1 It is given that $f(x) = ax^3 + 4a^2 - 2ax + 1$, where a is a constant. When $f(x)$ is divided by $x - 2a$, the remainder is $\frac{3}{2}$. Find two possible values of a. [4]

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2 On a school campus of 4000 students, one student was infected with a contagious virus. The spread of the virus through the students is given by $N = \frac{4000}{1+3999e^{-0.6t}}$, where N is the total number of students infected after t days.

After how many days will 40% of the population be infected? [4]

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3 A particle moves along the curve $y = \sqrt{3x - 2}$ in such a way that y-coordinate of the particle is increasing at a constant rate of 3 units per second. Find the y-coordinate of the particle at the instant that the x-coordinate of the particle is increasing at 4 units per second. [5]

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remove space

4 **(i)** Differentiate $(x^2+1)\ln(x^2+1)$ with respect to x. [3]

(ii) Hence find $\int x\ln(x^2+1)\,dx$. [2]

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5 (i) Given that $u = 3^t$, express $3^{t+1} + 3^{2-t} + 28 = 0$ as a quadratic equation in u.

(ii) Hence explain why there is no solution to $3^{t+1} + 3^{2-t} + 28 = 0$.

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6 **Do not use a calculator in this question.**

Solve the quadratic equation $\left(\sqrt{5}+3\right)x^2+3x+\left(\sqrt{5}-3\right)=0$, giving your answer in the form of $a+b\sqrt{5}$, where a and b are constants. [6]

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7 A curve has the equation $y = \cos x + \frac{1}{2}\sin 2x$ [for $0 < x < \frac{\pi}{2}$.] ✓ Removed

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Find, in exact form, the coordinates of the stationary point of the curve. for $0 < x < \frac{\pi}{2}$ [4]

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8 The equation of a curve is $y = 3x^2 + kx + k - 6$, where k is a constant.

(i) In the case where $k = 4$, find the set of values of x for $y + 3 \geq 0$. [3]

(ii) Show that, for all values of k, the curve cuts the x-axis at two distinct points. [3]

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9 **(i)** Sketch the parabola $y^2 = -4x$, for $-4 \leq x \leq 0$. [2]

(ii) The line $y = 2x + 4$ cuts the parabola $y^2 = -4x$ at the points A and B.
Find the coordinates of point A and point B. [4]

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10 **(a)** Solve $\log_x 16 = \log_2 x$. [4]

(b) Given that $\log_5 a = m$, express the following in terms of m.

(i) a^2 [1]

(ii) $\log_5\left(\frac{25}{a}\right)$ [2]

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11 **(i)** Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$. [4]

(ii) Hence solve the equation $\tan \frac{1}{2}x + \cot \frac{1}{2}x = 2\sqrt{2}$, for $0 \leq x \leq \pi$.
Leave your answer in terms of π. [5]

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12 A particle, travelling in a straight line, passes a fixed point O with velocity of -2 m/s. The acceleration, a m/s^2, of the particle, t seconds after passing O, is given by $a = \dfrac{12}{(4t+1)^2}$.

(i) Show that the particle comes instantaneously to rest when $t = 0.5$. [4]

(ii) Find the total distance travelled by the particle for the first 2 seconds. [4]

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13 The roots of the quadratic equation $2x^2 + 4x - 1 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Show that $\alpha + \beta = 4$. [3]

(ii) Find the quadratic equation with roots $\frac{2\alpha^2}{\beta}$ and $\frac{2\beta^2}{\alpha}$. [7]

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1 The equation of a curve is $y = \frac{3x+5}{x-4}$, where $x \neq 4$.

(i) Obtain an expression for $\frac{dy}{dx}$ and explain why y is always a decreasing function. [4]

(ii) By considering $\frac{d^2y}{dx^2}$, find the range of x for which $\frac{dy}{dx}$ is an increasing function. [3]

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2 **(i)** Show that $2x-1$ is a factor of $2x^3-x^2+18x-9$. [1]

(ii) Hence, explain why there is only one real root of the equation $2x^3-x^2+18x-9=0$. [2]

(iii) Express $\dfrac{7x+15}{2x^3-x^2+18x-9}$ in partial fractions. [5]

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3 **(a)** There is no term in x^2 in the expansion of $\left(1+kx+2x^2\right)\left(1-\frac{x}{2}\right)^6$. Find the value of constant k. [5]

(b) Find the term independent of x in the expansion of $\left(\frac{1}{2x^3}+3x^2\right)^{10}$. [3]

[Turn Over

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4 **(a)** It is given that $f(x) = k + 3\cos 2x$, where a is an integer. The point $\left(30°, \frac{1}{2}\right)$ lies on the graph.

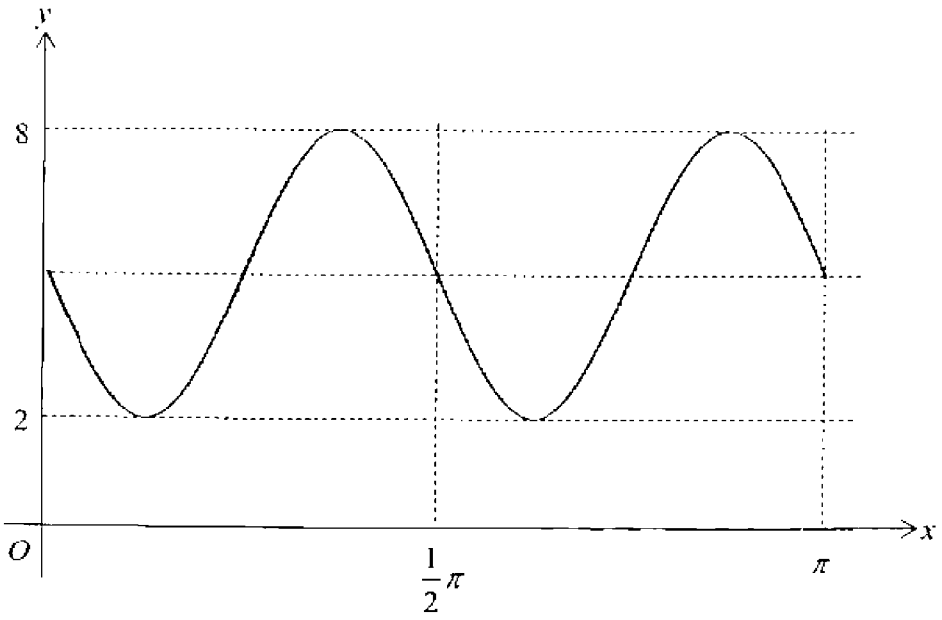
(i) Find the value of k. [1]

(ii) State the period and the amplitude of $f(x)$. [2]

(iii) Sketch the graph of $f(x)$ for $0° \leq x \leq 225°$. [3]

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(b)

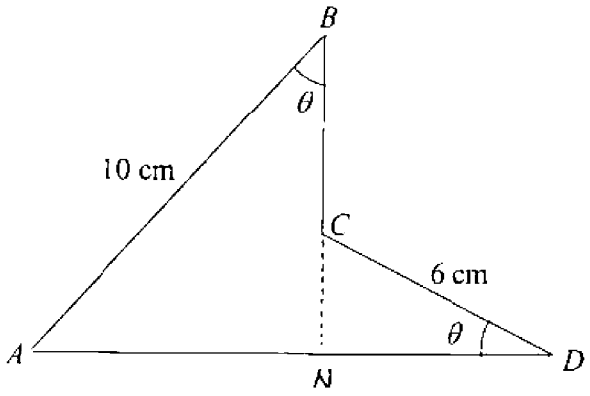


The diagram shows a sketch of the curve $y = a\sin(bx) + c$, for $0 \le x \le \pi$.

Find the value of a, b and c. [3]

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5



The diagram shows a quadrilateral $ABCD$. AD is a horizontal line and BC is a vertical line. It is given that angle ABC = angle ADC = $\theta°$, AB = 10 cm and CD = 6 cm. The perimeter of the quadrilateral is P cm.

(i) Show that $P = 16 + 4\sin\theta + 16\cos\theta$. [3]

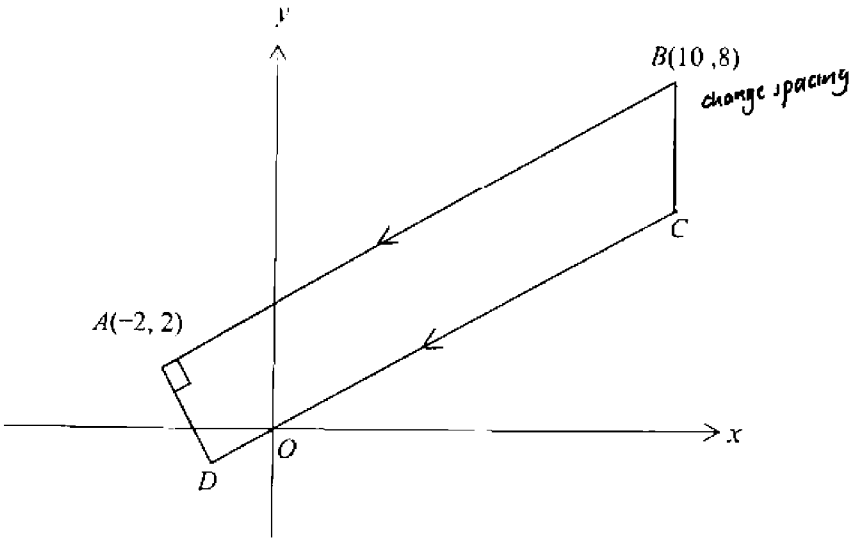
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(ii) Express P in terms of $k + R\cos(\theta + \alpha)$, where k is an integer, $R > 0$ and α is an acute angle. [4]

(iii) Find the value of θ for the pe[illegible]

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6



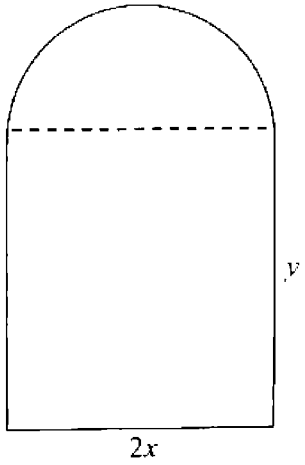
The diagram shows a trapezium $ABCD$ with vertices $A(-2, 2)$, $B(10, 8)$, C and D. The side AB is parallel to DC and the side BC is parallel to the y-axis. Angle $BAD = 90°$ and CD passes through the origin O.

(i) Find the coordinates of C and of D. [7]

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(ii) Hence find the area of trapezium $ABCD$. [2]

7



The diagram shows a window that is constructed in the shape of semicircle and rectangle. The perimeter of the window is 5 m. The length of the rectangle is $2x$ m and the breadth of the rectangle is y m.

(i) Express y in terms of x. [2]

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(ii) Show that the area of the window is $5x - \left(\frac{4+\pi}{2}\right)x^2$. [3]

(iii) Given that x can vary, find the value of x for which the area of the window is stationary. [4]

(iv) Explain why this value of x gives the largest area possible. [1]

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8 The points $A(-3, 0)$, $B(-2, 3)$ and $C(1, 2)$ lie on a circle.

(i) Prove that AC is a diameter of this circle. [3]

(ii) Find the equation of the circle. [3]

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(iii) The tangent to the circle at A cuts the y-axis at D. Find the coordinates of D. [3]

(iv) Find the coordinates of the point on the circle which is the furthest from the x-axis. [1]

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9 A company produces a disinfectant that destroys germs. After n minutes, there are y germs that are still alive. The table below shows experimental values of two variables y and n. One of the y-values has been recorded incorrectly.

n (minutes)	1	2	3	4	5
y (germs)	75	56	36	32	24

It is known that y and n are connected by the equation $y = ab^n$.

(i) On the graph paper on the next page, plot lg y against n and draw a straight line graph. The vertical lg y-axis should start at 1.3 and have a scale of 2 cm to 0.1. [3]

(ii) From your graph,

(a) identify the incorrect record and estimate its correct value of y, [2]

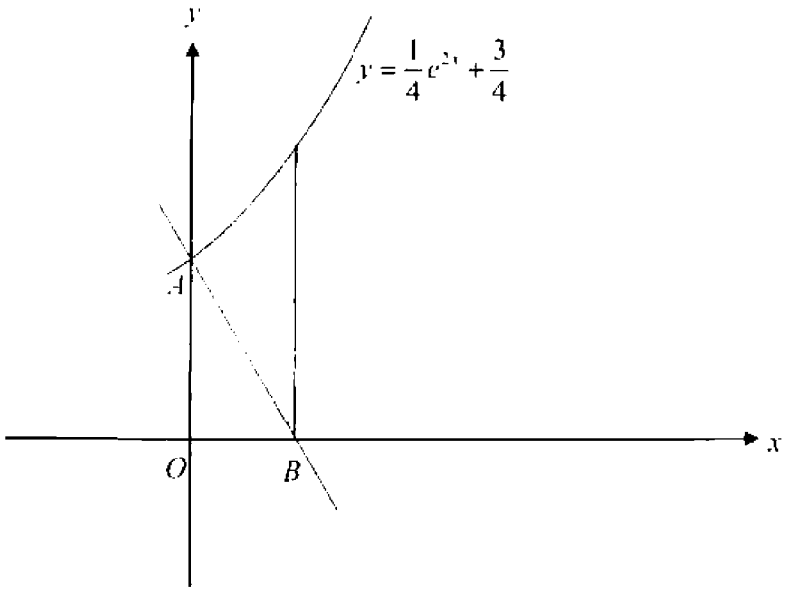
(b) find the value of a and b. [3]

(iii) After how many minutes the number of germs is half of its original number? [2]

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10



The diagram shows the graph of the curve $y = \frac{1}{4}e^{2x} + \frac{3}{4}$. The curve meets the y-axis at A.

The normal to the curve at A meets the x-axis at the point B.

Find the area of the shaded region enclosed by the curve, the line AB and the line through B parallel to the y-axis. Leave your answer in terms of e.

Show your working clearly. [10]

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[Space for Question 10]

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(i) Sketch the graph $y = |x^2 + 2x - 24|$, showing the coordinates of the turning point and the points where the curve meets the axes clearly. [4]

(ii) Find the range of values of k such that $|x^2+2x-24| = k$ has 4 solutions [1]

(iii) (ii) Solve $|x^2 + 2x - 24| = 24$. [3]

(iv) (iii) Hence solve $|x^2 + 2x - 24| < 24$. [2]

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1 It is given that $f(x) = ax^3 + 4a^2 - 2ax + 1$, where a is a constant. When $f(x)$ is divided by $x - 2a$, the remainder is $\frac{3}{2}$. Find two possible values of a. [4]

$f(2a) = a(2a)^3 + 4a^2 - 2a(2a) + 1$ [M1 : substituting $x = 2a$]

$= 8a^4 + 4a^2 - 4a^2 + 1$

$= 8a^4 + 1$ [B1]

$8a^4 + 1 = \frac{3}{2}$ [M1: understand that $f(2a)$ is remainder] ✓

$8a^4 = \frac{1}{2}$

$a^4 = \frac{1}{16}$

$a = \frac{1}{2}$ or $a = -\frac{1}{2}$ [A1]

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2 On a school campus of 4000 students, one student was infected with a contagious virus. The spread of the virus through the students is given by $N = \frac{4000}{1+3999e^{-0.6t}}$, where N is the total number of students infected after t days.

After how many days will 40% of the population be infected? [4]

40% of population = $\frac{40}{100} \times 4000 = 1600$ [B1]

$1600 = \frac{4000}{1+3999e^{-0.6t}}$

$1+3999e^{-0.6t} = \frac{4000}{1600}$ OR $1600(1+3999e^{-0.6t}) = 4000$

$1600 + 6398400e^{-0.6t} = 4000$

$1+3999e^{-0.6t} = 2.5$

$e^{-0.6t} = \frac{1}{2666}$

$e^{-0.6t} = \frac{1.5}{3999}$ [M1: attempt to isolate $e^{-0.6t}$] ✓

$-0.6t = \ln\left(\frac{1.5}{3999}\right)$ [M1: ln both sides] ✓

$t = 13.147$

Do not accept 13 or 13.1

∴ After 14 days [A1]

(accept 13.2)

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3 A particle moves along the curve $y = \sqrt{3x-2}$ in such a way that y-coordinate of the particle is increasing at a constant rate of 3 units per second. Find the y-coordinate of the particle at the instant that the x-coordinate of the particle is increasing at 4 units per second. [5]

$\frac{dy}{dx} = \frac{1}{2}(3x-2)^{-\frac{1}{2}} \times 3$

$= \frac{3}{2}(3x-2)^{-\frac{1}{2}}$ [B1]

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$3 = \frac{3}{2\sqrt{3x-2}} \times 4$ [M1] √

$3 = \frac{6}{\sqrt{3x-2}}$

$\sqrt{3x-2} = 2$

$3x-2 = 4$ [M1] √ square both sides

$x = 2$ [A1]

$\therefore y = \sqrt{3(2)-2}$

$= 2$ [A1]

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remove space

4 **(i)** Differentiate $(x^2+1)\ln(x^2+1)$ with respect to x. [3]

$u' = 2x$

$v' = \frac{1}{x^2+1} \times 2x = \frac{2x}{x^2+1}$ } [B1]

$\frac{dy}{dx} = u'v + v'u$

$= 2x \ln(x^2+1) + \frac{2x}{x^2+1} \times x^2+1$ [M1] ✓

$= 2x \ln(x^2+1) + 2x$ [A1]

remove space

(ii) Hence find $\int x\ln(x^2+1)\,dx$. [2]

$\int 2x \ln(x^2+1) + 2x \; dx = (x^2+1)\ln(x^2+1)$

$\int 2x \ln(x^2+1)\,dx + \int 2x \; dx = (x^2+1)\ln(x^2+1)$ [M1]

$\int 2x \ln(x^2+1)\,dx = (x^2+1)\ln(x^2+1) - x^2$

$\int x \ln(x^2+1)\,dx = \frac{1}{2}\left[(x^2+1)\ln(x^2+1) - x^2\right] + c$ [A1]

No 'A1' if no '+c'

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5. i)

$3^t \times 3 + 3^2 / 3^t + 28 = 0$

$3u + \frac{9}{u} + 28 = 0$

$3u^2 + 9 + 28u = 0$

ii)

$3u^2 + 28u + 9 = 0$

$(3u + 1)(u + 9) = 0$

$u = \frac{1}{3}$, $u = -9$

$3^t = \frac{1}{3}$, $3^t = -9$

No solution as 3^t can't be negative

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6 Do not use a calculator in this question.

Solve the quadratic equation $\left(\sqrt{5}+3\right)x^2+3x+\left(\sqrt{5}-3\right)=0$, giving your answer in the form of $a+b\sqrt{5}$, where a and b are constants. [6]

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a, b and c

$$x = \frac{-3 \pm \sqrt{3^2-4(\sqrt{5}+3)(\sqrt{5}-3)}}{2(\sqrt{5}+3)}$$

[M1: substitute into formula]

$$= \frac{-3 \pm \sqrt{9-4[5-9]}}{2\sqrt{5}+6}$$

[M1]

$$= \frac{-3 \pm \sqrt{25}}{2\sqrt{5}+6} = \frac{-3 \pm 5}{2\sqrt{5}+6}$$

[M1]

[M1: attempt to rationalise denominator]

$$\frac{2}{2\sqrt{5}+6} \times \frac{2\sqrt{5}-6}{2\sqrt{5}-6} \quad \longleftrightarrow \quad \frac{-8}{2\sqrt{5}+6} \times \frac{2\sqrt{5}-6}{2\sqrt{5}-6}$$

$$= \frac{4\sqrt{5}-12}{-16} \qquad = \frac{-16\sqrt{5}+48}{-16}$$

$$= -\frac{1}{4}\sqrt{5}+\frac{3}{4} \text{ [A1]} \qquad = \sqrt{5}-3 \text{ [A1]}$$

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7 A curve has the equation $y = \cos x + \frac{1}{2}\sin 2x$ [for $0 < x < \frac{\pi}{2}$.] ✓ Removed

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

$\frac{dy}{dx} = -\sin x + \cos 2x$

[B1] [B1]

(ii) Find, in exact form, the coordinates of the stationary point of the curve. for $0 < x < \frac{\pi}{2}$ [4]

$-\sin x + \cos 2x = 0$ [M1: $\frac{dy}{dx} = 0$]

$-\sin x + 1 - 2\sin^2 x = 0$

$2\sin^2 x + \sin x - 1 = 0$ [M1: apply double-angle formulae]

$2A^2 + A - 1 = 0$

$(2A - 1)(A + 1) = 0$

$A = \frac{1}{2}$ $A = -1$

<rej>

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ [A1] $y = \cos\frac{\pi}{6} + \frac{1}{2}\sin\frac{\pi}{3}$

$= \frac{1}{2}\sqrt{3} + \frac{1}{2} \times \frac{1}{2}\sqrt{3}$

$= \frac{3}{4}\sqrt{3}$

$\left(\frac{\pi}{6}, \frac{3}{4}\sqrt{3}\right)$ [B1]

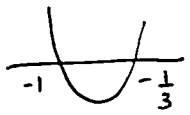
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8 The equation of a curve is $y = 3x^2 + kx + k - 6$, where k is a constant.

(i) In the case where $k = 4$, find the set of values of x for $y + 3 \geq 0$. [3]

$3x^2 + 4x - 2 + 3 \geq 0$ [B1 : subs. $k=4$]

$3x^2 + 4x + 1 \geq 0$

$(3x+1)(x+1) \geq 0$ OR  [M1]

$x \leq -1$ or $x \geq -\frac{1}{3}$ [A1]

(ii) Show that, for all values of k, the curve cuts the x-axis at two distinct points. [3]

$3x^2 + kx + k - 6 = 0$

$D = k^2 - 4(3)(k-6)$ [M1]

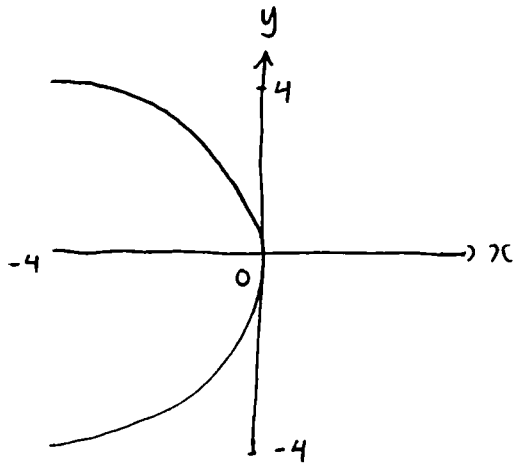
$= k^2 - 12k + 72$

$= (k-6)^2 + 36$ [A1]

Since $D > 0$ for any k, the curve cuts x-axis at two distinct points

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9 **(i)** Sketch the parabola $y^2 = -4x$, for $-4 \le x \le 0$. [2]



B1: shape

B1: endpoints and label x-y axis

(ii) The line $y = 2x + 4$ cuts the parabola $y^2 = -4x$ at the points A and B.
Find the coordinates of point A and point B. [4]

$(2x+4)^2 = -4x$ [B1]

$4x^2 + 16x + 16 = -4x$

$4x^2 + 20x + 16 = 0$ [M1: Correct expansion and simplification]

(-1) for wrong factorisation

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -4$ or $x = -1$ [A1]

A $(-4, -4)$ B $(-1, 2)$ [B1]

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10 **(a)** Solve $\log_x 16 = \log_2 x$. [4]

$\frac{\log_2 16}{\log_2 x} = \log_2 x$ [M1: change to base 2]

$\frac{4}{\log_2 x} = \log_2 x$ [B1]

$A^2 = 4$

$A = 2$ or -2

$\log_2 x = 2$ or $\log_2 x = -2$

$x = 4$ [A1] $\quad x = \frac{1}{4}$ [A1]

or $\log_x 16 = \frac{\log_2 x}{\log_2 2}$ [M1 change to base 2]

$4 \log_x 2 = \frac{1}{\log_x 2}$ [B1]

$B^2 = \frac{1}{4}$

$B = \frac{1}{2}$ or $-\frac{1}{2}$

$\log_x 2 = \frac{1}{2}$ or $\log_x 2 = -\frac{1}{2}$

$\sqrt{x} = 2$ $\quad x^{-\frac{1}{2}} = 2$

$x = 4$ [A1] $\quad x = \frac{1}{4}$ [A1]

(b) Given that $\log_5 a = m$, express the following in terms of m.

(i) a^2 [1]

5^{2m} or 25^m [B1]

(ii) $\log_5\left(\frac{25}{a}\right)$ [2]

$\log_5 25 - \log_5 a$ [M1]

$2 - m$ [A1]

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11 (i) Prove that $\tan x + \cot x = 2 \operatorname{cosec} 2x$. [4]

LHS $= \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}$ [B1: change to $\sin x$ and $\cos x$] or $\tan x + \dfrac{1}{\tan x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}$ [B1: combine two fractions] $= \dfrac{\tan^2 x + 1}{\tan x}$ [B1: combine two fractions]

$= \dfrac{1}{\frac{1}{2} \times 2 \sin x \cos x}$ ✓ [B1] $= \dfrac{\sec^2 x}{\tan x}$ ✓ [B1]

[B1] $= \begin{cases} \dfrac{2}{\sin 2x} = 2 \operatorname{cosec} 2x = \text{RHS} \\ \dfrac{1}{\cos^2 x \times \frac{\sin x}{\cos x}} \quad [B1] \\ = \dfrac{1}{\sin x \cos x} \end{cases}$

(ii) Hence solve the equation $\tan \frac{1}{2}x + \cot \frac{1}{2}x = 2\sqrt{2}$, for $0 \le x \le \pi$. [5]

Leave your answer in terms of π.

[B1]

$2 \operatorname{Cosec} x = 2\sqrt{2}$

$\operatorname{Cosec} x = \sqrt{2}$

$\sin x = \frac{1}{\sqrt{2}}$ [M1] ✓ understanding of $\operatorname{Cosec} x = \frac{1}{\sin x}$

$\alpha = \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$ [A1]

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12 A particle, travelling in a straight line, passes a fixed point O with velocity of -2 m/s. The acceleration, a m/s^2, of the particle, t seconds after passing O, is given by $a = \dfrac{12}{(4t+1)^2}$.

(i) Show that the particle comes instantaneously to rest when $t = 0.5$. [4]

$v = \int \frac{12}{(4t+1)^2}\,dt$

$v = \int 12(4t+1)^{-2}\,dt$

$v = \frac{12(4t+1)^{-1}}{-1\times 4} = \frac{-3}{4t+1} + c$ ✓ [B1]

when $t=0$, $v=-2$ $\quad -2 = -\frac{3}{1} + c$ [M1] ✓

$c = 1$

$\therefore \quad v = \frac{-3}{4t+1} + 1$

$v = 0$ [M1] ✓

$\frac{3}{4t+1} = 1$ ✓

$3 = 4t+1$ } [A1]

$4t = 2$

$t = \frac{1}{2}$ (shown)

OR

$t = 0.5$

$v = \frac{-3}{4(0.5)+1} + 1$ [M1]

$= \frac{-3}{3} + 1$

$v = 0$ (AG)

(ii) Find the total distance travelled by the particle for the first 2 seconds. [4]

$s = \int \frac{-3}{4t+1} + 1\,dt$

$s = \frac{-3\ln(4t+1)}{4} + t + c$ ... [B1]

when $t=0$, $s=0$

$0 = \frac{-3\ln 1}{4} + 0 + c$

$c = 0$ [B1]

t	s
0	0
$\frac{1}{2}$	−0.32396
2	0.35208

for $t = \frac{1}{2}$

for $t = 2$

Total distance

$= 0.32396 + 0.67604$

$=$ 1 metre [A1]

Accept 1.00 metre

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13 The roots of the quadratic equation $2x^2+4x-1=0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(i) Show that $\alpha+\beta=4$. [3]

$\frac{1}{\alpha}+\frac{1}{\beta}=-\frac{4}{2}$ } [B1] $\rightsquigarrow$ $\frac{\alpha+\beta}{\alpha\beta}=-2$

$\frac{1}{\alpha}\times\frac{1}{\beta}=-\frac{1}{2}$

$\alpha\beta=-2$ M1

$\frac{\alpha+\beta}{-2}=-2$ } [A1]

$\alpha+\beta=4$

(ii) Find the quadratic equation with roots $\frac{2\alpha^2}{\beta}$ and $\frac{2\beta^2}{\alpha}$. [7]

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=4^2-2(-2)$ [M1]√

$=20$

$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$

$=4(20-(-2))$ [M1]√

$=88$

SUM: $\frac{2\alpha^2}{\beta}+\frac{2\beta^2}{\alpha}=\left\{\frac{2\alpha^3+2\beta^3}{\alpha\beta}=\frac{2(\alpha^3+\beta^3)}{\alpha\beta}\right.$ [M1]

$\overset{[M1]\surd}{=}\frac{2\times88}{-2}=-88$

B1: attempt to find new sum and product

PRODUCT: $\frac{2\alpha^2\times2\beta^2}{\alpha\beta}=4\alpha\beta=\underbrace{4\times-2}_{[M1]}=-8$

$\therefore\ x^2+88x-8=0$ [A1]√ based on sum and product

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1 The equation of a curve is $y = \frac{3x+5}{x-4}$, where $x \neq 4$.

(i) Obtain an expression for $\frac{dy}{dx}$ and explain why y is always a decreasing function. [4]

$u' = 3 \quad v' = 1$

$\frac{dy}{dx} = \frac{3(x-4) - (3x+5)}{(x-4)^2}$ [M1]

$= \frac{3x - 12 - 3x - 5}{(x-4)^2}$

$= \frac{-17}{(x-4)^2}$ [A1]

As $(x-4)^2 > 0$ [B1], $\frac{-17}{(x-4)^2} < 0$. Since $\frac{dy}{dx} < 0$, y is decreasing function i.e. s.o.i

(ii) By considering $\frac{d^2y}{dx^2}$, find the range of x for which $\frac{dy}{dx}$ is an increasing function. [3]

$\frac{dy}{dx} = -17(x-4)^{-2}$

$\frac{d^2y}{dx^2} = \frac{34}{(x-4)^3}$ [B1]

$\frac{34}{(x-4)^3} > 0$ [M1] $\surd$ $\frac{d^2y}{dx^2} > 0$

$(x-4)^3 > 0$

$x - 4 > 0$

$x > 4$ [A1]

[Turn Over

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2 **(i)** Show that $2x-1$ is a factor of $2x^3-x^2+18x-9$. [1]

when $x = \frac{1}{2}$, $2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 18\left(\frac{1}{2}\right) - 9 = 0$ (B1)

Since remainder is zero, $2x-1$ is a factor

(ii) Hence, explain why there is only one real root of the equation $2x^3-x^2+18x-9=0$. [2]

$$\begin{array}{r|l} & x^2+9 \\ \hline 2x-1 & 2x^3-x^2+18x-9 \\ & 2x^3-x^2 \\ \hline & 18x-9 \\ & 18x-9 \\ \hline & 0 \end{array}$$

(B1)

$(2x-1)(x^2+9)=0$

$x=\frac{1}{2}$ $x^2+9=0$

no solution (B1)

$\therefore$ only one real root

(iii) Express $\frac{7x+15}{2x^3-x^2+18x-9}$ in partial fractions. [5]

$$\frac{7x+15}{(2x-1)(x^2+9)} = \frac{A}{2x-1} + \frac{Bx+C}{x^2+9} \quad (B1)$$

$7x+15 = A(x^2+9) + (Bx+C)(2x-1)$ [multiply by (B1)]

$x=\frac{1}{2}$ $18.5 = A(9.25)$

$A=2$ (A1)

$x=0$ $15 = 2(9) + C(-1)$

$15 = 18 - C$

$C=3$ (A1)

$x=1$ $22 = 10(2) + (B+3)$

$22 = 20 + B + 3$

$B=-1$ (B1)

$\therefore \frac{7x+15}{2x^3-x^2+18x-9} = \frac{2}{2x-1} + \frac{-x+3}{x^2+9}$

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3 **(a)** There is no term in x^2 in the expansion of $\left(1+kx+2x^2\right)\left(1-\frac{x}{2}\right)^6$. Find the value of constant k. [5]

$$\left(1+kx+2x^2\right)\left(1-\binom{6}{1}\left(\frac{x}{2}\right)+\binom{6}{2}\left(\frac{x}{2}\right)^2-\ldots\right)$$

$$\left(1+kx+2x^2\right)\left(1-6\left(\frac{x}{2}\right)+15\times\frac{x^2}{4}\right)$$

$$\left(1+kx+2x^2\right)\left(\overbrace{1-3x}^{\text{[B1]}}+\overbrace{\frac{15}{4}x^2}^{\text{[B1]}}-\ldots\right)$$

Coefficient of x^2: $\frac{15}{4}-3k+2=0$ [for understanding the meaning of 2 terms]

M1✓ finding coefficient of x^2

$5.75=3k$

$k=\frac{23}{12}$ [A1]

(b) Find the term independent of x in the expansion of $\left(\frac{1}{2x^3}+3x^2\right)^{10}$. [3]

$$T_{r+1}=\binom{10}{r}\left(\frac{1}{2x^3}\right)^{10-r}(3x^2)^r$$

[M1] $\frac{x^{2r}}{x^{30-3r}}=x^{2r-(30-3r)}$

$=x^{5r-30}$

$5r-30=0$

$r=6$

$T_7=\binom{10}{6}\left(\frac{1}{2x^3}\right)^4(3x^2)^6$ } [A1: finding T_7]

$=210\times\frac{1}{16}\times729$

$=9568.125$

[Turn Over

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4 **(a)** It is given that $f(x) = k + 3\cos 2x$, where a is an integer. The point $\left(30°, \frac{1}{2}\right)$ lies on the graph.

(i) Find the value of k. [1]

$\frac{1}{2} = k + 3\cos 60°$

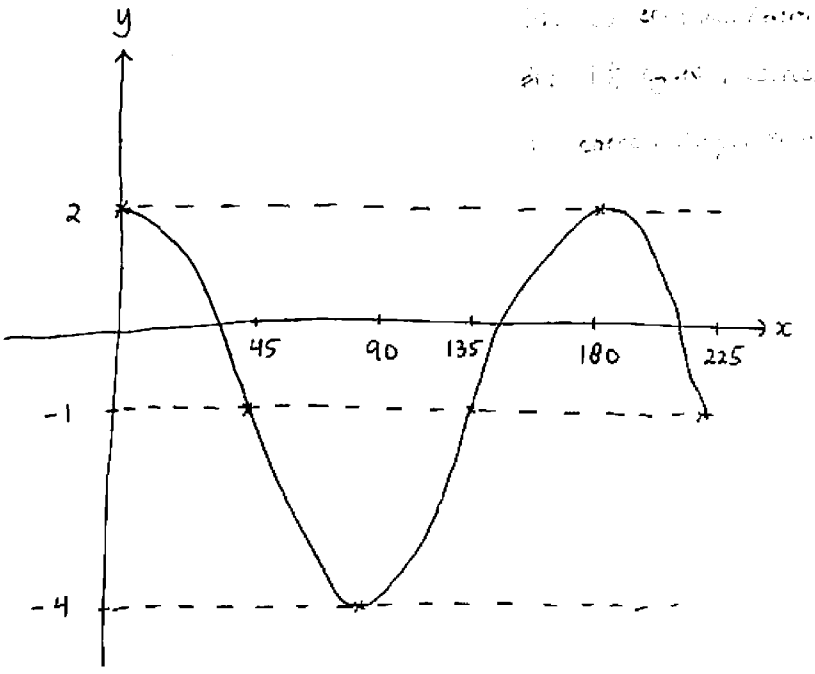
$k = -1$ [B1]

(ii) State the period and the amplitude of $f(x)$. [2]

period = $180°$ [B1]

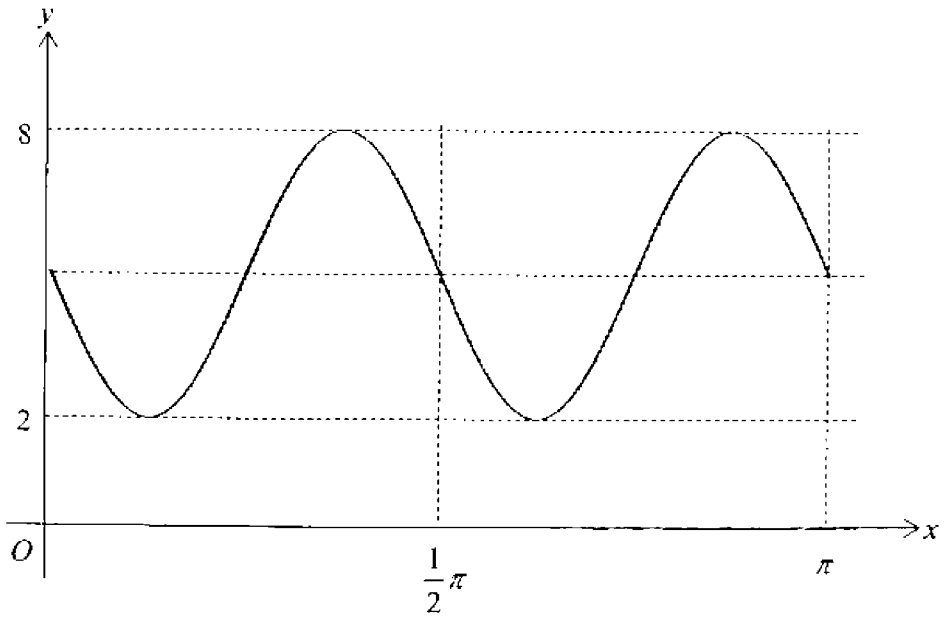
amplitude = 3 [B1]

(iii) Sketch the graph of $f(x)$ for $0° \leq x \leq 225°$. [3]



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(b)



The diagram shows a sketch of the curve $y = a\sin(bx) + c$, for $0 \le x \le \pi$.

Find the value of a, b and c. [3]

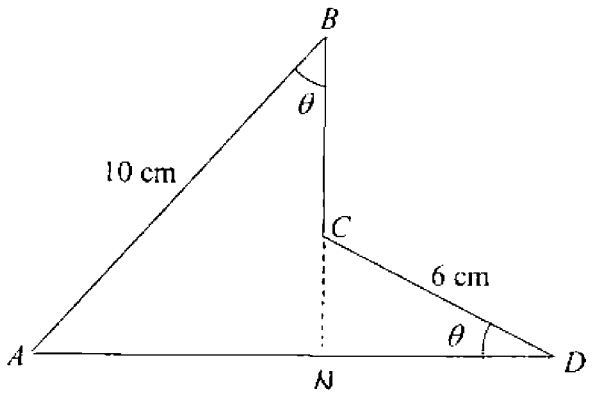
$c = 5$ [B1]

$a = -3$ [B1]

$b = 4$ [B1]

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5



The diagram shows a quadrilateral $ABCD$. AD is a horizontal line and BC is a vertical line. It is given that angle ABC = angle ADC = $\theta°$, AB = 10 cm and CD = 6 cm. The perimeter of the quadrilateral is P cm.

(i) Show that $P = 16 + 4\sin\theta + 16\cos\theta$. [3]

$\cos\theta = \frac{BN}{10}$ $\qquad$ $\sin\theta = \frac{CN}{6}$

$BN = 10\cos\theta$ $\qquad$ $CN = 6\sin\theta$ [B1]

$\sin\theta = \frac{AN}{10}$ $\qquad$ $\cos\theta = \frac{DN}{6}$ [B1]

$AN = 10\sin\theta$ $\qquad$ $DN = 6\cos\theta$

$P = 10 + 10\sin\theta + 6\cos\theta + 6 + (10\cos\theta - 6\sin\theta)$

$= 16 + 16\cos\theta + 4\sin\theta$ (shown)

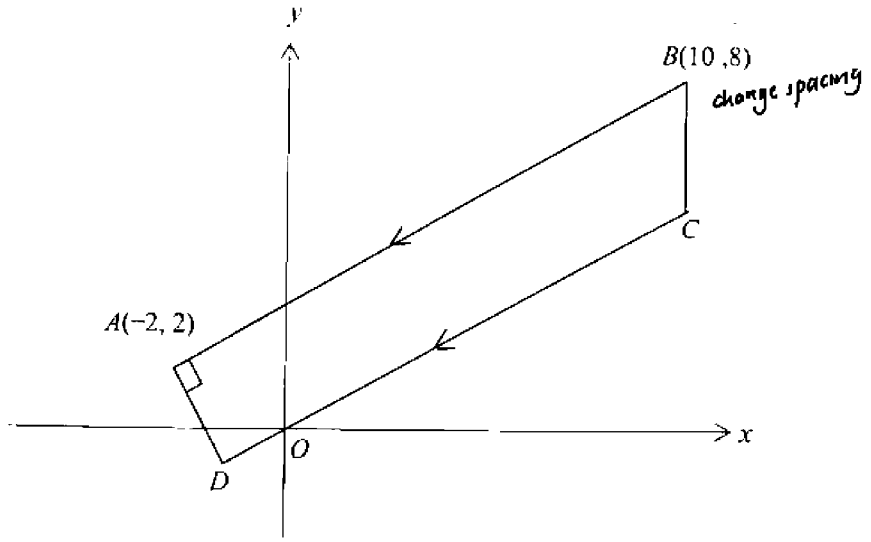
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(ii) Express P in terms of $k + R\cos(\theta - \alpha)$, where k is an integer, $R > 0$ and α is an acute angle. **[4]**

(iii) Find the value of θ for the point

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6



The diagram shows a trapezium $ABCD$ with vertices $A(-2, 2)$, $B(10, 8)$, C and D. The side AB is parallel to DC and the side BC is parallel to the y-axis. Angle $BAD = 90°$ and CD passes through the origin O.

(i) Find the coordinates of C and of D. [7]

$m_{AB} = \frac{8-2}{10-(-2)} = \frac{1}{2}$

$m_{CD} = \frac{1}{2}$ [B1]

Equation of CD : $y = \frac{1}{2}x$ [M1]

$y = \frac{1}{2}(10) = 5$

$\therefore$ C (10, 5) [A1]

OR

$m_{AD} = -2$

$D(h, \frac{1}{2}h)$

$\frac{\frac{1}{2}h - 2}{h+2} = -2$

$\frac{1}{2}h - 2 = -2h - 4$

$\frac{5}{2}h = -2$

$h = -\frac{4}{5}$ $\therefore$ $D(-\frac{4}{5}, -\frac{8}{5})$

$m_{AD} = -2$ [B1]

Equation of AD : $y = -2x + c$

$2 = -2(-2) + c$

$c = -2$

$y = -2x - 2$ [M1]

$-2x - 2 = \frac{1}{2}x$ [M1 : equating both equations]

$-2 = \frac{5}{2}x$

$x = -\frac{4}{5}$

$y = -\frac{2}{5}$

$\therefore$ $D(-\frac{4}{5}, -\frac{2}{5})$ [A1]

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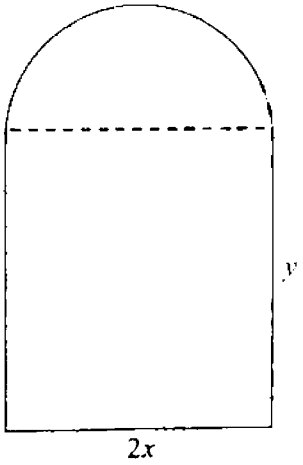
(ii) Hence find the area of trapezium $ABCD$. [2]

$$\text{Area} = \frac{1}{2}\left|\begin{matrix} -2 & -\frac{4}{5} & 10 & 10 & -2 \\ 2 & -\frac{2}{5} & 5 & 8 & 2 \end{matrix}\right|$$

$$= \frac{1}{2}\left(0.8 - 4 + 80 + 20 + \frac{8}{5} + \frac{20}{5} - 50 + 16\right) \quad \text{(M1)(A1)}$$

$$= 34.2 \text{ unit}^2 \quad \text{(A1)}$$

7



The diagram shows a window that is constructed in the shape of semicircle and rectangle. The perimeter of the window is 5 m. The length of the rectangle is $2x$ m and the breadth of the rectangle is y m.

(i) Express y in terms of x. [2]

$$2x + y + y + \frac{1}{2}\pi(2x) = 5 \quad \text{(M1)}$$

$$2x + 2y + \pi x = 5$$

$$2y = 5 - 2x - \pi x$$

$$y = \frac{5 - 2x - \pi x}{2} \quad \text{(A1)}$$

$$y = \frac{5}{2} - x - \frac{\pi}{2}x$$

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(ii) Show that the area of the window is $5x-\left(\frac{4+\pi}{2}\right)x^2$. [3]

Area $= 2xy + \frac{1}{2}\pi x^2$ (B1)

$= 2x\left(\frac{5}{2} - x - \frac{\pi}{2}x\right) + \frac{1}{2}\pi x^2$

$= 5x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$ (B1)

$= 5x - 2x^2 - \frac{1}{2}\pi x^2$

$= 5x - \left(2 + \frac{1}{2}\pi\right)x^2$ } (B1)

$= 5x - \left(\frac{4+\pi}{2}\right)x^2$

(iii) Given that x can vary, find the value of x for which the area of the window is stationary. [4]

$\frac{dA}{dx} = 5 - 2\left(\frac{4+\pi}{2}\right)x$ (B1) (B1)

$= 5 - (4+\pi)x$

$\frac{dA}{dx} = 0$ (M1)

$(4+\pi)x = 5$

$x = \frac{5}{4+\pi} = 0.700$ (A1)

(iv) Explain why this value of x gives the largest area possible. [1]

$\frac{d^2A}{dx^2} = -2\left(\frac{4+\pi}{2}\right) < 0$ (B1)

maximum

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8 The points $A(-3, 0)$, $B(-2, 3)$ and $C(1, 2)$ lie on a circle.

(i) Prove that AC is a diameter of this circle. [3]

$m_{AB} = \frac{3-0}{-2-(-3)} = 3$ [B1]

$m_{BC} = \frac{2-3}{1-(-2)} = -\frac{1}{3}$ [B1]

Since $m_{AB} \times m_{BC} = -1$, $\angle ABC = 90°$ } [B1]

$\therefore$ AC is diameter

(ii) Find the equation of the circle. [3]

$M_{AC} = \left(\frac{-3+1}{2}, \frac{0+2}{2}\right)$ [M1]

$M(-1, 1)$

$AC = \sqrt{(-3-1)^2 + (0-2)^2}$ [M1]

$= \sqrt{16+4}$

$= \sqrt{20}$

Radius $= \frac{\sqrt{20}}{2}$

$(x+1)^2 + (y-1)^2 = \left(\frac{\sqrt{20}}{2}\right)^2$

$(x+1)^2 + (y-1)^2 = 5$

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(iii) The tangent to the circle at A cuts the y-axis at D. Find the coordinates of D. [3]

$m_{AC} = \frac{2-0}{1-(-3)} = \frac{2}{4} = \frac{1}{2}$ [B1]

grad of tangent = -2

Equation: $y = -2x + c$ [M1 : finding equation of normal]

$0 = -2(-3) + c$

$c = -6$

$\therefore$ $D(0, -6)$ [A1]

(iv) Find the coordinates of the point on the circle which is the furthest from the x-axis. [1]

$(-1, 1+\sqrt{5})$ [A1]

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9 A company produces a disinfectant that destroys germs. After n minutes, there are y germs that are still alive. The table below shows experimental values of two variables y and n. One of the y-values has been recorded incorrectly.

n (minutes)	1	2	3	4	5
y (germs)	75	56	36	32	24

It is known that y and n are connected by the equation $y = ab^n$.

(i) On the graph paper on the next page, plot lg y against n and draw a straight line graph. The vertical lg y-axis should start at 1.3 and have a scale of 2 cm to 0.1. [3]

(ii) From your graph,

(a) identify the incorrect record and estimate its correct value of y, [2]

Incorrect record: $y = 36$ [B1]

$\lg y = 1.63$

$y = 10^{1.63} = 43 \pm 1$ [B1]

(b) find the value of a and b. [3]

$y = ab^n$

$\lg y = \underbrace{\lg a}_{c} + n\underbrace{\lg b}_{m}$ [M1]

gradient: $\frac{1.88 - 1.38}{1 - 5} = -\frac{1}{8}$

$\lg a = 2$ $\quad \lg b = -\frac{1}{8}$

$a = 100$ [B1] $\quad b = 0.750$ [B1]

[accept between $10^{1.9}$ and $10^{2.1}$]

based on the found gradient

(iii) After how many minutes the number of germs is half of its original number? [2]

$y = 100(0.75)^n$

$50 = 100(0.75)^n$

$\frac{1}{2} = 0.75^n$

$\lg \frac{1}{2} = n \lg 0.75$

$n = 2.41$ minutes

Changed Question:

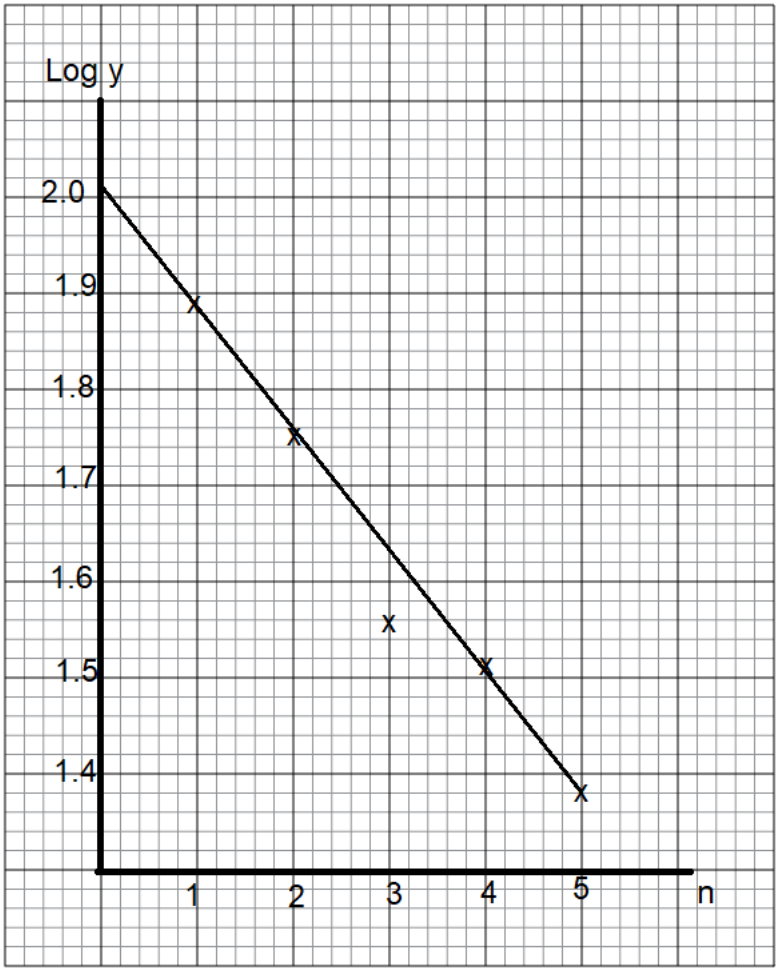
Using your graph, estimate after how many minutes would the number of germs be half of its original number?

$\lg 50 = 1.69897$ [M1]

Draw $y = \lg 50$

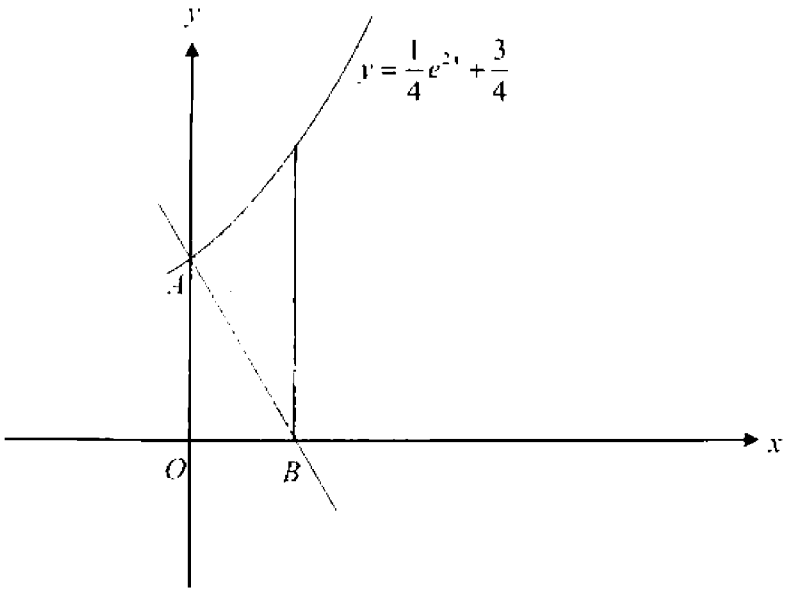
$n = 2.4$ [A1]

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10



The diagram shows the graph of the curve $y = \frac{1}{4}e^{2x} + \frac{3}{4}$. The curve meets the y-axis at A.

The normal to the curve at A meets the x-axis at the point B.

Find the area of the shaded region enclosed by the curve, the line AB and the line through B parallel to the y-axis. Leave your answer in terms of e.

Show your working clearly. [10]

A (0, 1) [B1]

$\frac{dy}{dx} = \frac{1}{2}e^{2x}$ [B1]

when $x = 0$, $\frac{dy}{dx} = \frac{1}{2}$

$\frac{1}{2} \times m_{normal} = -1$

$m_{normal} = -2$

$y = -2x + c$

$1 = c$

equation of normal: $y = -2x + 1$

when $y = 0$ $2x = 1$

$x = \frac{1}{2}$

$B(\frac{1}{2}, 0)$

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[Space for Question 10]

$$\int_0^{\frac{1}{2}} \frac{1}{4}e^{2x} + \frac{3}{4}\,dx$$ [M1: understanding to find area using integration]

$$\left[\underbrace{\frac{e^{2x}}{8} + \frac{3}{4}x}_{\text{B1 B1}}\right]_0^{\frac{1}{2}}$$

$$= \frac{e}{8} + \frac{3}{8} - \left(\frac{e^0}{8}\right)$$ [M1: Subs $x=\frac{1}{2}$ and $x=0$]

$$= \frac{3}{8} - \frac{1}{8} + \frac{e}{8}$$

$$= \frac{e}{8} + \frac{1}{4}$$

Area of $\triangle AOB = \frac{1}{2} \times \frac{1}{2} \times 1 = \frac{1}{4}$ [M1: ... coordinates of A and B]

$\therefore$ Shaded area $= \frac{e}{8} + \frac{1}{4} - \frac{1}{4}$

$= \frac{e}{8}$ units^2 [A1]

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(i) Sketch the graph $y = |x^2 + 2x - 24|$, showing the coordinates of the turning point and the points where the curve meets the axes clearly. [4]

$y = x^2 + 2x - 24$

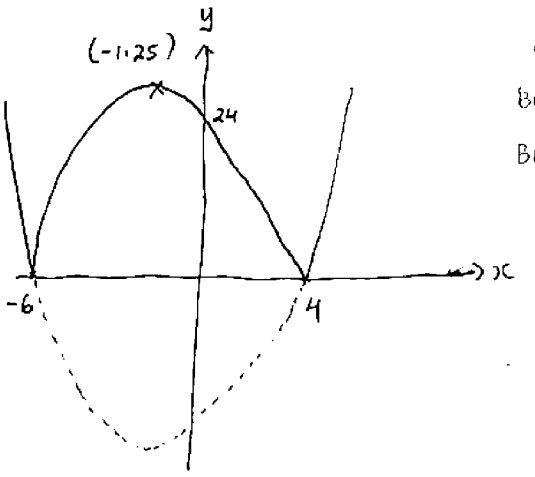
$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6$ or $x = 4$

Turning point $(-1, -25)$

y-int : -24



B1 : shape

B1 : turning point

B1 : y - intercept

M1 : finding x - intercepts

A1 : x - intercepts

(ii) Find the range of values of k such that $|x^2 + 2x - 24| = k$ has 4 solutions [1]

$0 < k < 25$ (B1)

(iii) (ii) Solve $|x^2 + 2x - 24| = 24$. [3]

$x^2 + 2x - 24 = 24$ or $x^2 + 2x - 24 = -24$ (M1)

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0$

$x = -8$ or 6 (A1)

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$ or -2 (A1)

$\therefore \quad x = -8, -2, 0, 6$

(iv) (iii) Hence solve $|x^2 + 2x - 24| < 24$. [2]

$-8 < x < -2$ or $0 < x < 6$ (B1)

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