Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL



ADDITIONAL MATHEMATICS
Paper 1

4047/01

26 August 2019 Monday 2 hours

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2019 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the spaces provided below each question.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
80

Setter: Mdm Manju Manoharan
Vetter: Mr Tan Lip Sing

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!\,r!}=\frac{n(n-1).....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1.$$
$$\sec^2 A=1+\tan^2 A.$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$
$$a^2=b^2+c^2-2bc\cos A.$$
$$\Delta=\frac{1}{2}ab\sin C.$$

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Answer all questions in the space provided.

1 The roots of the equation $x^2 - 12x + 25 = 0$ are $2\alpha + \beta$ and $2\beta + \alpha$.

(i) Find the value of $\alpha + \beta$ and $\alpha\beta$. [4]

(ii) Hence, form an equation whose roots are α and β. [1]

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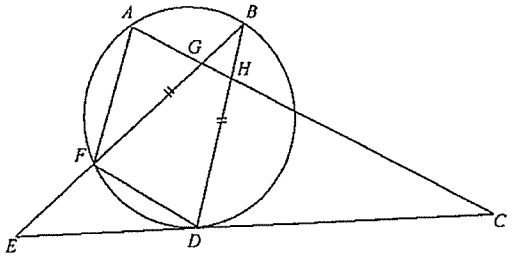
2 The line $y = x + 4$ meets the curve $y^2 = 7 - 2x$ at the points P and Q. Find the equation of the perpendicular bisector of PQ. [5]

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3 Express $\frac{5x+1}{(3-2x)(x^2+2)}$ in partial fractions. [5]

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4 The diagram below shows a circle with points A, B, D and F on its circumference where $BF = BD$.
The tangent to the circle at D meets BF extended at E and AH extended at C.



Prove that

(i) triangle EFD is similar to triangle EDB, [2]

(ii) $ED^2 - EF^2 = EF \times DB$. [3]

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5 (a) Given that $\frac{4^x}{2^{x+1}} = \frac{3}{5^x}$, evaluate 10^x, [3]

(b) A rectangular block has a square base of side $\left(1+\sqrt{3}\right)$cm. The volume of the block is $\left(14+8\sqrt{3}\right)\text{cm}^3$. **Without using a calculator,** find the height of the rectangle, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

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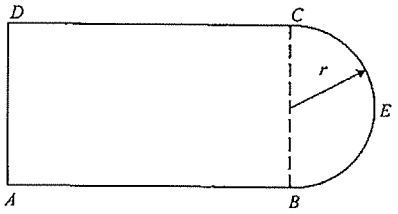
6 A curve has the equation $f(x)$, where $f'(x)=\frac{2x}{x^2-1}$ for $x>0$.

(i) Explain whether $f'(x)$ is an increasing or decreasing function. [3]

(ii) Given that $f(2)=8$, obtain an expression for $f(x)$, leaving your answer in exact form. [3]

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7 A dog park consists of a rectangle $ABCD$ and a semicircles of radius r m. The perimeter of the park is 200 m.

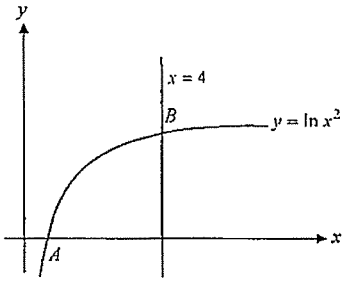


(i) Express the length of AB in terms of r. [2]

(ii) Given that r can vary, find the radius of the semicircle such that the dog park has the largest possible area. [4]

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8 The diagram below shows part of the curve $y = \ln x^2$ for $x > 0$.



(i) Find the exact coordinates of A and of B. [2]

(ii) Determine the area of the shaded region bounded by the curve, the line $x = 4$, and the x-axis. [4]

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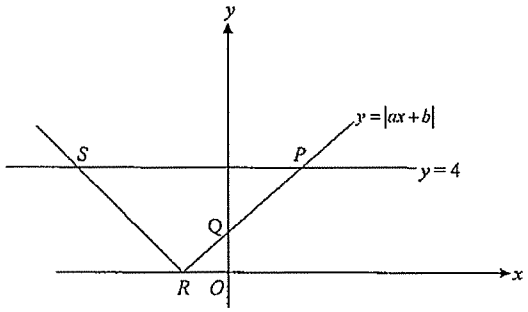
9 The equation of a circle, C is $x^2+y^2-kx+(k+2)y+c=0$, where k is a constant and $c \geq 0$.
The centre of the circle lies on the line $2x+5y+14=0$.

(i) Find the value of the constant k. [4]

(ii) Hence, find the range of values of c. [2]

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10



The diagram shows the line $y = 4$ and the graph of $y = |ax + b|$, where a and b are positive constants. The graph crosses the line $y = 4$ at the points P and S, crosses the y-axis at Q and meets the x-axis at R.

(i) Given that the x-coordinate of P is 1.5 and that $PQ : PR = 3 : 4$, find, with full explanation, the x-coordinate of S. [4]

(ii) Find the value of a and of b. [3]

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Hence,

(ii) state the period, in radians, and amplitude of $4\cos^2 x - 2\sin^2 x$, [2]

(iii) sketch the graph of $y = 4\cos^2 x - 2\sin^2 x$ for $0 \le x \le 2\pi$ radians. [2]



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12 (a) Find all the values of x between $0°$ and $360°$ for which $\sqrt{8}\sin(x-30°)=-1$. [3]

(b) Given that $2<x<6$, find the values of x for which $2\tan^2 x-13=\sec x$. [4]

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13 An object at A, with an initial displacement of 3 m from a fixed point O, travels in a straight line so that its velocity, v m/s , is given by $v = t^2 - 5t + 6$ where t is the time in seconds after leaving A.

(i) Find the values of t when the object comes to an instantaneous rest. [2]

(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [2]

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(iv) Find the average speed of the object in the first 5 seconds. [4]

END OF PAPER

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ADDITIONAL MATHEMATICS 4047/01
Paper 1
26 August 2019 Monday 2 hours

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2019 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

MARKING SCHEME

Setter: Mdm Manju Manoharan
Vetter: Mr Tan Lip Sing

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Answer all questions in the space provided.

1 The roots of the equation $x^2-12x+25=0$ are $2\alpha+\beta$ and $2\beta+\alpha$.

(i) Find the value of $\alpha+\beta$ and $\alpha\beta$. [4]

(ii) Hence, form an equation whose roots are α and β. [1]

(i) Sum of roots $= 2\alpha+\beta+2\beta+\alpha$ $3\alpha+3\beta=12$ $\therefore\ \alpha+\beta=4$ Pdt of roots $=(2\alpha+\beta)(2\beta+\alpha)$ $5\alpha\beta+2(\alpha^2+\beta^2)=25$ $5\alpha\beta+2[(\alpha+\beta)^2-2\alpha\beta]=25$ $\alpha\beta+2(\alpha+\beta)^2=25$ $\alpha\beta=25-2(4)^2$ $=-7$	B1 M1 M1 [Seen $(\alpha+\beta)^2-2\alpha\beta$] A1
(ii) Equation: $x^2-4x-7=0$	B1✓ [follow through]

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6 A curve has the equation $f(x)$, where $f'(x)=\frac{2x}{x^2-1}$ for $x>0$.

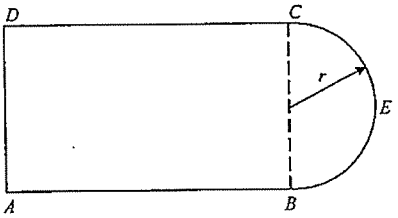
(i) Explain whether $f'(x)$ is an increasing or decreasing function. [3]

(ii) Given that $f(2)=8$, obtain an expression for $f(x)$. Leave your answer in exact form. [3]

(i) $f'(x)=\frac{2x}{x^2-1}$ $f''(x)=\frac{(x^2-1)(2)-(2x)(2x)}{(x^2-1)^2}$ $=\frac{-2x^2-2}{(x^2-1)^2}$ $=\frac{-2(x^2+1)}{(x^2-1)^2}$. For all $x>0$, $x^2+1>0$ and $(x^2-1)^2>0$ $\therefore \frac{-2(x^2+1)}{(x^2-1)^2}<0$ Hence, $f'(x)$ is a decreasing function for all $x>0$.	M1 [use of quotient rule accurately] M1[either numerator or denominator > 0 seen] A1 [with conclusion]
(ii) $f'(x)=\frac{2x}{x^2-1}$ $f(x)=\int\frac{2x}{x^2-1}dx$ $=\ln(x^2-1)+c$ When $f(2)=8$, $8=\ln(4-1)+c$ $c=8-\ln 3$ $\therefore f(x)=\ln(x^2-1)+8-\ln 3$	M1 M1 [Finding the constant, c] A1 [Only accept exact form]

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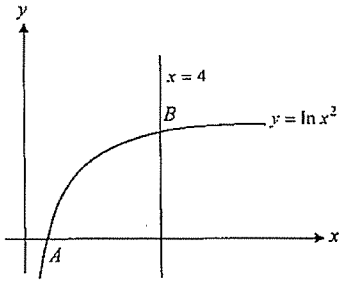
(i) Express the length of AB in terms of x. [2]

(ii) Given that x can vary, find the radius of the semicircle such that the dog park has the largest possible area. [4]

(i) $200 = 2AB + 2r + \frac{1}{2}(2\pi r)$ $AB = \frac{200 - 2r - \pi r}{2}$ $= 100 - r - \frac{1}{2}\pi r$	M1 A1
(ii) Area of park $= 2r(AB) + \frac{1}{2}(\pi r^2)$ $A = 2r\left(100 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}(\pi r^2)$ $A = 200r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$ $A = 200r - 2r^2 - \frac{1}{2}\pi r^2$ $\frac{dA}{dr} = 200 - 4r - \pi r$ When $\frac{dA}{dr} = 0$, $200 - 4r - \pi r = 0$ $r = \frac{200}{\pi + 4}$ $\frac{d^2A}{dr^2} = -4 - \pi < 0$ $\therefore A$ is maximum	M1 [with subst] M1 A1 M1

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8 The diagram below shows part of the curve $y = \ln x^2$ for $x > 0$.



(i) Find the exact coordinates of A and B. [2]

(ii) Determine the area of the shaded region bounded by the curve, the line $x = 4$, and the x-axis. [4]

(i) $y = \ln x^2$ When $x = 4, y = \ln 16 / 4 \ln 2$ When $y = 0, x = \pm 1$ A (1, 0) and B (4, ln 16)	B1, 1
(ii) $y = \ln x^2$ $x = e^{\frac{y}{2}}$ $\int_0^{\ln 16} e^{\frac{y}{2}} dy$ $= \left[2e^{\frac{y}{2}} \right]_0^{\ln 16}$ $= 2\left[e^{\ln 4} - 1 \right]$ $= 6$ Area of shaded region = $4 \ln 16 - 6$ $= 16 \ln 2 - 6$ sq units = 5.09 sq units	M1 [making x the subject] M1 [with correct limits from (i)] M1 [seen 4ln16 or equivalent from (i)] A1 [accept all proper exp of 16ln2]

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9 The equation of a circle, C is $x^2+y^2-kx+(k+2)y+c=0$, where k is a constant and $c \geq 0$. The centre of the circle lies on the line $2x+5y+14=0$.

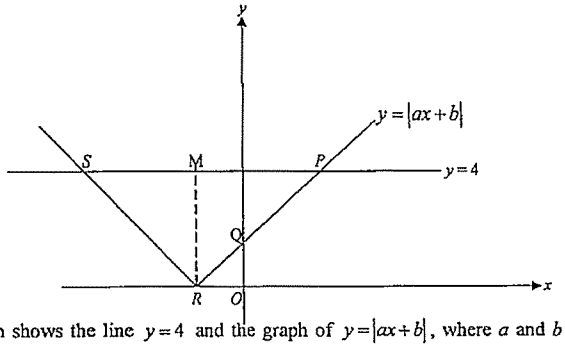
(i) Find the value of the constant k. [4]

(ii) Hence, find the range of values of c. [2]

(i) $x^2+y^2-kx+(k+2)y+c=0$ $2g=-k$ and $2f=k+2$ $g=-\frac{k}{2}$ $f=\frac{k+2}{2}$ $\therefore$ Centre $\left(\frac{k}{2},-\frac{k+2}{2}\right)$ Since the centre lies on $2x+5y+14=0$, $2\left(\frac{k}{2}\right)+5\left(-\frac{k+2}{2}\right)+14=0$ $5\left(-\frac{k+2}{2}\right)=-k-14$ $-5k-10=-2k-28$ $3k=18$ $k=6$	B1 [accurate to find g and f] M1 M1 [method shown to simplify] A1
(ii) Since $k=6$, $r=\sqrt{\left(\frac{6}{2}\right)^2+\left(-\frac{6+2}{2}\right)^2-c}$ $r=\sqrt{-c+25}$ $\therefore 0 \leq c < 25$	M1 A1

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The diagram shows the line $y=4$ and the graph of $y=|ax+b|$, where a and b are positive constants. The graph crosses the line $y=4$ at the points P and S, crosses the y-axis at Q and meets the x-axis at R.

(i) Given that the x-coordinate of P is 1.5 and that $PQ : PR = 3 : 4$, find, with full explanation, the x-coordinate of S. [4]

(ii) Find the value of a and of b. [3]

(i) $\frac{PQ}{PR}=\frac{1.5}{PM}$ $\frac{3}{4}=\frac{1.5}{PM}$ $PM=2$ units $\therefore x-$coord of $M=-0.5$ $PS=2PM=4$ units $\therefore x$-coord of $S=-2.5$	M1 [use of ratio] A1 [x-coord of M] M1[length of PS] A1
(ii) $R(-0.5,0)$ Gradient of PR $=\frac{0-4}{-0.5-1.5}=2$ Eqn of PR: $y-4=2(x-1.5)$ $y=2x+1$ $\therefore a=2,\ b=1$	 M1 [or finding gradient of SR $=-2$] M1 [subst into equation] A1

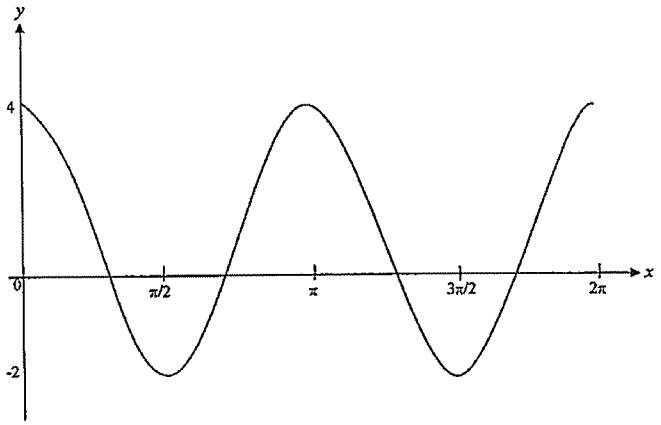
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11 (i) Show that $4\cos^2 x - 2\sin^2 x$ can be written as $a\cos 2x + b$, where a and b are integers. [2]

Hence,

(ii) state the period, in radians, and amplitude of $4\cos^2 x - 2\sin^2 x$, [2]

(iii) sketch the graph of $y = 4\cos^2 x - 2\sin^2 x$ for $0 \le x \le 2\pi$ radians. [2]

(i) $4\cos^2 x - 2\sin^2 x$ $= 2\left(2\cos^2 x\right) - 2\sin^2 x$ $= 2\left(\cos 2x + 1\right) - \left(1 - \cos 2x\right)$ $= 2\cos 2x + 2 - 1 + \cos 2x$ $= 3\cos 2x + 1$	M1 [Double angle formula on $\sin^2 x$ or $\cos^2 x$] A1
(ii) Amplitude = 3 Period = π	B1 B1
(iii)  C1 – Correct shape and amplitude C1 – Correct period	

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12 (a) Find all the values of x between 0° and 360° for which $\sqrt{8}\sin(x-30°)=-1$. [3]

(b) Given that $2<x<6$, find the values of x for which $2\tan^2 x-13=\sec x$. [4]

(a) $\sqrt{8}\sin(x-30°)=-1$ $\sin(x-30°)=-\frac{1}{\sqrt{8}}$ For $x-30°$ to exist in this interval, $-30°<x-30°<330°$ basic angle, $\alpha=\sin^{-1}\left(\frac{1}{\sqrt{8}}\right)=20.704°$ $x-30°=-20.704°,180°+20.704°,360-20.704°(rej)$ $x-30°=-20.704°,200.704°$ $x=9.295°,230.704°$ $x=9.3°,230.7°$	M1[basic angle without correct value] M1[for any one correct angle] A1 [correct to 1 d.p]
(b) $2\tan^2 x-13=\sec x$ $2(\sec^2 x-1)-13=\sec x$ $2\sec^2 x-15=\sec x$ $2\sec^2 x-\sec x-15=0$ $(2\sec x+5)(\sec x-3)=0$ $2\sec x+5=0$ or $\sec x=3$ $\sec x=-\frac{5}{2}$ or $\cos x=\frac{1}{3}$ $\cos x=-\frac{2}{5}$ $x=1.2309(rej), 2\pi-1.2309$ $x=\pi-1.159$ (rej), $\pi+1.159$ $x=4.30$, 5.05	M1 [use of identity] M1 M1 [solving for both 1 factors with correct fraction. NO negative] A1

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(i) Find the values of t when the object comes to an instantaneous rest. [2]

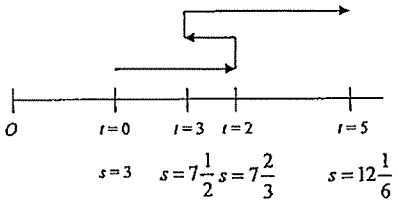
(ii) Find the acceleration of the object at $t = 5$ s. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the object from O after t seconds. [2]

(iv) Find the average speed of the object in the first 5 seconds. [4]

(i) When at instantaneous rest, $v = 0$ $v = t^2 - 5t + 6$ $t^2 - 5t + 6 = 0$ $(t-3)(t-2) = 0$ $t = 3$ or 2	M1[equate to 0] A1
(ii) $a = \frac{dv}{dt}$ $= 2t - 5$ When $t = 5$, $a = 2(5) - 5$ $= 5 \text{ m/s}^2$	M1 [Finding a] A1
(iii) $s = \int (t^2 - 5t + 6)\, dt$ $= \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$ When $t = 0$, $s = 3$, $c = 3$. $\therefore s = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + 3$	M1 A1

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(iv) When $t=2$, $s=\frac{2^3}{3}-\frac{5(2)^2}{2}+6(2)+3$ $=7\frac{2}{3}$ When $t=3$, $s=\frac{3^3}{3}-\frac{5(3)^2}{2}+6(3)+3$ $=7\frac{1}{2}$	M1 [finding 2 out 3 relevant s values]
When $t=5$, $s=\frac{5^3}{3}-\frac{5(5)^2}{2}+6(5)+3$ $=12\frac{1}{6}$	M1 [finding 3rd relevant s value]
 $s=3$ $s=7\frac{1}{2}$ $s=7\frac{2}{3}$ $s=12\frac{1}{6}$	
Average speed in first 5 s $=\frac{\left(7\frac{2}{3}-3\right)+\left(7\frac{2}{3}-7\frac{1}{2}\right)+\left(12\frac{1}{6}-7\frac{1}{2}\right)}{5}$ $=\frac{9.5}{5}$ $=1.9$ m/s	M1 [2 out 3 dist calculated correctly based on above values] A1

[Full 4 marks if equation used does not include $c=3$ from (iii)]

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