Find the equation of the tangent to the circle at $(3,-4)$. [3]

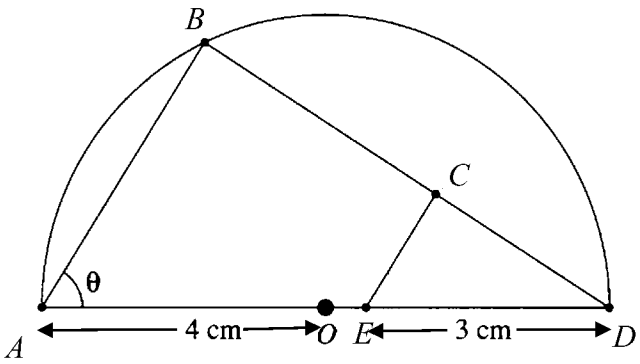
Gradient = $\dfrac{1-(-4)}{-1-3} = -\dfrac{5}{4}$ [M1]

$y-(-4) = \dfrac{4}{5}(x-3)$ [M1] attempt to find c

$y = \dfrac{4}{5}x - 6\dfrac{2}{5}$ [A1]

Downloaded from SingaKids.com

6



The diagram shows a triangle ABD. Points A, B and D lies on the circumference of the circle with centre, O, and diameter AD. AB is parallel to EC. Angle $BAD = \theta$, $ED = 3$ cm and $AO = 4$ cm.

(i) Show that $AB + BC + CE = 5\sin\theta + 11\cos\theta$. [3]

$$\sin\theta = \frac{CD}{3}$$

$$CD = 3\sin\theta$$

$$\sin\theta = \frac{BD}{8}$$

$$BD = 8\sin\theta$$

$$\cos\theta = \frac{CE}{3}$$

$$CE = 3\cos\theta$$

$$\cos\theta = \frac{BA}{8}$$ [M2, 1, 0] Minus 1 for each error

$$BA = 8\cos\theta$$

Downloaded from SingaKids.com

$AB + BC + CE = 8\cos\theta + (8\sin\theta - 3\sin\theta) + 3\cos\theta$

$= 11\cos\theta + 5\sin\theta$ [A1]

(ii) Express $5\sin\theta + 11\cos\theta$ in the form of $R\sin(\theta + \alpha)$. [4]

$11\cos\theta + 5\sin\theta = R\sin(\theta + \alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 5$
$R\sin\alpha = 11$ [M1] comparing coefficient

$\tan\alpha = \frac{11}{5}$ [M1] $\tan\alpha = \frac{11}{5}$ seen

$\alpha = \tan^{-1}\frac{11}{5}$

$= 65.556$

$= 65.6°$ [A1]

$R = \sqrt{11^2 + 5^2} = \sqrt{146}$ [A1]

$\sqrt{146}\sin(\theta + 65.6)$

(iii) Find the maximum value of $AB + BC + CE$ and the corresponding value of θ. [3]

Maximum = $\sqrt{146}$ [B1]

$\sin(\theta + 65.556) = 1$ [M1]
$\alpha = 90$
$\theta = 24.4°$ [B1]

(iv) Find the value of θ of which $AB + BC + CE = 9$ cm. [2]

Downloaded from SingaKids.com

$\sqrt{146}\sin(\theta+65.556)=9$

$\sin(\theta+65.556)=\dfrac{9}{\sqrt{146}}$ [M1]

$\alpha=48.145$

$\theta+65.556=48.145,180-48.145$

$\theta=66.3°$ [A1]

7 **(a)** Given that $f(x)=e^{5x}(2x-7)$, where $x>3.3$.

Show that $f(x)$ is an increasing function. [3]

$\dfrac{dy}{dx}=e^{5x}(2)+5(2x-7)e^{5x}$ [M1] accurate differentiation

$=e^{5x}(10x-33)$ [A1]

Since $e^{5x}>0$, $(10x-33)>0$ because $x>3.3$.

Therefore, $e^{5x}(10x-33)>0$ and $\dfrac{dy}{dx}>0$. $f(x)$ is an increasing function. [A1]

(b) A particle moves along the curve $y=\dfrac{\ln x}{x}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.4 units per second. Find the rate of change of the x-coordinate of the particle when $x=2$. [4]

$\dfrac{dy}{dx}=\dfrac{x(\frac{1}{x})-\ln x}{x^2}$ [M1] quotient rule

$=\dfrac{1-\ln x}{x^2}$ [A1]

$\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$

$0.4=\dfrac{1-\ln 2x}{x^2}\times\dfrac{dx}{dt}$ [M1] correct use of chain rule

Downloaded from SingaKids.com

When $x = 2$,

$$0.4 = \frac{1-\ln 2}{(2)^2} \times \frac{dx}{dt}$$

$\frac{dx}{dt}$ = 5.21 (3sf) units per second [A1]

(c) Given that $y = \sin x + p\cos^2 x$, where p is a constant, and that

$$\int_0^{\frac{\pi}{4}} y \, dx = 3 - \frac{\sqrt{2}}{2} + \pi .$$

Show that $p = 8$. [5]

$$\int_0^{\frac{\pi}{4}} \sin x + p\cos^2 x \, dx$$

$$= \int_0^{\frac{\pi}{4}} \sin x + p\left(\frac{\cos 2x + 1}{2}\right) dx \qquad \text{[M1] } \cos^2 x = \frac{\cos 2x + 1}{2} \text{ seen}$$

$$= \left[-\cos x + \frac{p}{2}\left(\frac{\sin 2x}{2}\right) + \frac{p}{2}x\right]_0^{\frac{\pi}{4}} \qquad \text{[M1] integration}$$

$$= \left[-\cos\frac{\pi}{4} + \frac{p}{4}\left(\sin\frac{\pi}{2}\right) + \frac{p}{2}\left(\frac{\pi}{4}\right)\right] - \left[-\cos 0 + \frac{p}{4}(\sin 0) + \frac{p}{2}(0)\right]$$

$$= -\frac{\sqrt{2}}{2} + \frac{p}{4} + \frac{p}{8}\pi + 1 \qquad \text{[A1] exact value of } \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$3 - \frac{\sqrt{2}}{2} + \pi = -\frac{\sqrt{2}}{2} + \frac{p}{4} + \frac{p}{8}\pi + 1$$

$$2 + \pi = \frac{p}{4} + \frac{p}{8}\pi \qquad \text{[M1]}$$

Downloaded from SingaKids.com

$1 = \frac{p}{8}$

$p = 8$ [A1]

8 **(a)** Solve the equation $2x^3 + 3x^2 - 13x + 8 = 0$. [4]

let f (x) $= 2x^3 + 3x^2 - 13x + 8$

$f(1) = 2(1)^3 + 3(1)^2 - 13(1) + 8 = 0$

$(x - 1)$ is a factor. [B1]

$$x - 1 \overline{)2x^3 + 3x^2 - 13x + 8}$$

$2x^2 + 5x - 8$

[M1] Long division

$(x-1)(2x^2 + 5x - 8) = 0$

$x = 1$ or

$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(2)(-8)}}{2(2)}$ [M1] quadratic formula

x = -3.61 or x = 1.11 [A1] all 3 correct x

(b) The term containing the highest power of x in a polynomial f(x) is $6x^3$. One of the roots of the equation f(x) = 0 is k and that $2x^2 + 3x + 4$ is a quadratic factor of f(x). f(x) leaves a remainder of 108 when divided by $x - 1$.

(i) Show that $k = -3$. [4]

$f(x) = 3(x-k)(2x^2 + 3x + 4)$ [B1]3 seen [B1] $(x-k)(2x^2 + 3x + 4)$ seen

$f(1) = 108$

$3(1-k)(2+3+4) = 108$ [M1]

$k = -3$ (shown) [A1]

Downloaded from SingaKids.com

(ii) Find the number of real roots of the equation f(x) = 0, justifying your answer. [2]

$3(x+3)(2x^2+3x+4)=0$

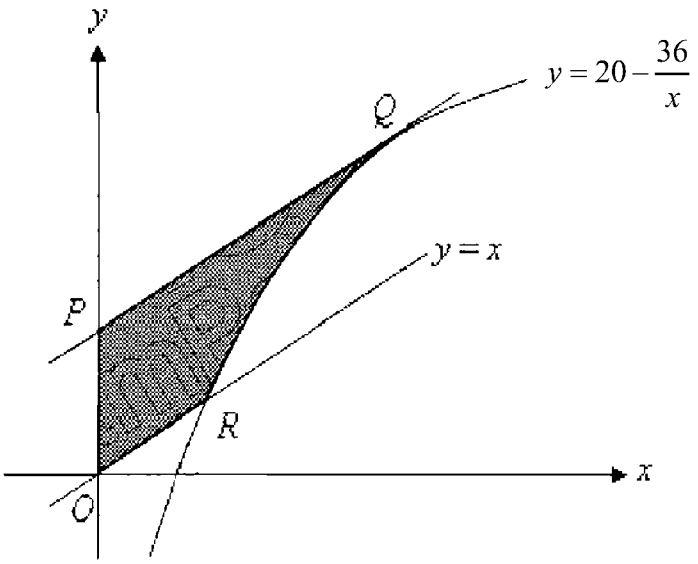
$x = -3$ or $2x^2+3x+4=0$

$b^2-4ac = 3^2-4(2)(4) = -23$ [M1] using discriminant or quadratic fomula

Since $b^2-4ac<0$, therefore $2x^2+3x+4=0$ has no real roots.

Hence, there is only 1 real root. [A1]

9



The diagram shows part of the curve $y = 20-\frac{36}{x}$ and line $y = x$.

OR and *PQ* are parallel lines. Find

(i) the coordinates of *R*, [3]

$x = 20-\frac{36}{x}$ [M1]use of simultaneous equation

$x^2 = 20x-36$

$x^2-20x+36=0$

$(x-18)(x-2)=0$ [M1]Factorisation or equivalent solving methods

x =18(rej) or $x = 2$

Downloaded from SingaKids.com

when $x = 2$,

$y = 2$

$R(2, 2)$ [A1]

(ii) the coordinates of Q, [3]

$\frac{dy}{dx} = 36x^{-2}$ [B1]

At Q, $\frac{dy}{dx} = 1$

$\frac{36}{x^2} = 1$ [M1] equating 1 as gradient

$x^2 = 36$

$x = 6$ or $x = -6$ (rej)

When $x = 6$,

y = 14

Q (6, 14) [A1]

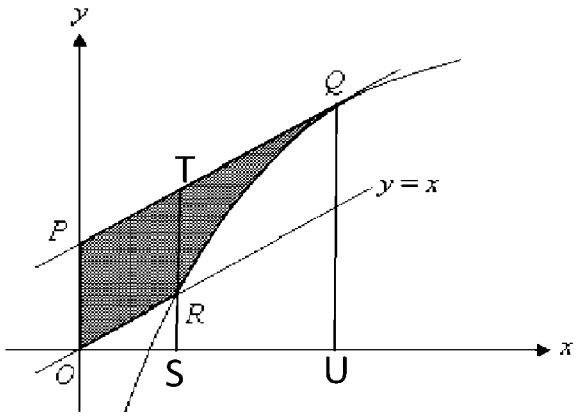
(iii) the area of the shaded region. [3]

Equation of PQ,

$m = 1$, $Q(6,14)$

$y - 14 = 1(x - 6)$ [M1] Attempting to find equation of line

$y = x + 8$



Downloaded from SingaKids.com

when $x = 3$, on line PQ perpendicular to line $x = 3$, (name point T)

$y = 2 + 8 = 10$

$T(2,10)$

Area of triangle $ORS = \frac{1}{2}(2)(2)$

$= 2$ unit^2

$\int_2^6 20 - \frac{36}{x} dx$

$= [20x - 36\ln x]_2^6$ [M1]accurate integration

$= [20(6) - 36\ln 6] - [20(2) - 36\ln 2]$

$= 40.449$ unit^2

Area of trapezium $OPQU = \frac{1}{2}(8+14)(6) = 66$ unit^2

Shaded region = 66 – 2 – 40.449 = 23.6 unit^2 (3sf) [A1]

10 A particle starts from rest, travels in a straight line so that t is the time in seconds after passing through a fixed point O. Its velocity, v m/s, is given by $v = 2t^2 - 15t + 18$.

(i) Find the acceleration(s) of the particle when it comes to instantaneous rest. [3]

$v = 0$

$(t - 6)(2t - 3) = 0$ [M1] concept of $v = 0$

$t = 6s \text{ or } t = 1.5s$

Downloaded from SingaKids.com

$$a = \frac{dv}{dt}$$

$a = 4t - 15\ m/s^2$ [M1]

$$at\ t = 6 \quad acceleration = 4(6) - 15 = 9m/s^2$$

$$at\ t = 1.5 \quad acceleration = 4(1.5) - 15 = -9\ m/s^2$$ A1

(ii) Find the minimum velocity of the particle. [3]

$$For\ \min velocity, \quad \frac{dv}{dt} = 0$$

$4t - 15 = 0$ [M1] concept of $\frac{dv}{dt}$ or $4t - 15 = 0$

$$t = 3.75s$$

$v = 2(3.75)^2 - 15(3.75) + 18 = -10.125$ m/s A1

$\frac{d^2y}{dx^2} = 4 > 0$

Therefore, v = -10.125 is minimum. [B1]

(iii) Calculate the average speed in the first 6 seconds. [5]

$$s = \int v\ dt$$

$$= \int 2t^2 - 15t + 18\ dt$$

$$= \frac{2}{3}t^3 - \frac{15}{2}t^2 + 18t + c$$ M1

$$when\ t = 0, s = 0 \quad \Rightarrow \quad c = 0$$

$$\therefore \quad s = \frac{2}{3}t^3 - \frac{15}{2}t^2 + 18t$$ A1

$$when\ \ t = 1.5, \ \ s = \frac{2}{3}(1.5)^3 - \frac{15}{2}(1.5)^2 + 18(1.5) = 12.375$$ M1

Downloaded from SingaKids.com

$$when\ t = 6,\ s = \frac{2}{3}(6)^3 - \frac{15}{2}(6)^2 + 18(6) = -18$$ M1

$$Total\ distance\ = 12.375 + 12.375 + 18 = 42.75\ m$$

Average speed = $\frac{42.75}{6}$ =7.125 m/s [A1]

11 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{x^b}{10^a}$, where a and b are constants. Measured values of x and y are given in the following table.

x	2	3	4	5
y	0.08	0.30	0.64	1.25

(i) Plot $\lg y$ against $\lg x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) By drawing a suitable line on your graph, solve the equation $y = x^{-2}$. [3]

--End of Paper--

Downloaded from SingaKids.com

Downloaded from SingaKids.com