1 Given that $\sqrt{216^x} = \frac{6^x}{6^{1-x}}$, find the value of $\sqrt{216^x}$. [4]

2 Given that $y = Me^x + Ne^{-5x}$, and that $\frac{dy}{dx} + 2y = e^x + e^{-5x}$, find the value of each of the constants M and N. [4]

3 The variables x and y are related in such a way that when $\frac{1}{y}$ is plotted against $\frac{1}{\sqrt{x}}$ a straight line is obtained.

It is given that $y = \frac{3}{5}$ when $x = 9$ and that $y = \frac{1}{4}$ when $x = \frac{1}{25}$.

Find the value of y when $x = 1$. [5]

4 **(a)** Show that the equation $x^2 + 2(p-2)x + (2p-5) = 0$ has real roots for all real values of p. [2]

(b) Explain why the quadratic expression $x^2 - 3x + 5$ is always positive. [3]

5 **(i)** Write down the first three terms in the expansion, in descending powers of x, of $\left(x^2 + \frac{m}{x}\right)^9$, where m is an integer. [1]

Given that the coefficient of x^3 in the expansion of $\left(x^2 + \frac{m}{x}\right)^9$ is -126,

(ii) show that $m = -1$. [2]

(iii) Hence, find the term independent of x in the expansion of $\left(3 - \frac{1}{x^3}\right)\left(x^2 + \frac{m}{x}\right)^9$. [4]

6 **(i)** Show that $4\cos^2 x - 2\sin^2 x = 3\cos 2x + 1$. [2]

Hence,

(ii) State the period and amplitude of $4\cos^2 x - 2\sin^2 x$, [2]

(iii) Sketch the graph of $y = 4\cos^2 x - 2\sin^2 x$ for $0 \le x \le 2\pi$. [3]

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7 **(a)** The roots of the quadratic equation $7x^2 + 5x - 4 = 0$ are α and β.

Find the quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [5]

(b) Show that the quadratic equation $2mx^2 + (8 - 4m)x + m + 1 = 0$ has real roots for $m \geq 9$. [3]

8 Water is poured, at a constant rate of 36π cm^3/s, into a hemispherical container of radius 9 cm.

When the depth of water is x cm, the volume, V cm^3, of water in the container is given by

$$V = \frac{1}{3}\pi x^2 (54 - x).$$

(i) Show that the time taken for the depth of water to reach 5 cm is 11.3 s. [3]

(ii) Find the rate of change of the depth of water at 11.3 s. [4]

9 **(a)** Express $\log_c(AB)$ in terms of p and q, given that

$p = \log_c A^3 B$ and $q = \log_c\left(\frac{B}{A^2}\right)$. [4]

(b) A cuboid box has a square base of length $\left(3 + 2\sqrt{3}\right)$ cm and its volume is $\left(150 + 87\sqrt{3}\right)$cm^3, find, in the form $p + q\sqrt{3}$, where p and q are integers,

(i) the height of the cuboid, [3]

(ii) the total length of its twelve edges. [1]

10 A curve is such that $\frac{dy}{dx} = \frac{x^2 + 4x - 2}{(x+2)^2}$.

(i) Given that the curve passes through the point $(0,3)$, find the equation of the curve. [3]

(ii) Find the x-coordinate of each of the stationary points of the curve. [2]

(iii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise, determine the nature of each stationary point. [3]

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11 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\frac{x}{2x+1}\right)\right]=\frac{1}{x(2x+1)}$. [3]

(ii) Hence, evaluate $\int_1^2 \frac{4x^2+3x+1}{x(2x+1)}\,\mathrm{d}x$. [5]

12 A particle travels along a straight line such that t is the time after passing through a fixed point O with velocity of v ms^{-1} given by $v=\frac{3t-2}{t+3}$.

(i) Find the initial velocity of the particle. [1]

(ii) Find the acceleration of the particle when it comes instantaneously at rest. [4]

(iii) Find the displacement of the particle from O in terms of t. [3]

(iv) Describe the motion of the particle for the first second from $t=0$ to $t=1$. [1]

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2020 Sec 4 A Math Prelim Paper 2
Clementi Town Sec

1 A circle has the equation $x^2 + y^2 + 4x - 8y = 80$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

(ii) Find the equation of the tangent to the circle at the point $P(4,12)$. [3]

2 Express $\dfrac{4x^3 - x^2 + 37x - 65}{(x^2 + 9)(2x - 1)}$ in partial fractions. [6]

3 A chicken is stored in a freezer at a temperature of $-18\ °C$. The chicken is removed from the freezer and left out to thaw. The temperature, $T\ °C$, of the chicken after it was removed from the freezer is given by the formula, $T = 22 - Ae^{-kt}$, where t is the time in hours after the chicken was removed from the freezer, and A and k are constants.

(i) Explain why $A = 40$. [1]

The temperature of the chicken was $0\ °C$ after 4 hours.

(ii) Show that the value of k, correct to 3 significant figures, is 0.149. [2]

(iii) The temperature range of between $5\ °C$ to $60\ °C$ is known as the Temperature Danger Zone, where food poisoning bacteria can grow to unsafe levels. If the chicken was removed from the freezer at 06 00, find the latest time, to the nearest minute, before the temperature of the chicken enters the Temperature Danger Zone. [3]

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(iv) Explain why the temperature of the chicken can never reach 22 °C. [1]

4 A curve has the equation $y = \dfrac{3x-1}{\sqrt{6x-1}}$, where $x > \dfrac{1}{6}$.

(i) Express $\dfrac{dy}{dx}$ in the form $\dfrac{ax}{\sqrt{(6x-1)^3}}$, where a is a constant. [4]

(ii) Evaluate $\displaystyle\int_2^{12} \frac{27x}{\sqrt{(6x-1)^3}}\,dx$. [3]

5 The function $f(x) = x^3 + ax^2 + b$, where a and b are constants, has a factor of $x-3$ and leaves a remainder of -30 when divided by $(x+2)$.

(i) Find the value of a and b. [4]

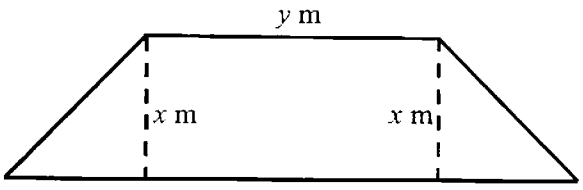
(ii) Determine, showing all necessary workings, the number of real roots of the equation $f(x) = 0$. [4]

6 Solve the equation

(i) $4 + \log_3(x+2) = 2\log_3(4x-1)$, [5]

(ii) $e^x = \dfrac{6}{e^x} + 1$ [4]

7



A gardener uses 100 m of fencing to enclose a plot of land with the shape shown above. The shape consists of two isosceles right angled triangles of base and height x m and a rectangle with sides x m and y m.

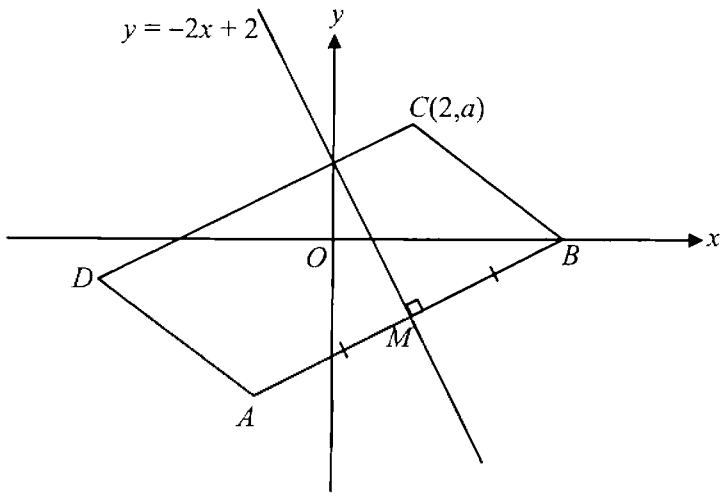
(i) Show that the area of the plot is $50x - \sqrt{2}x^2$ m^2. [4]

(ii) Given that x can vary, find the value of x for which the area of the plot is stationary. [3]

(iii) Explain why this value of x gives the gardener the largest area possible, and state the area of the plot corresponding to this value of x. [3]

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8 Solutions to this question by accurate drawing will not be accepted.



The diagram show a parallelogram $ABCD$ in which the point B is (6, 0) and C is $(2, a)$. The line with the equation $y = -2x + 2$ is the perpendicular bisector of AB, and intersects AB at point M.

(i) Show that $a = 3$. [3]

(ii) Show that the coordinates of A is $(-2, -4)$ and find the coordinates of D. [6]

(iii) Calculate the area of parallelogram $ABCD$. [2]

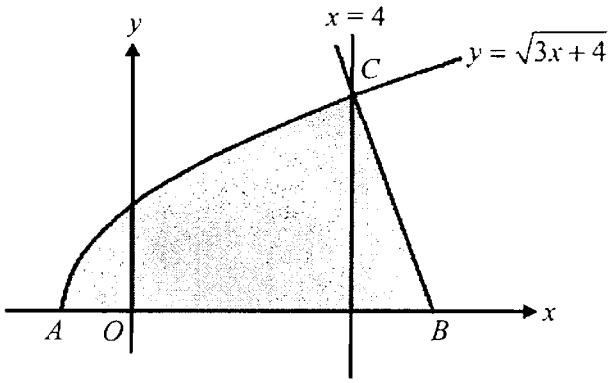
9 (i) Show that $\sin x \cos x(4 - \tan x - \cot x) = 2\sin 2x - 1$. [4]

(ii) Find, in terms of π, the value of $\int_0^{\frac{\pi}{6}} \sin x \cos x(4 - \tan x - \cot x)\, dx$. [5]

(iii) Find all the angles between $0°$ and $360°$ which satisfy the equation $\sin x \cos x(4 - \tan x - \cot x) = 0.5$. [3]

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10

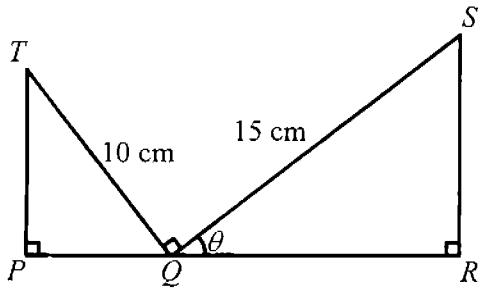


The diagram shows part of the curve $y = \sqrt{3x+4}$, meeting the x-axis at point A and the line $x = 4$ at point C. The normal to the curve at C meets the x-axis at point B. Find

(i) the coordinates of B, [6]

(ii) the area of the shaded region. [6]

11



The diagram shows 3 fixed points Q, S and T such that $QT = 10$ cm and $QS = 15$ cm and angle $SQT = 90°$. The lines PT and SR are perpendicular to PR and angle $RQS = \theta°$. The length PR is denoted by L cm.

(i) Show that $L = 10\sin\theta + 15\cos\theta$. [3]

(ii) Express L in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ for which $L = 13$. [4]

(iv) State the maximum value of L and the corresponding value of θ. [2]

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Solution

Qn	Solution	
1	$\sqrt{216^x} = \dfrac{6^x}{6^{1-x}}$ $6^{\frac{3}{2}x} = 6^{x-(1-x)}$ $\dfrac{3}{2}x = x - 1 + x$ $1 = \dfrac{1}{2}x$ $x = 2$ $\sqrt{216^x} = \sqrt{216^2}$ $= 216$	
2	$y = Me^x + Ne^{-5x}$ $\dfrac{dy}{dx} = Me^x - 5Ne^{-5x}$ $\dfrac{dy}{dx} + 2y = e^x + e^{-5x}$ $\dfrac{dy}{dx} = e^x + e^{-5x} - 2y$ $= e^x + e^{-5x} - 2\left(Me^x + Ne^{-5x}\right)$ $Me^x - 5Ne^{-5x} = e^x + e^{-5x} - 2Me^x - 2Ne^{-5x}$ $3Me^x - 3Ne^{-5x} = e^x + e^{-5x}$ $3M = 1$ $M = \dfrac{1}{3}$ $-3N = 1$ $N = -\dfrac{1}{3}$	
3	When $y = \dfrac{3}{5}$, $x = 9$ Sub into $\dfrac{1}{y} = m\left(\dfrac{1}{\sqrt{x}}\right) + c$ $\dfrac{1}{\left(\frac{3}{5}\right)} = m\left(\dfrac{1}{\sqrt{9}}\right) + c$	

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$\frac{5}{3}=\frac{m}{3}+c$

$m=5-3c$ ---(1)

When $y=\frac{1}{4}$, $x=\frac{1}{25}$

$\frac{1}{\left(\frac{1}{4}\right)}=m\left(\frac{1}{\sqrt{\frac{1}{25}}}\right)+c$

$4=5m+c$ ---(2)

Sub (1) into (2)

$4=5(5-3c)+c$

$4=25-15c+c$

$14c=21$

$c=\frac{3}{2}$

$m=5-3\left(\frac{3}{2}\right)=\frac{1}{2}$

The equation is $\frac{1}{y}=\frac{1}{2}\left(\frac{1}{\sqrt{x}}\right)+\frac{3}{2}$

when $x = 1$,

$\frac{1}{y}=\frac{1}{2}\left(\frac{1}{\sqrt{1}}\right)+\frac{3}{2}$

$y=\frac{1}{2}$

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4a	$x^2+2(p-2)x+(2p-5)=0$ Discriminant $=(2(p-2))^2-4(1)(2p-5)$ $=4(p^2-4p+4)-8p+20$ $=4p^2-16p+16-8p+20$ $=4p^2-24p+36$ $=4(p-3)^2$ $=[2(p-3)]^2$ Since $[2(p-3)]^2 \geq 0$, hence the eqn has real roots for all real values of *p*.	
4b	Since coefficient of $x^2>0$, Shape of graph is  Discriminant $=(-3)^2-4(1)(5)$ $=9-20$ $=-11<0$ Hence, it has no real roots. Therefore, x^2-3x+5 is always above the $x-$axis ⇨ x^2-3x+5 is always positive. **Alternative Solution** Since coefficient of $x^2>0$, Shape of graph is  Let $y=x^2-3x+5$ $\frac{dy}{dx}=2x-3$ $2x-3=0$ $x=\frac{3}{2}$ Then, $y=\frac{11}{4}$	

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	Since min of y is $\frac{11}{4}$, $\therefore x^2-3x+5$ is always positive.	
5i	$\left(x^2+\frac{m}{x}\right)^9 = x^{18}+\binom{9}{1}\left(x^2\right)^8\left(\frac{m}{x}\right)^1+\binom{9}{2}\left(x^2\right)^7\left(\frac{m}{x}\right)^2+....$ $= x^{18}+9mx^{15}+36m^2x^{12}+....$	
5ii	For $\left(x^2+\frac{m}{x}\right)^9$, general term is $(r+1)^{th}$ term $= \binom{9}{r}\left(x^2\right)^{9-r}(m)^r\left(x^{-1}\right)^r$ $= \binom{9}{r}(m)^r\left(x^{18-2r}\right)\left(x^{-r}\right)$ $= \binom{9}{r}(m)^r\left(x^{18-3r}\right)$ $18-3r=3$ $r=5$ $\therefore -126 = \binom{9}{5}m^5$ $-126 = 126m^5$ $m^5=-1$ $m=-1$	
5iii	For $\left(x^2+\frac{m}{x}\right)^9$, the term independent of x: $18-3r=0$ $r=6$ $\therefore \binom{9}{6}m^6 = 84(-1)^6 = 84$	

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	For $\left(3-\frac{1}{x^3}\right)\left(x^2+\frac{m}{x}\right)^9$, the term independent of *x:* $\left(3-\frac{1}{x^3}\right)\left(....-126x^3+84+....\right)^9$ $=3\times 84+126$ = 378	
6i	$4\cos^2 x-2\sin^2 x=3\cos 2x+1$ LHS $4\cos^2 x-2\sin^2 x$ $=2\left(2\cos^2 x\right)-\left(2\sin^2 x\right)$ $=2\left(\cos 2x+1\right)-\left(1-\cos 2x\right)$ $=2\cos 2x+2-1+\cos 2x$ $=3\cos 2x+1$ =RHS **Alternative** **RHS** $3\cos 2x+1$ $=3\left(\cos^2 x-\sin^2 x\right)+1$ $=3\cos^2 x-3\sin^2 x+1$ $=3\cos^2 x-3\sin^2 x+\left(\sin^2 x+\cos^2 x\right)$ $=4\cos^2 x-2\sin^2 x$ $=\text{LHS}$	
6ii	Amplitude = 3 Period = 180°	
6iii	[See last page]	
7a	$7x^2+5x-4=0$ $\alpha+\beta=-\frac{5}{7}$ $\alpha\beta=\frac{-4}{7}$ Sum of new roots:	

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$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}$$
$$=\frac{\beta^3+\alpha^3}{(\alpha\beta)^3}$$
$$=\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{(\alpha\beta)^3}$$
$$=\frac{(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)}{(\alpha\beta)^3}$$
$$=\frac{\left(-\frac{5}{7}\right)\left(\left(-\frac{5}{7}\right)^2-3\left(-\frac{4}{7}\right)\right)}{\left(-\frac{4}{7}\right)^3}$$
$$=\frac{545}{64}$$

Product of new roots:

$$\frac{1}{\alpha^3}\times\frac{1}{\beta^3}$$
$$=\frac{1}{(\alpha\beta)^3}$$
$$=\frac{1}{\left(-\frac{4}{7}\right)^3}$$
$$=-\frac{343}{64}$$

Hence, the new equation is :

$x^2-\left(\frac{545}{64}\right)x-\frac{343}{64}=0$ or

$64x^2-545x-343=0$

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7b	For $2mx^2+(8-4m)x+m+1=0$ to have real roots, $b^2-4ac\geq 0$ $(8-4m)^2-4(2m)(m+1)\geq 0$ $64-64m+16m^2-8m^2-8m\geq 0$ $8m^2-72m+64\geq 0$ $m^2-9m+8\geq 0$ $(m-8)(m-1)\geq 0$ $m\leq 1$ or $m\geq 8$ Since $2mx^2+(8-4m)x+m+1=0$ has real roots when $m\geq 8$, Then, it will also have real roots when $m\geq 9$.	
8i	$V=\frac{1}{3}\pi x^2(54-x)$ When $x=5$, $V=\frac{1}{3}\pi(5)^2(54-5)$ $=1282.817$ $36\pi \text{ cm}^3 -> 1 \text{ s}$ $1 \text{ cm}^3 -> \frac{1}{36\pi} \text{ s}$ $1282.817 \text{ cm}^3 -> 11.3426 \text{ s}$ $= 11.3\text{s}$ (3 s.f.) (Shown)	
8ii	$V=18\pi x^2-\frac{1}{3}\pi x^3$ $\frac{dV}{dx}=36\pi x-\pi x^2$ $\frac{dV}{dt}=36\pi$ $\frac{dV}{dx}\times\frac{dx}{dt}=\frac{dV}{dt}$	

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	$x = 5$ $\left(36\pi(5) - \pi(5)^2\right) \times \frac{\mathrm{d}x}{\mathrm{d}t} = 36\pi$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{36\pi}{155\pi}$ $= \frac{36}{155}$ cm/s	
9a	$p = \log_c A^3 B$ $p = 3\log_c A + \log_c B$ $p - 3\log_c A = \log_c B$ $q = \log_c \frac{B}{A^2}$ $q = \log_c B - 2\log_c A$ $q + 2\log_c A = \log_c B$ $p - 3\log_c A = q + 2\log_c A$ $5\log_c A = p - q$ $\log_c A = \frac{p-q}{5}$ $\log_c B = p - 3\left(\frac{p-q}{5}\right)$ $\log_c B = \frac{5p - 3p + 3q}{5}$ $\log_c B = \frac{2p+3q}{5}$ $\therefore \log_c AB = \frac{p-q}{5} + \frac{2p+3q}{5}$ $\log_c AB = \frac{3p+2q}{5}$	
9bi	$\therefore \left(3+2\sqrt{3}\right)^2 = 21 + 12\sqrt{3}$ Height $= \frac{150 + 87\sqrt{3}}{21 + 12\sqrt{3}}$	

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	$= \dfrac{50+29\sqrt{3}}{7+4\sqrt{3}}$ $= \dfrac{\left(50+29\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)}$ $= \dfrac{350-200\sqrt{3}+203\sqrt{3}-348}{49-48}$ $= \left(2+3\sqrt{3}\right)$ cm	
9bii	(b) the total length of edges $= 8\left(3+2\sqrt{3}\right)+4\left(2+3\sqrt{3}\right)$ $= 24+16\sqrt{3}+8+12\sqrt{3}$ $= \left(32+28\sqrt{3}\right)$ cm	
10i	$\dfrac{dy}{dx} = \dfrac{x^2+4x-2}{(x+2)^2}$ $y = \int \dfrac{x^2+4x-2}{(x+2)^2}\,dx$ $y = \int 1 - \dfrac{6}{(x+2)^2}\,dx$ $y = x - 6\left(\dfrac{(x+2)^{-1}}{(-1)(1)}\right)+c$ $y = x + \dfrac{6}{x+2}+c$ Sub $(0,3), c=0$ $\therefore y = x + \dfrac{6}{x+2}$	

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10ii	When $\frac{dy}{dx}=0$, $\frac{x^2+4x-2}{(x+2)^2}=0$ $x^2+4x-2=0$ $x=-4.45$ or $x=0.449$ (3 Sig Fig)	
10iii	$\frac{d^2y}{dx^2}$ $=\frac{(2x+4)(x+2)^2-(x^2+4x-2)(2)(x+2)}{(x+2)^4}$ $=\frac{2(x+2)(x^2+4x+4-x^2-4x+2)}{(x+2)^4}$ $=\frac{12(x+2)}{(x+2)^4}$ $=\frac{12}{(x+2)^3}$ When $x=-4.4494897$ $\frac{d^2y}{dx^2}=-0.816<0$ (Max Point) $x=0.4494897$ $\frac{d^2y}{dx^2}=0.816>0$ (Min Point)	

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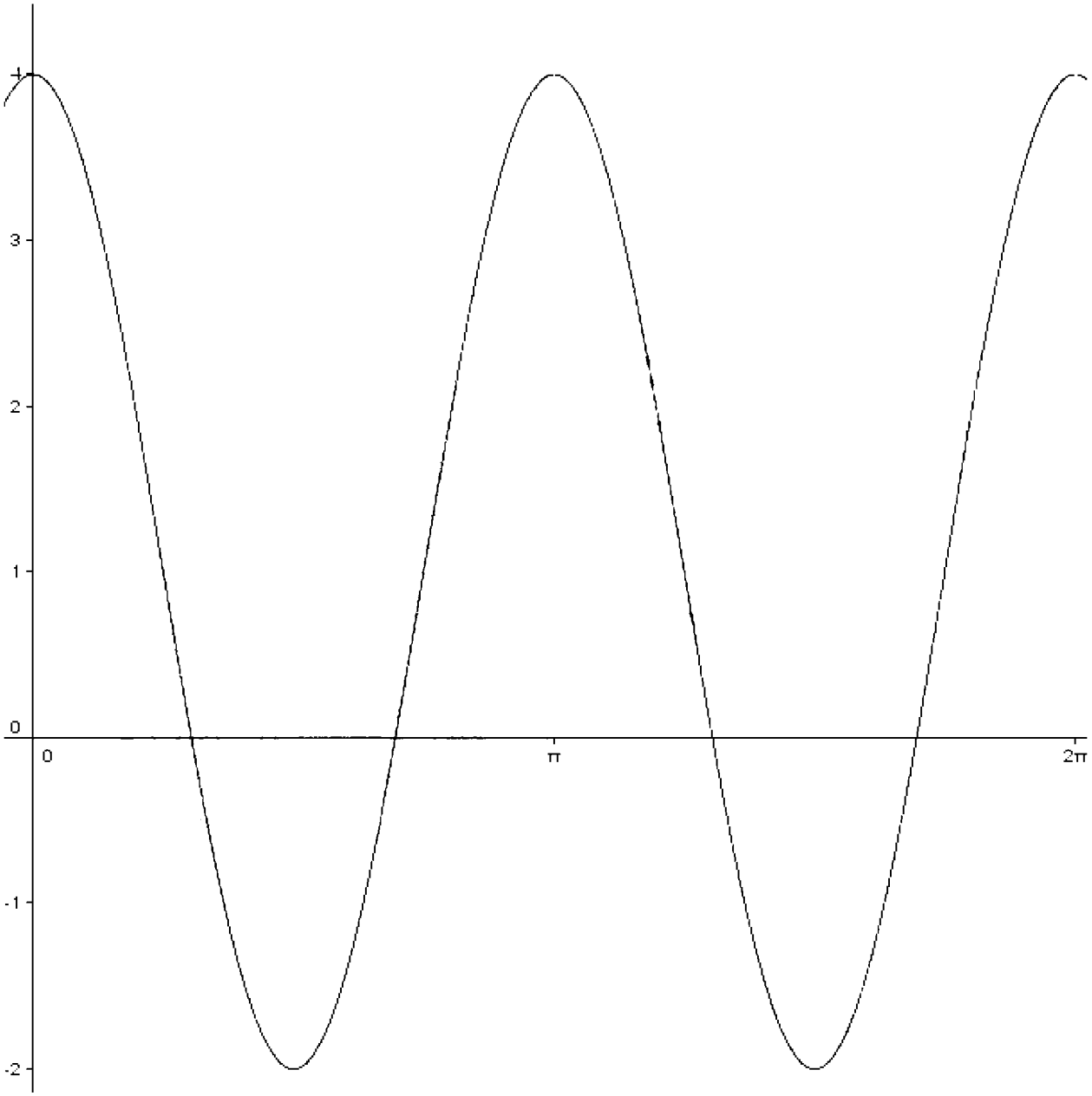
11i	$\frac{d}{dx}\left[\ln\left(\frac{x}{2x+1}\right)\right]$ $=\frac{d}{dx}[\ln x-\ln(2x+1)]$ $=\frac{1}{x}-\frac{2}{2x+1}$ $=\frac{1}{x(2x+1)}$	
11ii	$\int_1^2 \frac{4x^2+3x+1}{x(2x+1)}\,dx$ $=\int_1^2 \frac{4x^2+2x}{x(2x+1)}+\frac{1}{x(2x+1)}+\frac{x}{x(2x+1)}\,dx$ $=\int_1^2 2+\frac{1}{x(2x+1)}+\frac{1}{2x+1}\,dx$ $=[2x+\ln\left(\frac{x}{2x+1}\right)+\frac{1}{2}\ln(2x+1)]_1^2$ $=(4+\ln\frac{2}{5}+\frac{1}{2}\ln 5)-(2+\ln\frac{1}{3}+\frac{1}{2}\ln 3)$ $=2.44$ (to 3 sf)	
12i	When $t=0$ $v=\frac{3(0)-2}{(0)+3}=-\frac{2}{3}$ m/s	
12ii	$v=\frac{3t-2}{t+3}$ $a=\frac{(3)(t+3)-(3t-2)(1)}{(t+3)^2}$ $=\frac{3t+9-3t+2}{(t+3)^2}$ $=\frac{11}{(t+3)^2}$	

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	When instantaneously at rest, $v = 0$ $0 = \frac{3t-2}{t+3}$ $t = \frac{2}{3}$ $a = \frac{11}{\left(\frac{2}{3}+3\right)^2}$ $a = \frac{9}{11}$ m/s^2	
12iii	S $= \int v$ $= \int \frac{3t-2}{t+3}\,\mathrm{dt}$ $= \int 3 - \frac{11}{t+3}\,\mathrm{dt}$ $= 3t - 11\ln(t+3) + c$ Sub $t = 0$, $s = 0$ $\therefore c = 11\ln 3$ $S = 3t - 11\ln(t+3) + 11\ln 3$	
12iv	The particle travels from fixed point O at $t = 0$ and comes instantaneously at rest at $t = \frac{2}{3}$s where it changes direction and heads toward O.	

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6iii)



B1; shape

B1; max

B1; min

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2020 4EN5 Add Math Prelims P2 Solutions

Qn **Ans**

1(i) $x^2 + y^2 + 4x - 8y - 80 = 0$

$2g = 4$

$g = 2$

$2f = -8$

$f = -4$

$c = -80$

Centre $= (-2, 4)$

Radius $= \sqrt{(2)^2 + (-4)^2 - (-80)}$

$= 10$ units

Alternative method:

$x^2 + y^2 + 4x - 8y = 80$

$(x^2 + 4x) + (y^2 - 8y) = 80$

$(x+2)^2 + (y-4)^2 = 80 + 2^2 + 4^2$

$(x+2)^2 + (y-4)^2 = 100$

Centre $= (-2, 4)$

Radius $= 10$ units

1(ii) Let centre be C

Gradient of $CP = \dfrac{12-4}{4-(-2)}$

$= \dfrac{4}{3}$

Gradient of tangent at $P = -\dfrac{1}{\left(\frac{4}{3}\right)}$

$= -\dfrac{3}{4}$

Equation of tangent to the circle at P:

$(y-12) = -\dfrac{3}{4}(x-4)$

$y = -\dfrac{3}{4}x + 15$

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2 $(x^2+9)(2x-1) = 2x^3 - x^2 + 18x - 9$

$$
\begin{array}{r}
2 \\
2x^3 - x^2 + 18x - 9 \overline{)\ 4x^3 - x^2 + 37x - 65} \\
-(4x^3 - 2x^2 + 36x - 18) \\
\hline
x^2 + x - 47 \\
\hline
\end{array}
$$

$$\frac{x^2 + x - 47}{(x^2+9)(2x-1)} = \frac{Ax + B}{x^2 + 9} + \frac{C}{2x - 1}$$

Multiply throughout by $(x^2+9)(2x-1)$

$x^2 + x - 47 = (Ax + B)(2x - 1) + C(x^2 + 9)$

Substitute $x = \frac{1}{2}$

$-\frac{185}{4} = \frac{37}{4}C$

$C = -5$

$x^2 + x - 47 = (Ax + B)(2x - 1) - 5(x^2 + 9)$

Substitute $x = 0$

$-47 = -B - 45$

$B = 2$

$x^2 + x - 47 = (Ax + 2)(2x - 1) - 5(x^2 + 9)$

Substitute $x = 1$

$-45 = A + 2 - 50$

$A = 3$

(**Note:** Students may alternatively use comparing coefficient method or a mixture of both.)

$$\frac{4x^3 - x^2 + 37x - 65}{(x^2+9)(2x-1)} = 2 + \frac{3x + 2}{x^2 + 9} - \frac{5}{2x - 1}$$

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3(i) When $t = 0$, the temperature of the chicken $T = -18$
Therefore,
$-18 = 22 - Ae^{0}$
$-A = -40$
$A = 40$

3(ii) $T = 22 - 40e^{-kt}$
Substitute $T = 0$ and $t = 4$ into $T = 22 - 40e^{-kt}$
$0 = 22 - 40e^{-4k}$
$40e^{-4k} = 22$
$e^{-4k} = \dfrac{22}{40}$
$-4k = \ln\left(\dfrac{11}{20}\right)$
$k = \dfrac{\ln\left(\dfrac{11}{20}\right)}{-4}$
$k = 0.149459$
$k = 0.149$ (shown)

3(iii) Substitute $T = 5$ into $T = 22 - 40e^{-0.149t}$
$5 = 22 - 40e^{-0.149t}$
$40e^{-0.149t} = 17$
$e^{-0.149t} = \dfrac{17}{40}$
$-0.149t = \ln\left(\dfrac{17}{40}\right)$
$t = \dfrac{\ln\left(\dfrac{17}{40}\right)}{-0.149}$
$t = 5.7427$
$t = 5$ hours 44.6 minutes
$t \approx 5$ hours 44 minutes
Latest time before the temp of the chicken enters TDZ $= 11\ 44$

3(iv) $T = 22 - 40e^{-0.149t}$
$e^{-0.149t} > 0$
$-40e^{-0.149t} < 0$
$22 - 40e^{-0.149t} < 22$
$T < 22$

For all values of t, T will always be less than 22. As such, the temperature of the chicken can never reach $22\ ^{\circ}\text{C}$.

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4(i) $y = \dfrac{3x-1}{\sqrt{6x-1}}$

$$\frac{dy}{dx} = \frac{(6x-1)^{\frac{1}{2}}\frac{d}{dx}(3x-1) - (3x-1)\frac{d}{dx}(6x-1)^{\frac{1}{2}}}{\left((6x-1)^{\frac{1}{2}}\right)^2}$$

$$\frac{dy}{dx} = \frac{3(6x-1)^{\frac{1}{2}} - (3x-1)\left(\frac{1}{2}\right)(6)(6x-1)^{-\frac{1}{2}}}{6x-1}$$

$$\frac{dy}{dx} = \frac{3\sqrt{6x-1} - \frac{3(3x-1)}{\sqrt{6x-1}}}{6x-1}$$

$$\frac{dy}{dx} = \frac{\frac{3(6x-1) - 3(3x-1)}{\sqrt{6x-1}}}{6x-1}$$

$$\frac{dy}{dx} = \frac{18x - 3 - 9x + 3}{\sqrt{(6x-1)^3}}$$

$$\frac{dy}{dx} = \frac{9x}{\sqrt{(6x-1)^3}}$$

4(ii) $\int_2^{12} \dfrac{27x}{\sqrt{(6x-1)^3}}\,dx$

$$= 3\int_2^{12} \frac{9x}{\sqrt{(6x-1)^3}}\,dx$$

$$= 3\left[\frac{3x-1}{\sqrt{6x-1}}\right]_2^{12}$$

$$= 3\left(\frac{3(12)-1}{\sqrt{6(12)-1}} - \frac{3(2)-1}{\sqrt{6(2)-1}}\right)$$

$$= 7.94$$

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5(i) By factor theorem,

$f(3) = 0$

$3^3 + a(3)^2 + b = 0$

$9a + b = -27 - - - (1)$

By remainder theorem,

$f(-2) = 0$

$(-2)^3 + a(-2)^2 + b = -30$

$4a + b = -22 - - - (2)$

$(1) - (2)$

$5a = -5$

$a = -1$

Sub. $a = -1$ into (2)

$-4 + b = -22$

$b = -18$

5(ii)

$$
\begin{array}{r}
x^2 + 2x + 6 \\
x - 3 \overline{)\; x^3 - x^2 - 18} \\
-\,(x^3 - 3x^2) \\
\hline
2x^2 - 18 \\
-\,(2x^2 - 6x) \\
\hline
6x - 18 \\
-\,(6x - 18) \\
\hline
0 \\
\hline
\end{array}
$$

$x^3 - x^2 - 18 = 0$

$(x - 3)(x^2 + 2x + 6) = 0$

[M1: by long division, synthetic division, or comparing coefficient]

$x = 3$ or $x^2 + 2x + 6 = 0$

For $x^2 + 2x + 6 = 0$

Discriminant $b^2 - 4ac$

$= (2)^2 - 4(1)(6)$

$= -20 < 0$

Since $b^2 - 4ac < 0$, there are no real roots for $x^2 + 2x + 6 = 0$.

Therefore, $f(x) = 0$ only has one real root.

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6(i) $4+\log_3(x+2)=2\log_3(4x-1)$

$4+\log_3(x+2)=\log_3(4x-1)^2$

$\log_3 3^4+\log_3(x+2)=\log_3(4x-1)^2$

$\log_3 81(x+2)=\log_3(4x-1)^2$

$81(x+2)=(4x-1)^2$

$81x+162=16x^2-8x+1$

$16x^2-89x-161=0$

$(16x+23)(x-7)=0$

$x=7 \quad \text{or} \quad x=-\frac{23}{16}\text{(rej)}$

6(ii) $e^x=\frac{6}{e^x}+1$

$e^{2x}+e^x-6=0$

Substitute $u=e^x$

$u^2+u-6=0$

$(u+3)(u-2)=0$

$u=2 \quad \text{or} \quad u=-3 \text{ (rej)}$

$e^x=2$

$x=\ln 2$

$x=0.693$

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7(i) By Pythagoras Theorem,

Diagonal of trapezium $= \sqrt{x^2 + x^2}$

$= \sqrt{2x^2}$

$2y + 2x + 2\sqrt{2x^2} = 100$

$y = 50 - x - \sqrt{2}x$

Area of plot

$= 2 \times \frac{1}{2} \times x \times x + y \times x$

$= x^2 + \left(50 - x - \sqrt{2}x\right) \times x$

$= x^2 + 50x - x^2 - \sqrt{2}x^2$

$= 50x - \sqrt{2}x^2$ m^2 (shown)

7(ii) Let $A = 50x - \sqrt{2}x^2$

$\frac{dA}{dx} = 50 - 2\sqrt{2}x$

$\frac{dA}{dx} = 0$

$50 - 2\sqrt{2}x = 0$

$x = 17.7$

7(iii) $\frac{d^2A}{dx^2} = -2\sqrt{2} < 0$

Since $\frac{d^2A}{dx^2} < 0$, by the second derivative test, the area is maximum when $x = 17.7$.

Sub. $x = 17.7$ into $A = 50x - \sqrt{2}x^2$

$A = 50(17.678) - \sqrt{2}(17.678)^2$

$A = 442$ m^2

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8(i) Gradient of perpendicular bisector of AB and $CD = -2$

Gradient of $CD = -\frac{1}{-2}$

$= \frac{1}{2}$

OR

Gradient of CD = Gradient of AB

$= \frac{0-(-4)}{6-(-2)}$

$= \frac{1}{2}$

Eqn of CD: $y = \frac{1}{2}x + 2$

Sub. $y = a$, $x = 2$

$a = \frac{1}{2}(2) + 2$

$a = 3$

8(ii) Eqn of AB:

$y - 0 = \frac{1}{2}(x - (6))$

$y = \frac{1}{2}x - 3$

*No M1 awarded if student used A to find gradient of AB

Sub. $y = \frac{1}{2}x - 3$ into $y = -2x + 2$

$-2x + 2 = \frac{1}{2}x - 3$

$x = 2$

Sub. $x = 2$ into $y = -2x + 2$

$y = -2$

$M = (2, -2)$

Let the coordinates of A be (x, y)

$\frac{x+6}{2} = 2$

$x = -2$

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$$\frac{y+0}{2} = -2$$
$$y = -4$$

$A = (-2, -4)$
$D = (-6, -1)$

8(iii) Area of parallelogram $ABCD$

$$= \frac{1}{2}\begin{vmatrix} -2 & 6 & 2 & -6 & -2 \\ -4 & 0 & 3 & -1 & -4 \end{vmatrix}$$
$$= 40 \text{ units}^2$$

9(i) LHS

$$= \sin x \cos x(4 - \tan x - \cot x)$$

$$= \sin x \cos x\left(4 - \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right)$$

$$= 4\sin x \cos x - \sin^2 x - \cos^2 x$$

$$= 2(2\sin x \cos x) - (\sin^2 x + \cos^2 x)$$

$$= 2\sin 2x - (\sin^2 x + \cos^2 x)$$

$$= 2\sin 2x - 1$$

$$= \text{RHS}$$

9(ii)

$$\int_0^{\frac{\pi}{6}} \sin x \cos x(4 - \tan x - \cot x)\, dx$$

$$= \int_0^{\frac{\pi}{6}} 2\sin 2x - 1\, dx$$

$$= \left[-\cos 2x - x\right]_0^{\frac{\pi}{6}}$$
$$= \left(-\cos 2\left(\frac{\pi}{6}\right) - \frac{\pi}{6}\right) - \left(-\cos 2(0) - 0\right)$$

$$= -\frac{1}{2} - \frac{\pi}{6} + 1$$

$$= \frac{1}{2} - \frac{\pi}{6}$$

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9(iii) $\sin x \cos x(4 - \tan x - \cot x) = 0.5$

$2\sin 2x - 1 = 0.5$

$\sin 2x = 0.75$

$\alpha = \sin^{-1}(0.75)$

$\alpha = 58.0°$

$0° < x < 360°$

$0° < 2x < 720°$

Since $\sin 2x$ is positive, $2x$ is in either the 1st or 2nd quadrant.

$2x = 58.0°$ or $180° - 58.0°$ or $360° + 58.0°$ or $360° + 180° - 58.0°$

$2x = 58.0°$ or $122.0°$ or $418.0°$ or $482.0°$

$x = 29.0°$ or $61.0°$ or $209.0°$ or $241.0°$

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10(i) Sub. $x = 4$ into $y = \sqrt{3x+4}$

$y = 4$

$C = (4, 4)$

$y = (3x+4)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(3x+4)^{-\frac{1}{2}}(3)$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x+4}}$

Sub. $x = 4$ into $\frac{dy}{dx} = \frac{3}{2\sqrt{3x+4}}$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{2\sqrt{3(4)+4}}$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{8}$

Gradient of the normal to the curve at $C = -\frac{8}{3}$

Equation of normal to the curve at C:

$y - 4 = -\frac{8}{3}(x-4)$

$y = -\frac{8}{3}x + \frac{44}{3}$

Sub. $y = 0$ into $y = -\frac{8}{3}x + \frac{44}{3}$

$x = \frac{11}{2}$

$B\left(\frac{11}{2}, 0\right)$

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10(ii) Sub. $y = 0$ into $y = \sqrt{3x+4}$

$x = -\frac{4}{3}$

$A\left(-\frac{4}{3}, 0\right)$

Area bounded by the curve $y = \sqrt{3x+4}$, x axis and $x = 4$

$= \int_{-\frac{4}{3}}^{4} (3x+4)^{\frac{1}{2}} \, dx$

$= \left[\frac{(3x+4)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)} \right]_{-\frac{4}{3}}^{4}$

$= \left[\frac{2(3x+4)^{\frac{3}{2}}}{9} \right]_{-\frac{4}{3}}^{4}$

$= \left(\frac{2(3(4)+4)^{\frac{3}{2}}}{9} \right) - \left(\frac{2\left(3\left(-\frac{4}{3}\right)+4\right)^{\frac{3}{2}}}{9} \right)$

$= \frac{128}{9}$ units^2

Area of triangle bounded by BC, x axis and $x = 4$

$= \frac{1}{2}(4)\left(\frac{11}{2} - 4\right)$

$= 3$ units^2

Shaded area

$= \frac{128}{9} + 3$

$= \frac{155}{9}$ units^2

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11(i) $\angle PQT = 180° - 90° - \theta°$

$= 90° - \theta°$

$\angle QTP = \theta°$

$\sin \angle QTP = \dfrac{PQ}{QT}$

$\sin\theta = \dfrac{PQ}{10}$

$PQ = 10\sin\theta$

$\cos\theta = \dfrac{QR}{QS}$

$\cos\theta = \dfrac{QR}{15}$

$QR = 15\cos\theta$

$L = PQ + QR$

$L = 10\sin\theta + 15\cos\theta$ (shown)

11(ii) Let $10\sin\theta + 15\cos\theta = R\sin(\theta + \alpha)$

$10\sin\theta + 15\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 10$ ---(1)

$R\sin\alpha = 15$ ---(2)

(2) ÷ (1)

$\tan\alpha = \dfrac{15}{10}$

$\alpha = \tan^{-1}\dfrac{3}{2}$

$\alpha = 56.310°$

Sub. $\alpha = 56.310°$ into (2)

$R\sin 56.310° = 15$

$R = \dfrac{15}{\sin 56.310°}$

$R = 18.028$

$L = 18.0\sin(\theta + 56.3°)$

11(iii) $18.028\sin(\theta + 56.310) = 13$

$0° < \theta < 90°$

$56.310° < \theta + 56.310 < 146.310°$

$\sin(\theta + 56.310) = \dfrac{13}{18.028}$

$\alpha = \sin^{-1}\left(\dfrac{13}{18.028}\right)$

$\alpha = 46.146°$

$\theta + 56.310$ is in the 1st or 2nd quadrant

$\theta + 56.310 = 180° - 46.146°$

$\theta = 77.5°$

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11(iv) Maximum value of $L = 18.0$
Corresponding value of θ
$= 90° - 56.3°$
$= 33.7°$

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