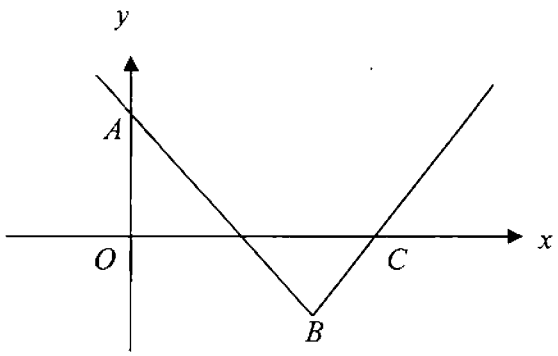
1 A cone has curved surface area $\pi\left(6+\sqrt{2}\right)$ cm^2 and slant height $\left(3+\sqrt{2}\right)$ cm.

Without using a calculator, find the diameter of the base of the cone. Give your answer in the form $\left(a+b\sqrt{2}\right)$ cm, where a and b are constants. [4]

2 Given that the roots of $3x^2+4x-6=0$ are $3\alpha+\beta$ and $3\beta+\alpha$, find a quadratic equation whose roots are α and β. [5]

3 The diagram shows part of the graph of $y=-2+|5-2x|$.



(i) Find the coordinates of the points A, B and C. [3]

(ii) Hence, find the range of values of x for which $y \le 0$. [1]

4 The function f is defined by $f(x)=x^3-2x^2+kx-3$ where k is a constant.

(i) Given that $f(x)$ is exactly divisible by $x-3$, find the value of k. [2]

(ii) Show that the equation $f(x)=0$ has only one real root. [4]

(iii) Use your answers to parts **(i)** and **(ii)** to solve the equation $4^{3y}-2(4^{2y})+k(4^y)=3$. [2]

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5 The number of bacteria, P, present in a culture is related to the time, T hours, by $P = h(10^{kT})$, where h and k are constants. The table below shows corresponding values of P and T.

T	1	3	5	7
P	110	550	2730	13520

(i) Draw the graph of $\lg P$ plotted against T, using a scale of 1 cm for 1 unit on the T–axis and a scale of 4 cm for 1 unit on the $\lg P$–axis. [3]

(ii) Use the graph to estimate the value of each of the constants h and k. [3]

(iii) Explain what is the significance of the value of h found in **(ii)**. [1]

(iv) Use your graph to find the number of the bacteria present when T = 4. [1]

6 The line $x = 1$ passes through the centre A of a circle, C_1, with a radius of 10.

The equation of the tangent to the circle at the lowest point, B is $y = -15$.

(i) State the coordinates of A. [1]

(ii) Find the equation of C_1. [1]

(iii) This circle intersects the y-axis at points M and N.

Find the length of MN in the form $a\sqrt{11}$ where a is an integer. [3]

(iv) A second circle, C_2, is a reflection of C_1 in the line $x = k$ where k is an integer.

The equation of C_2 is $x^2 + y^2 - 30x + 10y + 150 = 0$.

Find the value of k. [2]

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7 **(i)** Prove the identity $\cos^4 A - \sin^4 A = \cos 2A$. [3]

(ii) Hence, solve the equation $\cos^4 A - \sin^4 A = 2\sin A - \frac{1}{2}$, $0° < A < 360°$. [4]

(iii) Given that $8\cos^4 x - 8\sin^4 x = \frac{3}{\text{cosec } 2x}$, **without using a calculator**, find the value of $\tan 2x$. [2]

8 A cyclist travels along a straight horizontal road at a velocity v m/s where $v = 40e^{-0.2t} - 5$, t seconds after passing a point P.

(i) Find the velocity of the cyclist at P. [1]

(ii) Find an expression, in terms of t, for the acceleration of the cyclist. [1]

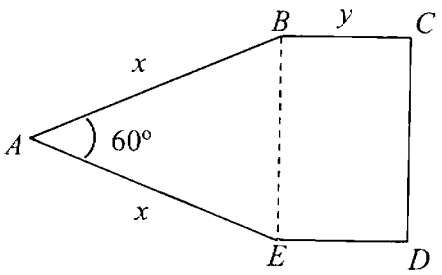
(iii) Hence, explain what happens after the cyclist passes the point P. [1]

(iv) Given that he stops at Q, find the time taken to travel from P to Q. [2]

(v) Calculate the distance PQ in metres. [3]

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9 A preschool principal decides to use 100 m of fencing to create a playground for the children. The design of the playground is made up of a triangle ABE and a rectangle $BCDE$ as shown in the diagram. $AB = AE = x$ m and $BC = y$ m.



(i) Express y in terms of x. [2]

(ii) Show that the area, A m^2, of the playground is given by

$$A = \frac{\sqrt{3}-6}{4}x^2 + 50x.$$ [2]

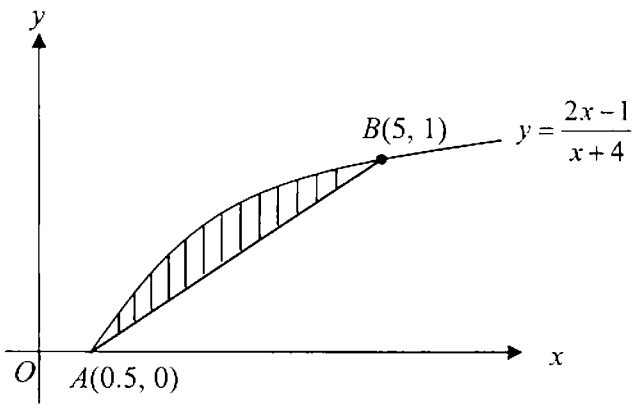
(iii) Given that x can vary, find the value of x which gives a stationary value of A. [4]

(iv) Hence, explain why the children or teachers might be happy with this design. [2]

10 The equation of a curve is given by $y = \frac{2x-1}{x+4}$, $x \neq -4$.

(i) Explain with working, why the curve has no turning points. [3]

The diagram below shows part of the curve $y = \frac{2x-1}{x+4}$. Points $A(0.5, 0)$ and $B(5, 1)$ lie on the curve.



(ii) Find the equation of the normal at B. [3]

(iii) By expressing $\frac{2x-1}{x+4}$ in the form $a + \frac{b}{x+4}$, where a and b are constants, find, showing full working, the area of the shaded region. [5]

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11 The curve for which $\frac{dy}{dx} = 3x - \frac{k}{x^2}$, where k is a constant, is such that the tangent at $(2, -3)$ passes through the point $(0, 3)$. Find

(i) the value of k, [2]

(ii) the equation of the curve. [4]

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2020 Sec 4 A-Math Prelim Paper 2 - Ang Mo Kio Sec

1 It is given that $y = e^{-\frac{1}{2}x^2}$, find $\frac{dy}{dx}$ and hence evaluate $\int_0^1 1 - xe^{-\frac{1}{2}x^2}\,dx$, giving your answer in exact value. [6]

2 Given that $\cos A = \frac{7}{25}$, $\tan B = -\frac{5}{12}$ and the angles A and B are in the same quadrant, find without using a calculator, the value of

(i) $\sec B$, [2]

(ii) $\tan(A+B)$, [2]

(iii) $\cos\left(\frac{A}{2}\right)$. [3]

3 Given that $y = x\sin 2x$, find the value of the constant k such that

$$4xy + x\left(\frac{d^2y}{dx^2}\right) - 2\left(\frac{dy}{dx}\right) = k\sin 2x.$$ [7]

4 The equation of a curve is $y = \ln\sqrt{5-2x}$.

(i) Find $\frac{dy}{dx}$ and hence find the coordinates of point P on the curve at which the normal to the curve is parallel to $y - 4x = 1$. [5]

(ii) Find the coordinates of point Q on the curve given that its x-coordinate increases at the rate of 0.2 units per second and its y-coordinate decreases at the rate of 0.1 units per second. [3]

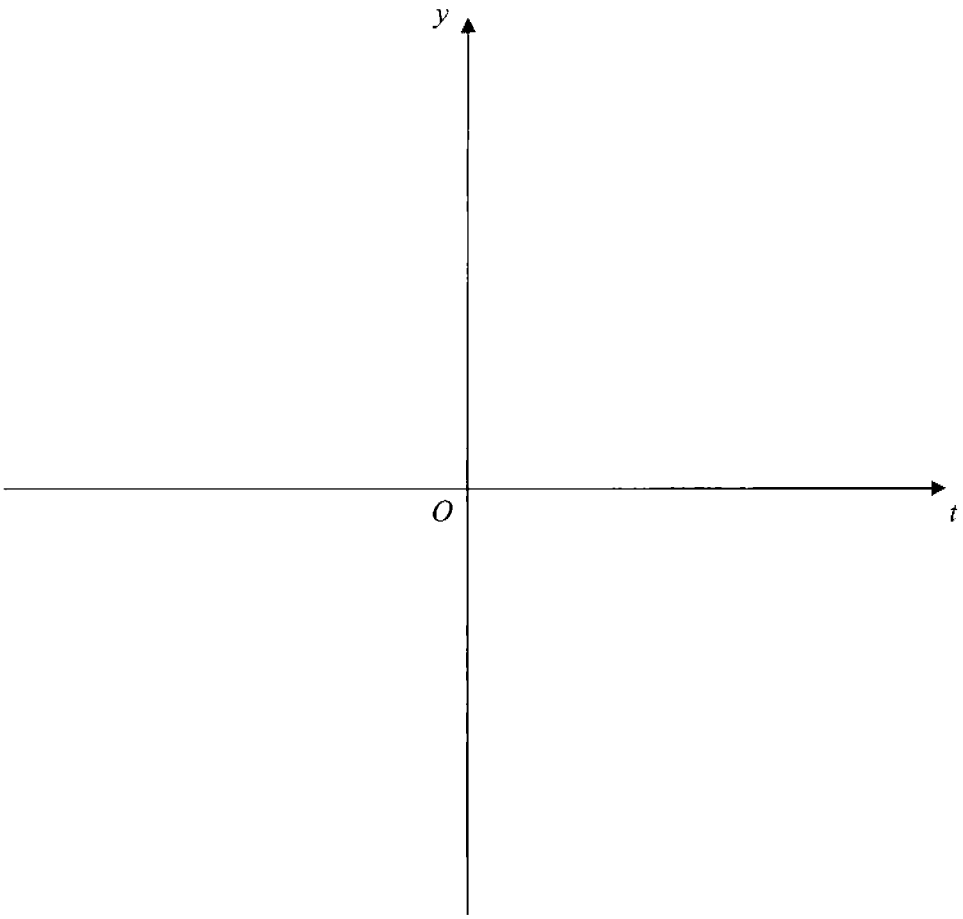
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5 The number of people infected by a virus after t days is given by $P(t) = P_0 e^{kt}$, where P_0 is the number of people infected at a particular time and k is a constant.

(i) Calculate the value of k if, after 9 days, the number of people infected is 10 times its original number. [3]

(ii) Find the value of P_0 if the number of people infected after 30 days is 468 000, giving your answer correct to nearest whole number. [2]

(iii) Sketch the graph of $y = P(t)$ on the axes provided. [2]



(iv) Explain why the function $P(t)$ may not be suitable to calculate the total number of people infected as t becomes very large. [1]

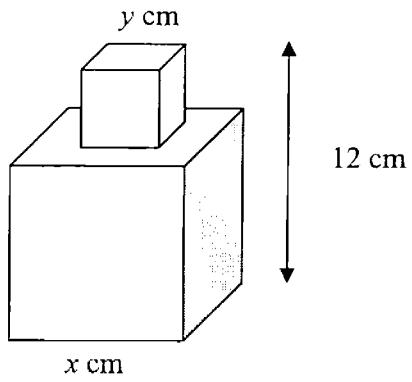
6 **(i)** Given $6\lg\left(x\sqrt{y}\right) = 8 + \lg x^2 - \lg y$, express y in terms of x. [5]

(ii) Solve the equation $\left(\log_5 x\right)^2 = 27\log_x 5$. [4]

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7 Find the value of k for which the coefficient of x^3 in the expansion $(1+kx)^6+(2-3x)^5$ is -920. [5]

8 The diagram shows a solid made up of a smaller cube of side y cm placed on top of a larger cube of side x cm. The height of the solid is 12 cm and its volume is 756 cm^3.



(i) Show that $x^2-xy+y^2=63$. [2]

(ii) Hence find the volume of the smaller cube. [4]

9 It is given that $f(x)=3\cos x$ and $g(x)=-\sin 2x$ for $0\le x\le 2\pi$.

(i) State the amplitude and the period, in radians, of function g. [2]

(ii) Solve the equation $f(x)=g(x)$. [4]

(iii) Sketch, on the same axes provided, the graphs of $y=f(x)$ and $y=g(x)$. [4]



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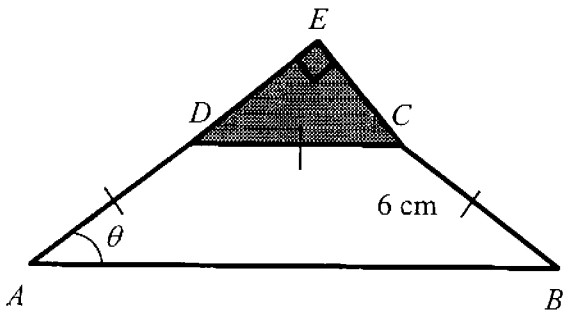
(iv) Hence state the number of solution(s) for the equation $f(x)-3=g(x)$. [1]

10 **(i)** Find the range of values of x for which $2x(x-3)\le 3-5x$. [3]

(ii) Find the range of values of k for which the line $2y+k=x$ intersects the curve $y=2x+\frac{6}{x}$ at 2 distinct points. [5]

(iii) A curve has the equation $y=px^2+(2p+3)x+p-3$. Find the range of values of p for which the curve lies completely below the x-axis. [3]

11 The diagram shows a trapezium $ABCD$ where AB is parallel to DC, $AD=DC=CB=6$ cm and angle $DAB=\theta$. CDE is a triangle with angle $CED=90°$ and ADE is a straight line.



(i) Show that the length, P cm, of the perimeter of figure $ABCED$ is given by $P=18+18\cos\theta+6\sin\theta$. [3]

(ii) Express P in the form $P=18+R\cos(\theta-\alpha)$, where R is a positive constant and $0°<\alpha<90°$. [4]

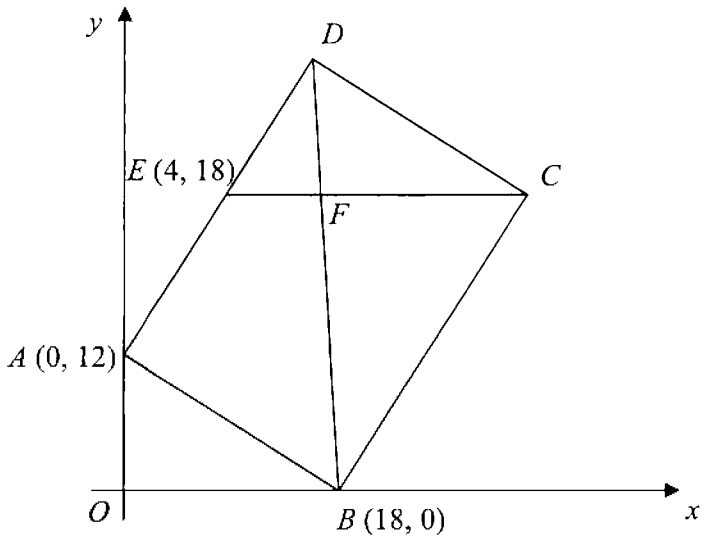
Given that θ varies,

(iii) find the maximum value of P and state the corresponding value of θ, [2]

(iv) find the value of θ for which $P=30$ cm. [2]

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12 The diagram shows a rectangle $ABCD$, where A is (0, 12) and B is (18, 0). E is a point on AD such that EC is parallel to the x-axis and the coordinates of E is (4, 18). Diagonal BD cuts EC at the point F.



Find

(i) the coordinates of C, [3]

(ii) the coordinates of F, [6]

(iii) the area of triangle ABF. [2]

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2020 4E/5N A Math Paper 1 Marking Scheme

Qn No.	Solution	Marks
1 [4]	$\pi rl = \pi\left(6+\sqrt{2}\right)$ $r = \frac{6+\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$ $r = \frac{18-6\sqrt{2}+3\sqrt{2}-2}{3^2-(\sqrt{2})^2}$ $r = \frac{16-3\sqrt{2}}{7}$ Diameter $2r = \frac{32-6\sqrt{2}}{7}$ $= \frac{32}{7} - \frac{6}{7}\sqrt{2}$	M1 M1 (conjugate) M1 A1
2 [5]	$3x^2+4x-6=0$ $(3\alpha+\beta)+(3\beta+\alpha) = -\frac{4}{3}$ $4\alpha+4\beta = -\frac{4}{3}$ $\alpha+\beta = -\frac{1}{3}$ $(3\alpha+\beta)(3\beta+\alpha) = \frac{-6}{3}$ $9\alpha\beta+3\alpha^2+3\beta^2+\alpha\beta = -2$ $10\alpha\beta+3\left[(\alpha+\beta)^2-2\alpha\beta\right] = -2$ $3(\alpha+\beta)^2+4\alpha\beta = -2$ $3\left(-\frac{1}{3}\right)^2+4\alpha\beta = -2$ $4\alpha\beta = -2-\frac{1}{3}$ $\alpha\beta = -\frac{7}{12}$ Hence equation is $x^2+\frac{1}{3}x-\frac{7}{12}=0$ or $12x^2+4x-7=0$	M1 (sum of roots) M1 (product of roots) M1 M1(attempt to find $\alpha+\beta$ and $\alpha\beta$ A1

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3(i) [3]	Let $x=0,\ y=-2+\lvert 5-2(0)\rvert=3$ $\therefore A(0,3)$ Let $5-2x=0 \Rightarrow x=2.5,\ y=-2$ $\therefore B(2.5,-2)$ Let $y=0,\ -2+\lvert 5-2x\rvert=0$ $5-2x=2$ or -2 $x=1.5$ or $x=3.5$ $\therefore C(3.5,0)$	B1 B1 B1
(ii) [1]	$1.5 \le x \le 3.5$	B1
4(i) [2]	$\mathrm{f}(x)=x^3-2x^2+kx-3$ $\mathrm{f}(3)=0$ $3^3-2(3)^2+3k-3=0$ $k=-2$	M1 A1
(ii) [4]	$\mathrm{f}(x)=x^3-2x^2-2x-3=0$ $(x-3)(x^2+bx+1)=0$ Comparing $x^2, b-3=-2$ $b=1$ $\therefore \mathrm{f}(x)=(x-3)(x^2+x+1)=0$ $x-3=0$ or $x^2+x+1=0$ $x=3$ $\quad b^2-4ac=1^2-4(1)(1)$ $=-3<0$ No real roots hence f(x) has only one real root $x=3$.	M1 (comparing coeff or long division) M1 (correct factors) M1(must equate to 0) A1
(iii) [2]	$(4^y)^3-2(4^y)^2+k(4^y)-3=0$ let $4^y=x$ $\lg 4^y=\lg 3$ $y=\dfrac{\lg 3}{\lg 4}=0.792$	M1 A1

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5(i) [3]

T	1	3	5	7
$lg\ P$	2.04	2.74	3.44	4.13

Best fit line & all points correct [B2]
- Minus one mark for any incorrect point

Label axes correctly with the correct scale [B1]

(ii) [3]

$P = h(10^{kT})$

$\lg P = \lg h + \lg 10^{kT}$

$\lg P = kT + \lg h$

k = gradient

$= \dfrac{4.0 - 1.7}{6.6 - 0}$

$\therefore k = 0.348$ (accept 0.3 to 0.4)

M1

A1

$\lg \underline{h} = \lg P$-intercept

$\lg h = 1.7$ (accept 1.6 to 1.8)

$h = 10^{1.7} = 50.1$

A1

(iii) [1]

Value of h represents the initial/ original number of bacteria.

B1

(iv) [1]

$T = 4, \lg P = 3.1$

$P = 1258.925412$

$P = 1260$

B1 (Explain how to get P from lg P)
Accept T=11 too

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6(i) [1]	Centre A lies on $x=1$ and line from centre is perpendicular to the tangent at point B, the vertical length from A to B is 10 units. (r=10) Therefore , A is $(1,-5)$	B1
(ii)	$(x-1)^2+(y+5)^2=100$	B1
(iii) [3]	M and N are y-intercepts, let $x=0$, $(0-1)^2+(y+5)^2=100$ $(y+5)^2=100-1$ $y+5=\pm\sqrt{99}$ $y=-5+3\sqrt{11}$ or $-5-3\sqrt{11}$ Length $MN=(-5+3\sqrt{11})-(-5-3\sqrt{11})$ $=6\sqrt{11}$	M1 M1 A1
(iv) [2]	$x^2+y^2-30x+10y+150=0$ Either by formula or completing the square method, Centre of $C_2=(15,-5)$ Midpoint of the 2 centres $=\left(\frac{15+1}{2},-5\right)=(8,-5)$ Hence $x=8=k$	M1 A1
7(i) [3]	$\cos^4 A-\sin^4 A$ $=(\cos^2 A-\sin^2 A)(\cos^2 A+\sin^2 A)$ $=(\cos^2 A-\sin^2 A)(1)$ $=\cos 2A$	M1(Factorise) M1($\cos^2 A+\sin^2 A=1$) M1($\cos^2 A+\sin^2 A=\cos 2A$
(ii) [4]	$2\cos^4 A-2\sin^4 A=4\sin A-1$ $2\cos 2A=4\sin A-1$ $4\sin^2 A+4\sin A-3=0$ $(2\sin A+3)(2\sin A-1)=0$ $\sin A=-\frac{3}{2}(NA)$ or $\sin A=\frac{1}{2}$ basic $\angle=\sin^{-1}\left(\frac{1}{2}\right)=30^\circ$ $A=30^\circ$ or 150°	M1(use identity from (i)) M1(factorise eqn in sin A) M1(rej -1.5) A1
(iii) [2]	$8(\cos^4 x-\sin^4 x)=3\sin 2x$ $8\cos 2x=3\sin 2x$ $\tan 2x=\frac{8}{3}=2\frac{2}{3}$	M1 A1

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8(i) [1]	$v = 40e^{-0.2t} - 5$ At P, $t = 0$, $v = 40e^0 - 5 = 35$ $\therefore$ velocity is 35 m/s	B1
(ii) [1]	$a = \frac{dv}{dt} = 40(-0.2)e^{-0.2t}$ $a = -8e^{-0.2t}$	B1
(iii) [1]	The negative acceleration shows that he slows down after passing point P.	B1
(iv) [2]	At Q, $v = 0$ $v = 40e^{-0.2t} - 5 = 0$ $e^{-0.2t} = \frac{5}{40}$ $\ln e^{-0.2t} = \ln \frac{1}{8}$ $-0.2t = \ln \frac{1}{8}$ $t = \ln \frac{1}{8} \div (-0.2) = 10.3972$ $t = 10.4s$	M1 A1
(v) [3]	$s = \frac{40e^{-0.2t}}{-0.2} - 5t + c$ $s = -200e^{-0.2t} - 5t + c$ When $t = 0, s = 0$, $0 = -200 + c \Rightarrow c = 200$ $\therefore s = -200e^{-0.2t} - 5t + 200$ When $t = 10.3972$, $s = PQ$ $= -200e^{-0.2(10.3972)} - 5(10.3972) + 200$ $= 123.0139615 \simeq 123m$ (3sf)	M1 M1 A1
OR	$PQ = \int_0^{10.3972} \left(40e^{-0.2t} - 5\right) dt$ $= \left[\frac{40e^{-0.2t}}{-0.2} - 5t\right]_0^{10.3972}$ $= \left[-200e^{-0.2t} - 5t\right]_0^{10.3972}$ $= \left[-200e^{-0.2(10.3972)} - 5(10.3972)\right] - \left[-200e^0 - 0\right]$ $= 123.0139615$ $= 123m$	

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9(i) [2]	Perimeter = 100 $3x+2y=100$ $2y=100-3x$ $y=\dfrac{100-3x}{2}$	 M1 A1
(ii) [2]	$A=\dfrac{1}{2}x^2\sin 60^o+xy$ $A=\dfrac{1}{2}x^2\left(\dfrac{\sqrt{3}}{2}\right)+x\left(\dfrac{100-3x}{2}\right)$ $A=\dfrac{\sqrt{3}}{4}x^2+\dfrac{100x-3x^2}{2}$ $A=\dfrac{\sqrt{3}}{4}x^2+50x-\dfrac{3}{2}x^2$ $A=\dfrac{\sqrt{3}-6}{4}x^2+50x$	M1 M1
(iii) [4]	$A=\dfrac{\sqrt{3}-6}{4}x^2+50x$ $\dfrac{dA}{dx}=\dfrac{\sqrt{3}-6}{2}x+50$ $\dfrac{dA}{dx}=0$ $\dfrac{\sqrt{3}-6}{2}x=-50$ $x=-50\div\left(\dfrac{\sqrt{3}-6}{2}\right)$ $x=23.4m$	 M1, M1 M1 A1
(iv) [2]	$\dfrac{dA}{dx}=\dfrac{\sqrt{3}-6}{2}x+50$ $\dfrac{d^2A}{dx^2}=\dfrac{\sqrt{3}-6}{2}$ $=-2.13<0$ $\therefore$ Area is a maximum Children/ Teacher might be happy as this design will end up with a **maximum** area which means the biggest **land area** for them to play.	 M1(any appropriate method to prove nature of Area) A1

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10(i) [3]	$y=\dfrac{2x-1}{x+4},\ x\neq -4$ $\dfrac{dy}{dx}=\dfrac{2(x+4)-(2x-1)}{(x+4)^2}$ $=\dfrac{2x+8-2x+1}{(x+4)^2}$ $=\dfrac{9}{(x+4)^2}$ >0 since $(x+4)^2>0$ $\dfrac{dy}{dx}\neq 0,\ \therefore$ curve has no turning points.	M1 M1(correct dy/dx) M1(state $(x+4)^2>0$)
(ii) [3]	At $B(5, 1)\ \dfrac{dy}{dx}=\dfrac{9}{(5+4)^2}=\dfrac{1}{9}$ Gradient of normal = -9 Eqn of normal: $y-1=-9(x-5)$ $y=-9x+46$	M1 M1 A1
(iii) [5]	by long division $\dfrac{2x-1}{x+4}=2-\dfrac{9}{x+4}$ Area of shaded region $=\int_{0.5}^{5}2-\dfrac{9}{x+4}\,dx-\dfrac{1}{2}(5-0.5)(1)$ $=\left[2x-9\ln(x+4)\right]_{0.5}^{5}-\dfrac{9}{4}$ $=\left[(10-9\ln 9)-(1-9\ln 4.5)\right]-\dfrac{9}{4}$ $=0.512$ units^2	A1 M1,M1 M1 A1

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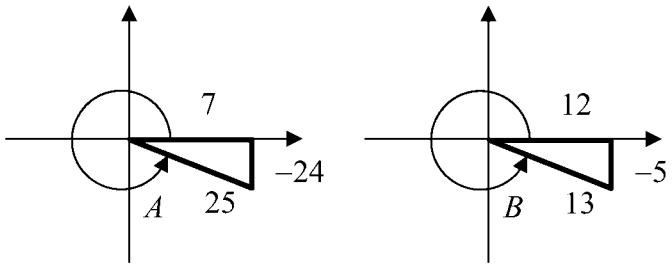
11(i) [2]	gradient of tangent $= \frac{3-(-3)}{0-2} = -3$ At $(2,-3)$, $\frac{dy}{dx} = 3(2) - \frac{k}{2^2} = -3$ $6 - \frac{k}{4} = -3$ $\frac{k}{4} = 9$ $k = 36$	M1 A1
(ii) [4]	$y = \int 3x - \frac{36}{x^2} dx$ $y = \frac{3x^2}{2} - \frac{36x^{-1}}{-1} + c$ subst $(2,-3)$, $-3 = \frac{3}{2}(2)^2 + 36(2)^{-1} + c$ $c = -27$ Eqn of curve: $y = \frac{3x^2}{2} + \frac{36}{x} - 27$	M1, M1(+c) M1(show subt of pt) A1

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AMKSS 2020 Prelim 4E5N AM P2 Answer Scheme

	Answer	Marks
1 **[6]**	$\frac{dy}{dx}=\left(-\frac{1}{2}(2x)\right)e^{-\frac{1}{2}x^2}=-xe^{-\frac{1}{2}x^2}$ $\int_0^1 1-xe^{-\frac{1}{2}x^2}\,dx$ $=\int_0^1 1\,dx+\int_0^1 -xe^{-\frac{1}{2}x^2}\,dx$ $=\left[x\right]_0^1+\left[e^{-\frac{1}{2}x^2}\right]_0^1$ $=\left[1-0\right]+\left[e^{-\frac{1}{2}}-e^0\right]$ $=1+e^{-\frac{1}{2}}-1$ $=e^{-\frac{1}{2}}$	M1, A1 (or B2) M1 (integrate $\int_0^1 1\,dx$), M1 (integrate using $\frac{dy}{dx}$) M1 (evaluate from $x=0$ to $x=1$) A1

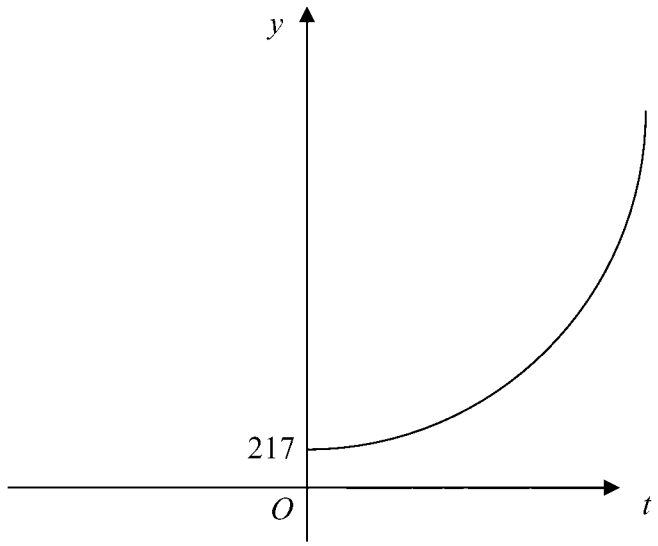
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2(i) **[2]**	 $\sec B = \dfrac{1}{\frac{12}{13}}$ $= \dfrac{13}{12}$	M1 (apply identity and correct cos B) A1
2(ii) **[2]**	$\tan A = -\dfrac{24}{7}$ $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $= \dfrac{\left(-\frac{24}{7}\right) + \left(-\frac{5}{12}\right)}{1 - \left(-\frac{24}{7}\right)\left(-\frac{5}{12}\right)}$ $= \dfrac{323}{36}$	M1 (apply identity and correct tan A) A1
2(ii) **[3]**	$\cos A = 2\cos^2\left(\dfrac{A}{2}\right) - 1$ $\dfrac{7}{25} = 2\cos^2\left(\dfrac{A}{2}\right) - 1$ $\cos^2\left(\dfrac{A}{2}\right) = \dfrac{16}{25}$ $\cos\left(\dfrac{A}{2}\right) = \dfrac{4}{5}$ (Rejected) or $-\dfrac{4}{5}$ (since $135° < \dfrac{A}{2} < 180°$)	M1 (apply identity and sub in cos A) M1 A1
3 **[7]**	$y = x\sin 2x$ $\dfrac{dy}{dx} = (\sin 2x)(1) + x(2\cos 2x)$ $= \sin 2x + 2x\cos 2x$ $\dfrac{d^2y}{dx^2} = 2\cos 2x + \left[(\cos 2x)(2) + 2x(-2\sin 2x)\right]$ $= 2\cos 2x + 2\cos 2x - 4x\sin 2x$ $= 4\cos 2x - 4x\sin 2x$ $4x(x\sin 2x) + x(4\cos 2x - 4x\sin 2x) - 2(\sin 2x + 2x\cos 2x)$ $= 4x^2\sin 2x + 4x\cos 2x - 4x^2\sin 2x - 2\sin 2x - 4x\cos 2x$ $= -2\sin 2x$ $k = -2$	M1, M1 (for each term) M1 for $\dfrac{d}{dx}(\sin 2x) = 2\cos 2x$, M1, M1 for each term in product rule M1 (subs and expand) A1

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4(i) **[5]**	$y = \ln\sqrt{5-2x} = \frac{1}{2}\ln(5-2x)$ $\frac{dy}{dx} = \frac{1}{2}\left(\frac{-2}{5-2x}\right)$ $= \frac{-1}{5-2x}$ Alternatively, $y = \ln\sqrt{5-2x} = \ln(5-2x)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{\frac{1}{2}(5-2x)^{-\frac{1}{2}}(-2)}{(5-2x)^{\frac{1}{2}}}$ $= \frac{-1}{5-2x}$ $y - 4x = 1$ Gradient of normal = 4 $\frac{dy}{dx} = -\frac{1}{4}$ $\frac{-1}{5-2x} = -\frac{1}{4}$ $4 = 5-2x$ $2x = 1$ $x = \frac{1}{2}$ $y = \ln\sqrt{5-2\left(\frac{1}{2}\right)} = \ln 2$ $P\left(\frac{1}{2}, \ln 2\right)$	M1 A1 M1 M1 A1
4(i) **[3]**	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $-0.1 = \frac{-1}{5-2x} \times 0.2$ $-0.5 + 0.2x = -0.2$ $0.2x = 0.3$ $x = 1.5$ $y = \ln\sqrt{2}$ $Q\left(1.5,\ \ln\sqrt{2}\right)$	M1 (apply chain rule and use $\frac{dy}{dt} = -0.1$) M1 A1

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5(i) **[3]**	$10P_0 = P_0e^{k(9)}$ $10 = e^{9k}$ $\ln 10 = 9k \ln e$ $k = 0.2558427881$ $= 0.256$ (3 s.f.)	M1 M1 (take ln on both sides) A1
5(ii) **[2]**	$468000 = P_0e^{0.2558(30)}$ $P_0 = 217.2263574$ $= 217$	M1 A1
5(iii) **[2]**		B1 (correct shape above t-axis and only for $t > 0$) B1 (correct y-intercept based on their (ii) answer)
5(iv) **[1]**	The number of people infected becomes infinite as t becomes very large / is always increasing. Not acceptable: Too large, cannot be calculated, math error etc.	B1

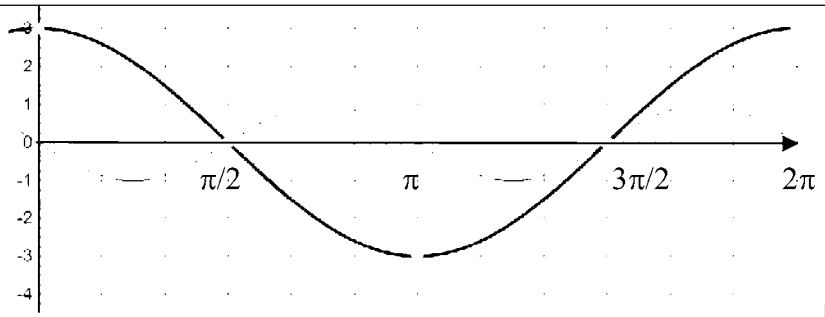
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6(i) **[5]**	$6\lg x\sqrt{y} = 8 + \lg x^2 - \lg y$ $\lg\left(x\sqrt{y}\right)^6 - \lg x^2 + \lg y = 8$ $\lg x^6 y^3 - \lg x^2 + \lg y = 8$ $\lg \dfrac{\left(x^6 y^3\right) y}{x^2} = 8$ $\lg x^4 y^4 = 8$ $4\lg xy = 8$ $\lg xy = 2$ $xy = 10^2$ $y = \dfrac{100}{x}$	M1 (apply law 3: $6\lg x\sqrt{y} = \lg\left(x\sqrt{y}\right)^6$) M1 (combine lg) M1 (simplify indices) M1 (change to index form) A1
6(ii) **[4]**	$\left(\log_5 x\right)^2 = 27\log_x 5$ $\left(\log_5 x\right)^2 = 27\left(\dfrac{\log_5 5}{\log_5 x}\right)$ $\left(\log_5 x\right)^2 = \dfrac{27}{\log_5 x}$ $\left(\log_5 x\right)^3 = 27$ $\log_5 x = 3$ $x = 5^3 = 125$	M1 (change of base) M1 (simplify $\log_5 5 = 1$ and form cubic) M1 (find $\log_5 x$) A1

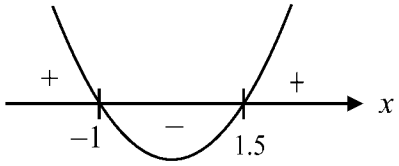
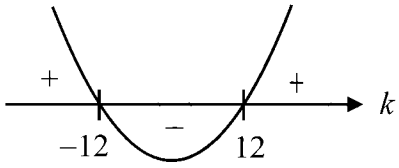
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7 **[5]**	$(1+kx)^6+(2-3x)^5$ Term in $x^3 = {}^6C_3(kx)^3+{}^5C_3(2)^2(3x)^3$ ${}^6C_3(k)^3+{}^5C_3(2)^2(3)^3=-920$ $20k^3-1080=-920$ $k^3=8$ $k=2$	B1, B1 M1 (equate to sum of coeff to –920) M1 (simplify) A1
8(i) **[2]**	$x^3+y^3=756$ $(x+y)(x^2-xy+y^2)=756$ $12(x^2-xy+y^2)=756$ $x^2-xy+y^2=63$ (Shown)	M1 (factorise using sum of cubes) M1 (subs in volume and $x+y$)
8(ii) **[4]**	$x^2-xy+y^2=63...(1)$ $x+y=12...(2)$ From (2): $x=12-y...(3)$ (3) subs into (1): $(12-y)^2-(12-y)y+y^2=63$ $144-24y+y^2-12y+y^2+y^2=63$ $3y^2-36y+81=0$ $y^2-12y+27=0$ $(y-3)(y-9)=0$ $y=3$ or $y=9$ (rejected) Volume of smaller cube = $3^3=27\ \text{cm}^3$.	M1 (eliminate x or y) M1 (correct quadratic equation) M1 (factorise/use quadratic formula) A1

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9(i) **[2]**	amplitude = 1; period = π	B1, B1
9(ii) **[4]**	$3\cos x = -\sin 2x$ $3\cos x + \sin 2x = 0$ $3\cos x + 2\sin x\cos x = 0$ $\cos x(3 + 2\sin x) = 0$ $\cos x = 0$ or $\sin x = -1.5$ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ (rejected)	M1 (use identity) M1 (factorise) A1, A1
9(iii) **[4]**		For $y = 3\cos x$ B1 for 1 cycle of cosine graph with amplitude 3. B1 for correct intercepts and max/min points. For $y = -\sin 2x$ B1 for 2 cycle of inverted sine graph with amplitude 1. B1 for correct x-intercepts.
9(iv) **[1]**	3 solutions	B1

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10(i) **[3]**	$2x(x-3) \le 3-5x$ $2x^2-6x \le 3-5x$ $2x^2-x-3 \le 0$ $(2x-3)(x+1) \le 0$  $-1 \le x \le 1.5$	M1 (expand and simplify RHS = 0) M1 (factorise) A1
10(ii) **[5]**	$2y+k=x \ldots (1)$ $y=2x+\frac{6}{x} \ldots (2)$ Subs. (2) into (1): $2\left(2x+\frac{6}{x}\right)+k=x$ $4x+\frac{12}{x}-x+k=0$ $3x+\frac{12}{x}+k=0$ $3x^2+kx+12=0$ $(k)^2-4(3)(12)>0$ $k^2-144>0$ $(k+12)(k-12)>0$  $k<-12$ or $k>12$	M1 (eliminate x or y) A1 (correct quadratic) M1 (any use of discriminant (correct a, b, c) M1 (factorise) A1
10(iii) **[3]**	Curve lies completely below the x-axis $y=px^2+(2p+3)x+p-3$ has no real roots $(2p+3)^2-4(p)(p-3)<0$ $4p^2+12p+9-4p^2+12p<0$ $24p+9<0$ $24p<-9$ $p<-\frac{3}{8}$	 M1 (correct a, b, c) M1 (discriminant < 0) A1

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11(i) **[3]**	Let F be a point on AB such that DF is perpendicular to AB. $\cos\theta = \frac{AF}{6}$ $AF = 6\cos\theta$ $\angle EDC = \angle DAB = \theta$ (corresponding angles) $\cos\theta = \frac{DE}{6}$ $DE = 6\cos\theta$ $\sin\theta = \frac{CE}{6}$ $CE = 6\sin\theta$ $P = AB + BC + CE + ED + DA$ $= 6\cos\theta + 6 + 6 + 6\cos\theta + 6\sin\theta + 6\cos\theta + 6$ $= 18 + 18\cos\theta + 6\sin\theta$	M1 (minus 1 if no working shown) M1 M1
11(ii) **[4]**	$18\cos\theta + 6\sin\theta = R\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $R\cos\alpha = 18$ $R\sin\alpha = 6$ $R = \sqrt{18^2 + 6^2} = \sqrt{360}$ (accept 19.0) $\tan\alpha = \frac{R\sin\alpha}{R\cos\alpha} = \frac{6}{18}$ $\alpha = 18.43°$ $P = 18 + \sqrt{360}\cos(\theta - 18.4°)$ (1 d.p.)	M1 M1 M1 A1
11(iii) **[2]**	Max $P = 18 + \sqrt{360}$ $= 36.97366596 = 37.0$ cm (3s.f.) $\cos(\theta - 18.4°) = 1$ $\theta - 18.4° = 0°$ $\theta = 18.4°$	A1 A1
11(iv) **[2]**	$18 + \sqrt{360}\cos(\theta - 18.43°) = 30$ $\cos(\theta - 18.43°) = \frac{12}{\sqrt{360}}$ $\theta - 18.43° = 50.76°$ $\theta = 69.20342834 = 69.2°$(1 d.p.)	M1 A1

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12(i) **[4]**	Let coordinates of C be (p, q) Gradient of $AE = \frac{18-12}{4-0} = \frac{3}{2}$ Gradient of $BC = \frac{18-0}{p-18}$ $\frac{18-0}{p-18} = \frac{3}{2}$ $36 = 3p - 54$ $3p = 90$ $p = 30$ C (30, 18)	M1 M1 (equate gradient/substitute into equation) A1
12(ii) **[6]**	Midpoint of AC $= \left(\frac{0+30}{2}, \frac{12+18}{2}\right) = (15,15)$ Let coordinates of D be (a, b) $\left(\frac{a+18}{2}, \frac{b}{2}\right) = (15,15)$ $\frac{a+18}{2} = 15 \qquad \frac{b}{2} = 15$ $a = 12 \qquad b = 30$ D (12, 30) Gradient of $BD = \frac{30-0}{12-18} = -5$ Equation of BD: $y - 0 = -5(x - 18)$ $y = -5x + 90$ $18 = -5x + 90$ $5x = 72$ $x = 14.4$ F (14.4, 18)	M1 M1 M1 (if no working shown for D, max 4 out of 6) M1 M1 A1
12(iii) **[2]**	Area of triangle ABF $= \frac{1}{2}\begin{vmatrix} 0 & 18 & 14.4 & 0 \\ 12 & 0 & 18 & 12 \end{vmatrix}$ $= = \frac{1}{2}\left[(0+324+172.8)-(216+0+0)\right]$ $= 140.4$ units^2	M1 A1

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