Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Wednesday
15 Aug 2018

MATHEMATICS
Paper 1

4048/01
2 h

INSTRUCTIONS TO CANDIDATES
Write your name, class and index number on the question paper.
Write in dark blue or black ink on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give your answer in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

INFORMATION FOR CANDIDATES
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Marks
80

This question paper consists of 17 printed pages.

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Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

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Answer **all** the questions.

1 Write the following in order of size, **largest** first.

$$-\frac{7}{8} \quad , \quad -8 \times 10^{-1} \quad , \quad -0.\dot{8}\dot{1}$$

Answer : ……..... [1]

---

2 **(a)** Calculate $\dfrac{\sqrt{4.73}-3.28}{\sqrt[3]{99.7}\times 1.25}$.

Write down the first four digits on your calculator display.

(b) The number 102 999 corrected to ***n*** significant figures is 103 000.
Write down the largest possible value of ***n***.

Answer : **(a)** …..…….. [1]

(b) …..…….. [1]

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3 One hundred identical drops of oil have a total mass of 4550 milligrams.
1 milligram = 1×10^{-3} grams.
It is given that 1 cm^3 of oil weighs 0.91 grams.
Calculate the volume of one drop, in m^3, giving your answer in standard form.

Answer : ……..…….... m^3 [2]

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4 The numbers 98, 784 and 1176, written as the products of their prime factors, are

$98 = 2 \times 7^2$, $784 = 2^4 \times 7^2$ and $1176 = 2^3 \times 3 \times 7^2$.

(a) Find the largest possible integer, w, which divides 98, 784 and 1176 exactly.

(b) Find the smallest integer, p, such that $\sqrt[3]{\frac{784 \times 1176}{p}}$ is a whole number.

Answer : **(a)** $w =$ ……..……..……... [1]

(b) $p =$ …...…..…… [1]

---

5 The map of The Singapore Zoo has a scale of 1 : 3000.

(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres.

(b) The actual area of the Baboon Enclosure is 0.0225 km^2. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres.

Answer : **(a)** ...…...…..……..... km [1]

(b) ……..……..…….. cm^2 [1]

---

6 Factorise $p^2 + 5p + 6$.

Hence, explain why $p^2 + 5p + 6$ is always even for all positive integer values of p.

Answer : …..……..…….....

……..

……..

……..

.……[2]

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7 A tank is filled at a constant rate with water by 7 taps in 45 seconds.

(a) If two additional taps are used, how long does it take to fill the same tank?

(b) If the volume of the tank is V cm^3, express the rate of flow of water from each tap in terms of V.

Answer : **(a)** ……..…… s [1]

(b) ……..…… cm^3/s [1]

---

8 Anne planned to travel to Japan for holidays. She wanted to change 2800 Singapore Dollars (SGD) into Japanese Yen (JPY).

One day she went to *XYZ* Shopping Mall and saw two money changer shops there. Should Anne change her money at Shop *A* or Shop *B*? Justify your answer with clear calculation.

Money Changer Shop *A*
Exchange rate :
1 SGD = 80.32 JPY

Money Changer Shop *B*
Exchange rate :
100 JPY = 1.25 SGD

Answer : …...……

……. [3]

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9 Solve the equation $\frac{1}{4y} = 2^{-6}$.

Answer : $y =$ …..…….... [2]

---

10 The income tax payable in 2017 was calculated as follows:

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable ($)
First $20,000	0	0
Next 10,000	2	200
First $30,000	–	200
Next 10,000	3.50	350
First $40,000	–	550
Next $40,000	7	2,800
First $80,000	–	3,350
Next $40,000	11.5	4,600

Mr Lim paid an income tax amount of $2545 for his income earned in 2017.
Calculate his amount of chargeable income in 2017.

Answer : $ …….……..…….... [3]

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11 **(a)** Express $-x^2+4x-5$ in the form of $a(x+h)^2+k$.

Answer : **(a)** ……..……..…….... [2]

(b) **Hence**, sketch the graph of $y=-x^2+4x-5$ on the axes below. Indicate clearly the turning point, the values where the graph crosses the *x*- and *y*- axes (if any).

Answer : **(b)** [2]



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(c) **Hence**, explain why the equation $-x^2+4x-5=0$ has no solution.

Answer : **(c)** ……..……

……..……. [1]

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12 The following diagram shows 2 geometrically similar bottles of perfumes of the same brand.

Bottle ***A***
135 ml
$48





Bottle ***B***
320 ml
$95

(a) Is the cost of the perfume directly proportional to the quantity of the perfume? Explain with clear calculation.

Answer : **(a)** ……

..……

……..…… [3]

(b) Write down the ratio of the height of Bottle ***A*** to that of Bottle ***B***.

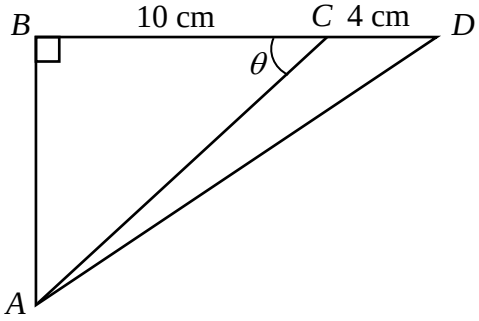
(c) It is given that the surface area of Bottle ***A*** is 270 cm^2. Calculate the surface area of Bottle ***B***.

Answer : **(b)** ……. : …….. [1]

(c) ..……...…… cm^2 [2]

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13 In the diagram, not drawn to scale, BCD is a straight line. Given that BC = 10 cm, CD = 4 cm, and $\cos\theta = \frac{4}{5}$.



Find

(a) the length of AC,

(b) the value of $\tan\angle ACD$, giving your answer as a fraction in its simplest form,

(c) the exact value of AD^2,

(d) the shortest distance of C to AD.

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Answer : **(a)** ...…..…….…..... cm [1]

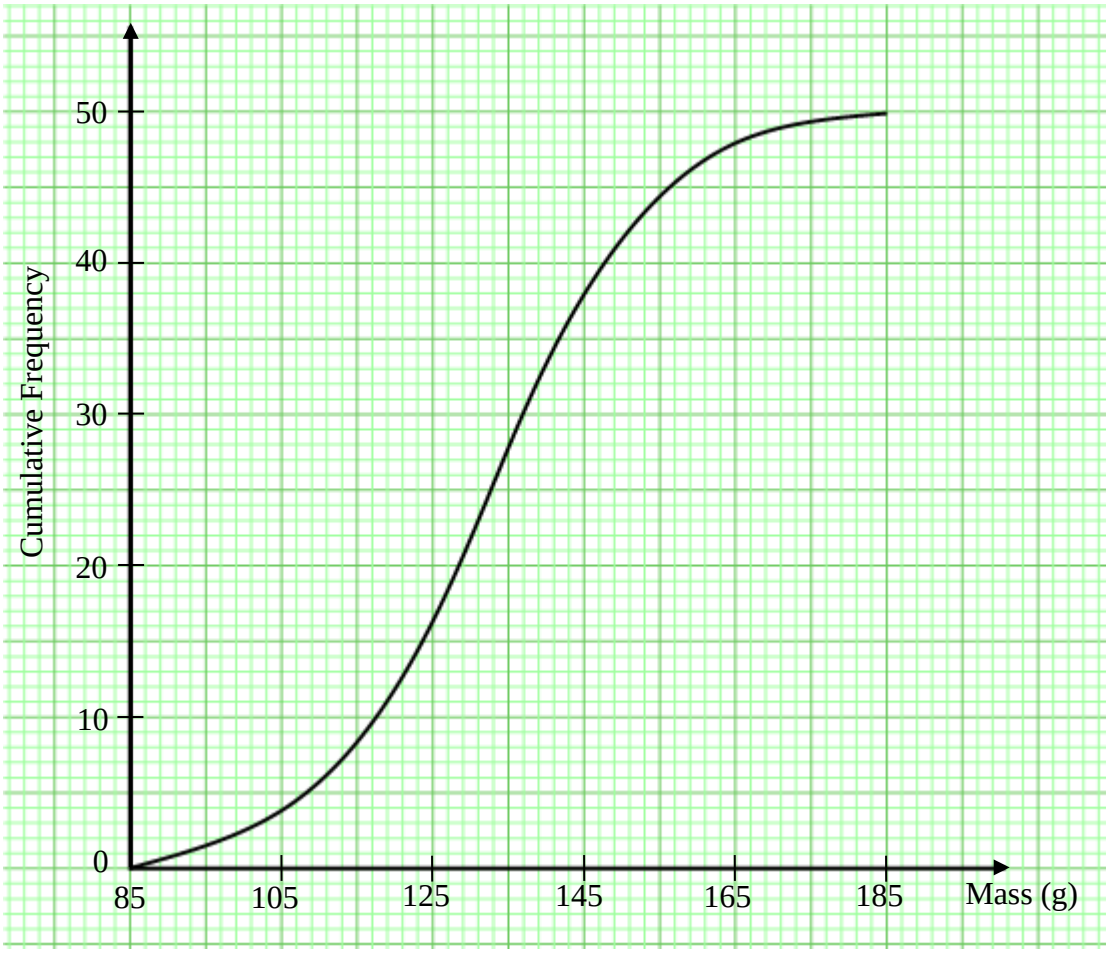
(b) ……..…….... [2]

(c) …...…….. [1]

(d) …...……..... cm [2]

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14 **(a)** The cumulative frequency curve below shows the distribution of the masses of 50 apples.



(i) The masses of these apples was tabulated into a grouped frequency table as follows. Find the values of p and of q.

Mass (x g)	Number of apples
$x < 85$	0
$85 \le x \le 105$	4
$105 \le x \le 125$	p
$125 \le x \le 145$	22
$145 \le x \le 165$	q
$165 \le x \le 185$	2

Answer : **(a)(i)** p = …..…….

q = ……. [1]

(ii) Using your grouped frequency table, calculate an estimate of

(a) the mean mass of the apples,

(b) the standard deviation.

Answer : **(a)(ii)(a)** ……..…… g [1]

(b) ……. g [1]

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(b) Another set of 50 apples have the same median but a smaller standard deviation. Describe how the cumulative frequency curve will differ from the given curve.

Answer : **(b)** ……

……

…… [1]

---

15 The table below shows a number sequence.

Pattern	No. of terms (N)	Value (V)
1^2	1	1
$1^2 - 2^2$	2	–3
$1^2 - 2^2 + 3^2$	3	6
$1^2 - 2^2 + 3^2 - 4^2$	4	–10
$1^2 - 2^2 + 3^2 - 4^2 + 5^2$	5	a
$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$	6	b
⋮		

(a) State the value of a and b.

(b) Form an equation for V in terms of N when

(i) N is an odd number.

(ii) N is an even number.

(c) Hence, find the value of $17^2 - 18^2 + 19^2 - 20^2 \ldots + 125^2$.

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Answer : **(a)** a = ……. , b = ……. [2]

(b)(i) ……. [1]

(ii) ……. [1]

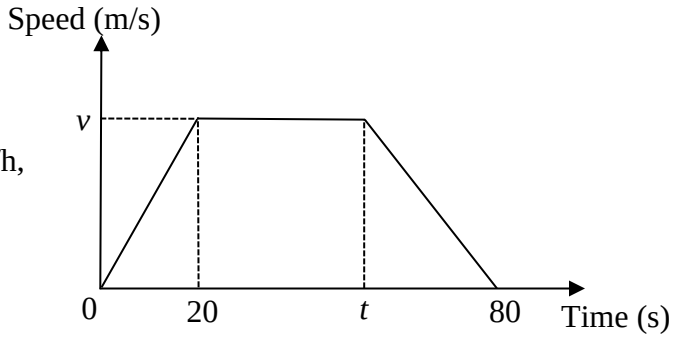
(c) ……. [2]

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16 The diagram below shows the speed-time graph of a bus journey.
The bus accelerated from rest at 1.25 m/s^2 to a speed of v m/s in 20 seconds, and travelled at this speed until t seconds before it came to a stop at t = 80 seconds. The total distance travelled for the whole journey was 1.45 km.

(a) Find the values of
  (i) v,
  (ii) t.

(b) Calculate the average speed, in km/h, of the bus for the whole journey.



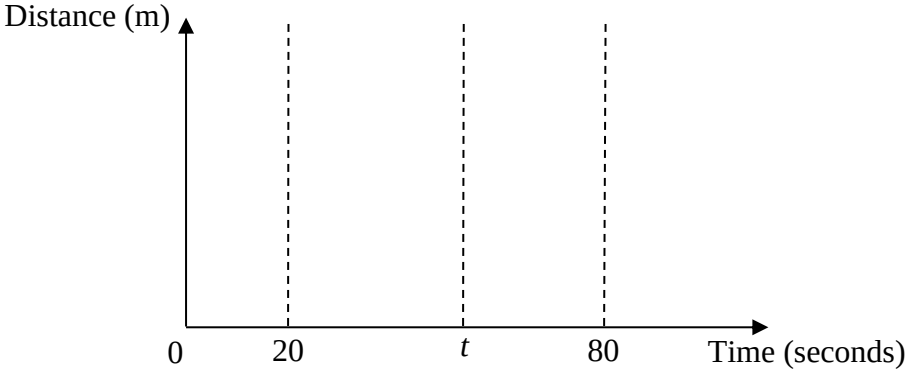
Answer : **(a)(i)** v = ……... [1]

(ii) t = ...…… [2]

(b) …… km/h [2]

(c) On the axes below, draw the distance-time graph of the bus journey, marking and stating the distance travelled for each time duration clearly on the vertical axis.

Answer : **(c)** [2]



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17 Given that $\varepsilon = \{x : x \text{ is a positive integer and } x \leq 12\}$,

$A = \{x : x \text{ is an odd integer and } 3x + 5 > 11\}$,

$B = \{x : x \text{ is a factor of } 28\}$,

$C = \{x : x \text{ is a composite number}\}$.

(a) List the elements of

(i) $A \cup B$,

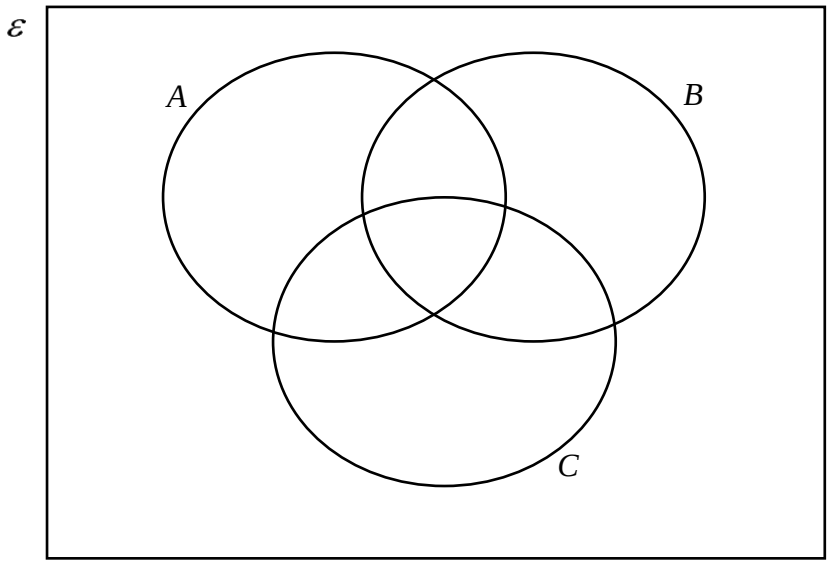
(ii) $B \cap C$,

Answer : **(a)(i)** $A \cup B$ = ……[1]

(ii) $B \cap C$ =……[1]

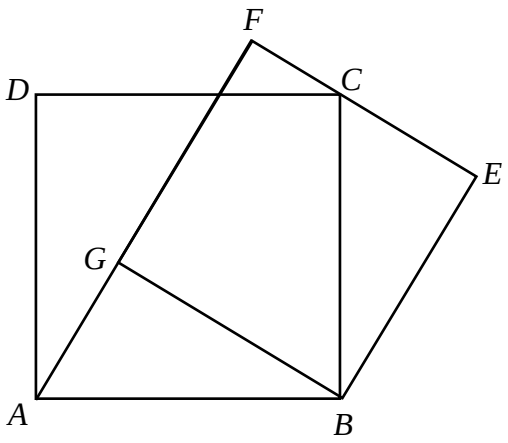
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(b) Represent the elements of ε and sets *A*, *B* and *C* in the Venn Diagram below. [1]



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18 In the diagram below, *ABCD* is a square and *BEFG* is a rectangle.
AGF and *FCE* are straight lines.



(a) Show that ΔABG is congruent to ΔCBE.

Answer : **(a)** ……

……..

……..……

..…....……

…....……..

……...

…....……

…....……

…....……

…… [3]

(b) Alice commented that *BEFG* cannot be a square. Do you agree with her, giving your reasons?

Answer : **(b)** ……

…….

…....……

…....……

…....…… [1]

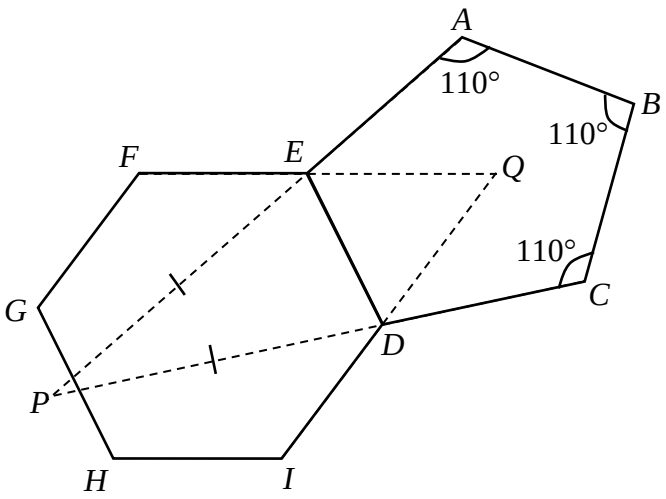
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19 The diagram shows a pentagon *ABCDE* and a regular hexagon *DEFGHI* sharing a common side *DE*. *AE* and *CD* are produced to meet at *P* such that ΔDEP is an isosceles triangle. *FE* and *ID* are produced to meet at *Q*. $\angle EAB = \angle ABC = \angle BCD = 110°$.
Stating your reasons clearly, find

(a) $\angle AED$,

(b) $\angle DPE$,

(c) $\angle PDQ$.



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Answer : **(a)** $\angle AED$ = …… ° [2]

(b) $\angle DPE$ = …… ° [2]

(c) $\angle PDQ$ = …… ° [2]

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20 A box contains a number of blue, green and red balls. There are 27 blue and green balls altogether. A ball is selected at random from the box. The probability of drawing a green ball is twice the probability of drawing a blue ball. The probability of drawing a red ball is $\frac{4}{7}$.

(a) Show that the probability of drawing a **green** ball is $\frac{2}{7}$. [2]

(b) Find the number of red balls in the bag.

Answer : **(b)** ……... [1]

(c) Two balls are selected at random from the box, **with replacement**.
Find the probability that both balls are blue.

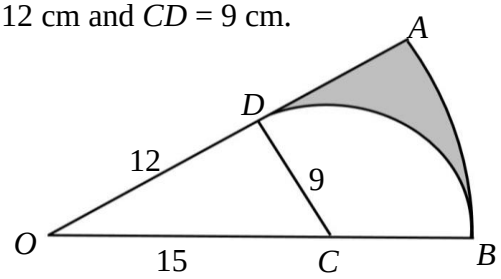
Answer : **(c)** ……... [1]

(d) Two balls are selected at random from the box, **without replacement**.
Find the probability that both balls are different colours.

Answer : **(d)** ……... [2]

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21 The figure shows a sector OAB with centre O, and an arc BD of another circle with centre C. It is given that OC = 15 cm, OD = 12 cm and CD = 9 cm.



(a) Prove that ΔODC is a right-angled triangle.

……

……

……

……

…… [1]

(b) Find

(i) $\angle AOB$, in radian,

(ii) the perimeter of the shaded region,

(iii) the area of the shaded region.

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Answer : **(b)(i)** ……. radian [1]

(ii) ……..…… cm [2]

(iii) ……..…..…… cm^2 [2]

~ END OF PAPER ~

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Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

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Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

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Answer **all** the questions.

1 Write the following in order of size, **largest** first.

$-\frac{7}{8}$, -8×10^{-1} , $-0.\dot{8}\dot{1}$

-0.875 , -0.8 , $-0.8181\ldots$

Answer : -8×10^{-1} , $-0.\dot{8}\dot{1}$, $-\frac{7}{8}$ **[B1]** [1]

---

2 **(a)** Calculate $\frac{\sqrt{4.73}-3.28}{\sqrt[3]{99.7}\times 1.25}$ $= -0.190667\ldots$

Write down the first four digits on your calculator display.

(b) The number 102 999 corrected to ***n*** significant figures is 103 000.
Write down the largest possible value of ***n***.

Answer : **(a)** -0.190 **[B1]** [1]

(b) 5 **[B1]** [1]

---

3 One hundred identical drops of oil have a total mass of 4550 milligrams.
1 milligram = 1×10^{-3} grams.
It is given that 1 cm^3 of oil weighs 0.91 grams.
Calculate the volume of one drop, in m^3, giving your answer in standard form.

Volume of 1 drop of oil $= \frac{4550\times 10^{-3}}{0.91\times 100}$ **[M1]**

$= 0.05\ cm^3$

$= 0.05 \times 10^{-6}\ m^3$

$= 5 \times 10^{-8}\ m^3$

Answer : 5×10^{-8} m^3 **[A1]** [2]

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4 The numbers 98, 784 and 1176, written as the products of their prime factors, are

$98 = 2 \times 7^2$, $784 = 2^4 \times 7^2$ and $1176 = 2^3 \times 3 \times 7^2$.

(a) Find the largest possible integer, w, which divides 98, 784 and 1176 exactly.

(b) Find the smallest integer, p, such that $\sqrt[3]{\frac{784 \times 1176}{p}}$ is a whole number.

(a) HCF, $w = 2 \times 7^2$

$= 98$

(b) $\frac{784 \times 1176}{p} = \frac{(2^4 \times 7^2) \times (2^3 \times 3 \times 7^2)}{p}$

$= \frac{2^7 \times 3 \times 7^4}{p}$

$= 2^6 \times 7^3$

$p = 2 \times 3 \times 7$

$= 42$

Answer : **(a)** $w =$ 98 **[B1]** [1]

(b) $p =$ 42 **[B1]** [1]

---

5 The map of The Singapore Zoo has a scale of 1 : 3000.

(a) If the length from the entrance to Reptile Garden is 4.5 cm on the map, calculate the actual distance, in kilometres.

(b) The actual area of the Baboon Enclosure is 0.0225 km^2. Calculate the area on the map that represents the enclosure, giving your answer in square centimetres.

(a) 1 cm : 3000 cm

1 cm : 0.03 km

4.5 cm : 0.135 km

(b) 1 cm : 0.03 km

1 cm^2 : 0.0009 km^2

25 cm^2 : 0.0225 km^2

Answer : **(a)** 0.135 **[B1]** km [1]

(b) 25 **[B1]** cm^2 [1]

---

6 Factorise $p^2 + 5p + 6$.

Hence, explain why $p^2 + 5p + 6$ is always even for all positive integer values of p.

Answer : $p^2 + 5p + 6 = (p + 2)(p + 3)$ **[M1]**

When p = odd integer , $p + 3$ = even.

When p = even integer , $p + 2$ = even. **[A1]**

Since one of the factors is always even, hence $p^2 + 5p + 6$ is always even.

[2]

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7 A tank is filled at a constant rate with water by 7 taps in 45 seconds.

(a) If two additional taps are used, how long does it take to fill the same tank?

(b) If the volume of the tank is V cm^3, express the rate of flow of water from each tap in terms of V.

(a) Time taken $= \frac{7}{9} \times 45$

$= 35$ s

(b) Rate of flow of water per tap $= \frac{V}{7 \times 45}$ or $= \frac{V}{9 \times 35}$

$= \frac{V}{315}$ cm^3/s

Answer : **(a)** 35 **[A1]** s [1]

(b) $\frac{V}{315}$ **[A1]** cm^3/s [1]

---

8 Anne planned to travel to Japan for holidays. She wanted to change 2800 Singapore Dollars (SGD) into Japanese Yen (JPY).

One day she went to *XYZ* Shopping Mall and saw two money changer shops there. Should Anne change her money at Shop *A* or Shop *B*? Justify your answer with clear calculation.

Money Changer Shop *A*
Exchange rate :
1 SGD = 80.32 JPY

Money Changer Shop *B*
Exchange rate :
100 JPY = 1.25 SGD

At Shop *A*, 2800 SGD $=$ 2800×80.32 JPY **[M1]**

$= 224896$ JPY

At Shop *B*, 2800 SGD $= \frac{2800}{1.25} \times 100$ JPY **[M1]**

$= 224000$ JPY

Difference $=$ $224896 - 224000$

$= 896$ JPY

Answer : Anne should change her money at **Shop A** because she will **get** **[A1]**
extra 896 JPY. [3]

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9 Solve the equation $\frac{1}{4y} = 2^{-6}$.

$\frac{1}{4y} = 2^{-6}$

$\frac{1}{4y} = \frac{1}{2^6}$

$4y = 2^6$ **[M1]**

$y = \frac{2^6}{2^2}$

$= 16$

Answer : $y =$ 35 **[A1]** [2]

---

10 The income tax payable in 2017 was calculated as follows:

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable ($)
First $20,000	0	0
Next 10,000	2	200
First $30,000	–	200
Next 10,000	3.50	350
First $40,000	–	550
Next $40,000	7	2,800
First $80,000	–	3,350
Next $40,000	11.5	4,600

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Mr Lim paid an income tax amount of $2545 for his income earned in 2017. Calculate his amount of chargeable income in 2017.

Let his amount of chargeable income be $\$x$.

$550 + (x - 40\,000) \times 0.07 = 2545$ **[M1]**

$x = \frac{2545 - 550}{0.07} + 40\,000$ **[M1]**

$= 68\,500$

Answer : $ 68 500 **[A1]** [3]

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11 **(a)** Express $-x^2+4x-5$ in the form of $a(x+h)^2+k$.

$$-x^2+4x-5=-\left(x^2-4x+5\right)$$
$$=-\left[x^2-4x+\left(\frac{-4}{2}\right)^2-\left(\frac{-4}{2}\right)^2+5\right]$$
$$=-\left[(x-2)^2+1\right]$$
$$=-(x-2)^2-1$$

[M1]

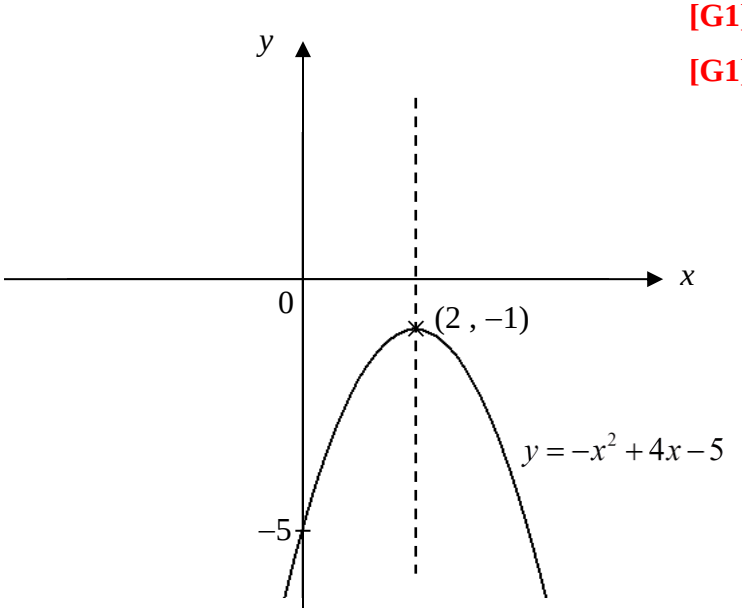
Answer : **(a)** $-(x-2)^2-1$ **[A1]** [2]

(b) **Hence**, sketch the graph of $y=-x^2+4x-5$ on the axes below. Indicate clearly the turning point, the values where the graph crosses the *x*- and *y*- axes (if any).

Answer : **(b)** [2]

[G1] – Correct shape & position

[G1] – Correct label turning point & *y*-intercept



(c) **Hence**, explain why the equation $-x^2+4x-5=0$ has no solution.

Answer : **(c)** The graph of $y=-x^2+4x-5$ will never cross the *x*-axis. **[B1]**

[1]

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12 The following diagram shows 2 geometrically similar bottles of perfumes of the same brand.

Bottle ***A***
135 ml
$48





Bottle ***B***
320 ml
$95

(a) Is the cost of the perfume directly proportional to the quantity of the perfume? Explain with clear calculation.

Costs per ml for Bottle $A = \frac{48}{135}$
$= \$0.36$ (2 d.p.) **[M1]**

Costs per ml for Bottle $B = \frac{95}{320}$
$= \$0.30$ (2 d.p.) **[M1]**

Answer : **(a)** Since **cost per ml not a constant**, the cost of the perfume **[A1]**
.....
not directly proportional to the quantity of the perfume.
.....
..... [3]

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(b) Write down the ratio of the height of Bottle ***A*** to that of Bottle ***B***.

(c) It is given that the surface area of Bottle ***A*** is 270 cm^2. Calculate the surface area of Bottle ***B***.

(b) $\frac{H_A}{H_B} = \sqrt[3]{\frac{135}{320}}$
$= \frac{3}{4}$

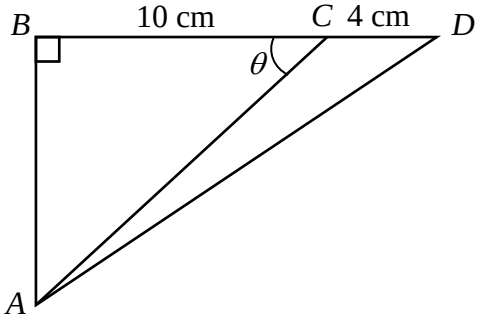
(c) $\frac{S_B}{270} = \left(\frac{4}{3}\right)^2$
Surface of Bottle $B = \left(\frac{4}{3}\right)^2 \times 270$ **[M1]**
$= 480$ cm^2

Answer : **(b)** 3 : 4 **[A1]** [1]

(c) 480 cm^2 **[A1]** [2]

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13 In the diagram, not drawn to scale, BCD is a straight line. Given that BC = 10 cm, CD = 4 cm, and $\cos\theta = \frac{4}{5}$.



Find

(a) the length of AC,

(b) the value of $\tan\angle ACD$, giving your answer as a fraction in its simplest form,

(c) the exact value of AD^2,

(d) the shortest distance of C to AD.

(a) $\cos\theta = \frac{10}{AC} = \frac{4}{5}$

$AC = \frac{10\times 5}{4} = 12.5$ cm

(b) $AB = \sqrt{12.5^2 - 10^2} = 7.5$ cm **[M1]**

$\tan\angle ACD = -\tan\angle ACB$

$= -\frac{7.5}{10}$

$= -\frac{3}{4}$

(c) $AD^2 = 7.5^2 + 14^2 = 252.25$

(d) Let the shortest distance be h cm.

Method 1

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 7.5$ **[M1]**

$h = \frac{4\times 7.5}{\sqrt{252.25}}$

$= 1.89$ cm (3 s.f.)

Method 2

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\sin(180°-\theta)$ **[M1]**

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\sin\theta$

$\frac{1}{2}\times\sqrt{252.25}\times h = \frac{1}{2}\times 4\times 12.5\times\frac{7.5}{12.5}$

$h = \frac{4\times 7.5}{\sqrt{252.25}}$

$= 1.89$ cm (3 s.f.)

Answer : **(a)** 12.5 **[A1]** cm [1]

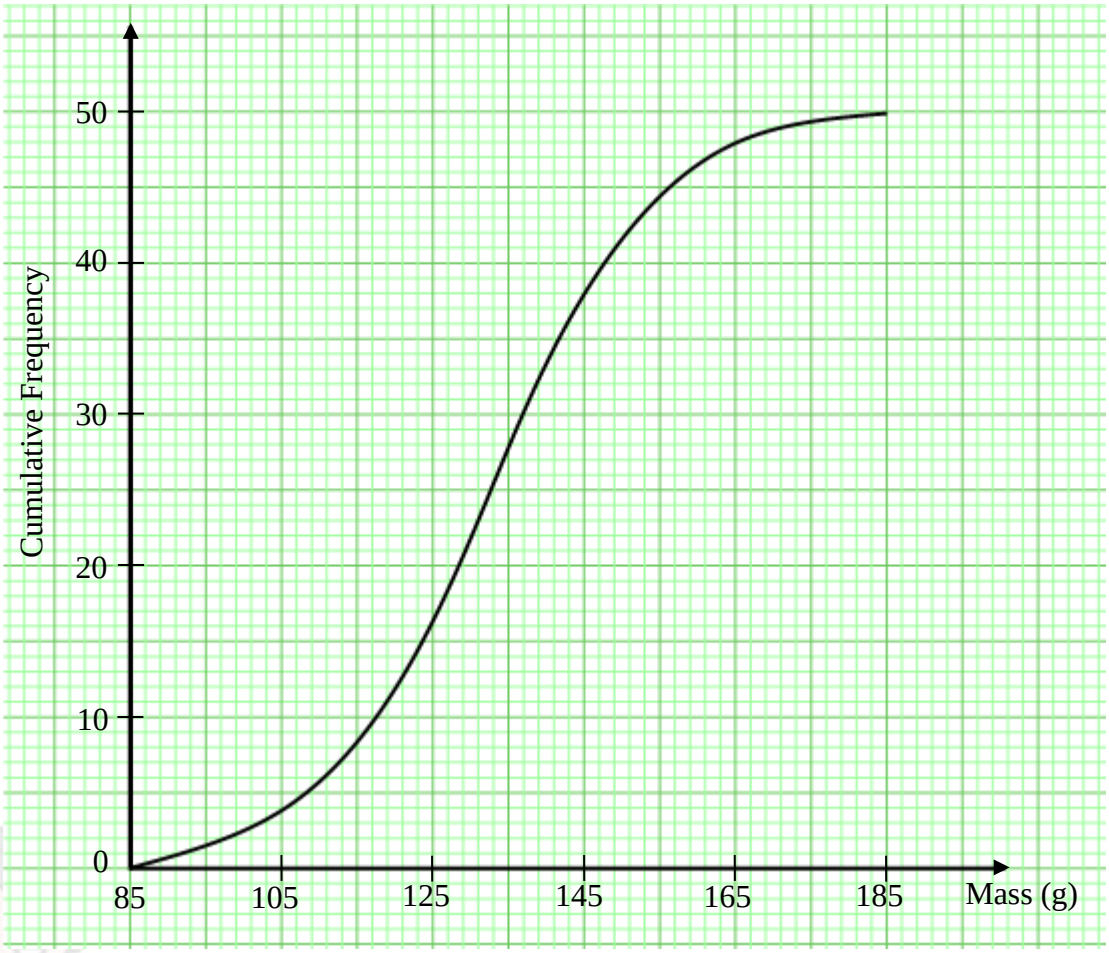
(b) $-\frac{3}{4}$ **[A1]** [2]

(c) 252.25 **[A1]** [1]

(d) 1.89 **[A1]** cm [2]

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14 **(a)** The cumulative frequency curve below shows the distribution of the masses of 50 apples.



(i) The masses of these apples was tabulated into a grouped frequency table as follows. Find the values of p and of q.

Mass (x g)	Number of apples
$x < 85$	0
$85 \le x \le 105$	4
$105 \le x \le 125$	p
$125 \le x \le 145$	22
$145 \le x \le 165$	q
$165 \le x \le 185$	2

Answer : **(a)(i)** $p =$ 12 ..... **[B1 – Both must be correct]**

$q =$ 10 ..... [1]

(ii) Using your grouped frequency table, calculate an estimate of

(a) the mean mass of the apples,

(b) the standard deviation.

Answer : **(a)(ii)(a)** 132.6 ..... **[B1]** g [1]

(b) 19.0 ..... **[B1]** g [1]

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(b) Another set of 50 apples have the same median but a smaller standard deviation. Describe how the cumulative frequency curve will differ from the given curve.

Answer : **(b)** The **second curve** will be **steeper** as smaller standard deviation shows that it has a narrower spread of data. **[B1]**

[1]

---

15 The table below shows a number sequence.

Pattern	No. of terms (N)	Value (V)
1^2	1	1
$1^2 - 2^2$	2	–3
$1^2 - 2^2 + 3^2$	3	6
$1^2 - 2^2 + 3^2 - 4^2$	4	–10
$1^2 - 2^2 + 3^2 - 4^2 + 5^2$	5	a
$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2$	6	b
⋮		

(a) State the values of a and b.

(b) Form an equation for V in terms of N when

(i) N is an odd number.

(ii) N is an even number.

(c) Hence, find the value of $17^2 - 18^2 + 19^2 - 20^2 \ldots + 125^2$.

(c) $V = \left[\frac{125\,(125+1)}{2}\right] - \left[-\frac{16\,(16+1)}{2}\right]$ **[M1]**

$= 7875 - (-136)$

$= 8011$

Answer : **(a)** $a =$ 15 **[B1]**, $b =$ –21 **[B1]** [2]

$V = \frac{1}{2}N^2 + \frac{1}{2}N$ **(b)(i)** $V = \frac{N(N+1)}{2}$ **[B1]** [1]

$V = -\frac{1}{2}N^2 - \frac{1}{2}N$ **(ii)** $V = -\frac{N(N+1)}{2}$ **[B1]** [1]

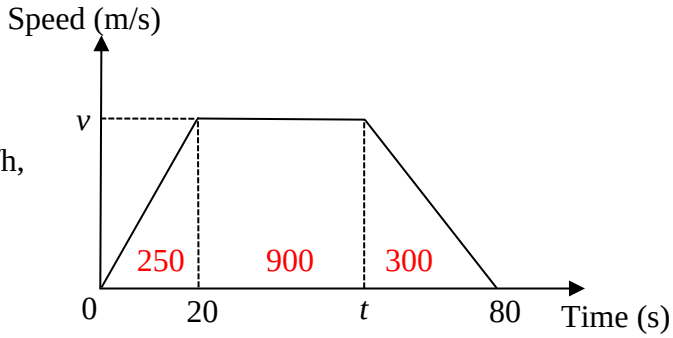
(c) 8011 **[A1]** [2]

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16 The diagram below shows the speed-time graph of a bus journey.
The bus accelerated from rest at 1.25 m/s^2 to a speed of v m/s in 20 seconds, and travelled at this speed until t seconds before it came to a stop at t = 80 seconds. The total distance travelled for the whole journey was 1.45 km.

(a) Find the values of
(i) v,
(ii) t.

(b) Calculate the average speed, in km/h, of the bus for the whole journey.



(a) (i) $\frac{v}{20} = 1.25$

$v = 1.25 \times 20$

$= 25$

(a) (ii) $\frac{1}{2}(20)(25) + (t-20)(25) + \frac{1}{2}(80-t)(25) = 1450$ **[M1]**

$250 + 25t - 500 + 1000 - 12.5t = 1450$

$12.5t = 700$

$t = 56$

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(b) Average speed $= \frac{1.45 \text{ km}}{\frac{80}{3600} \text{ h}}$ **[M1]**

$= 65.25$ km/h

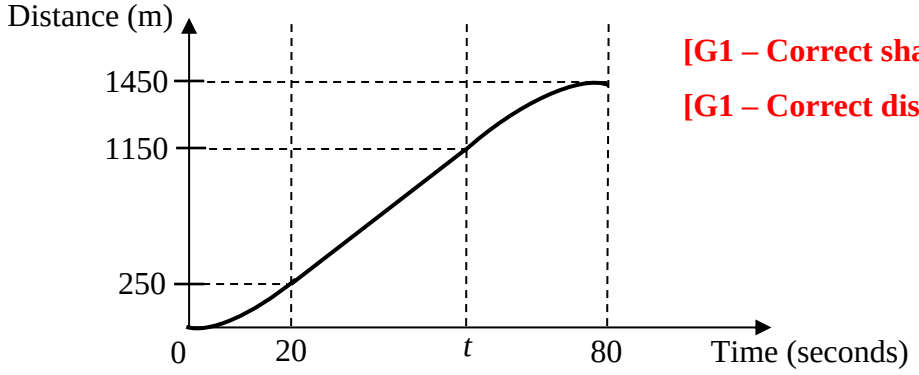
Answer : **(a)(i)** $v =$ 25 **[A1]** [1]

(ii) $t =$ 56 **[A1]** [2]

(b) 65.25 **[A1]** km/h [2]

(c) On the axes below, draw the distance-time graph of the bus journey, marking and stating the distance travelled for each time duration clearly on the vertical axis.

Answer : **(c)** [2]



[G1 – Correct shape]

[G1 – Correct distance]

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17 Given that $\varepsilon = \{x : x \text{ is a positive integer and } x \leq 12\}$, **= { 1, 2, 3, ….. , 12 }**

$A = \{x : x \text{ is an odd integer and } 3x + 5 > 11\}$, **= { 3, 5, 7, 9, 11 }**

$B = \{x : x \text{ is a factor of } 28\}$, **= { 1, 2, 4, 7 }**

$C = \{x : x \text{ is a composite number}\}$. **= { 4, 6, 8, 9, 10, 12 }**

(a) List the elements of

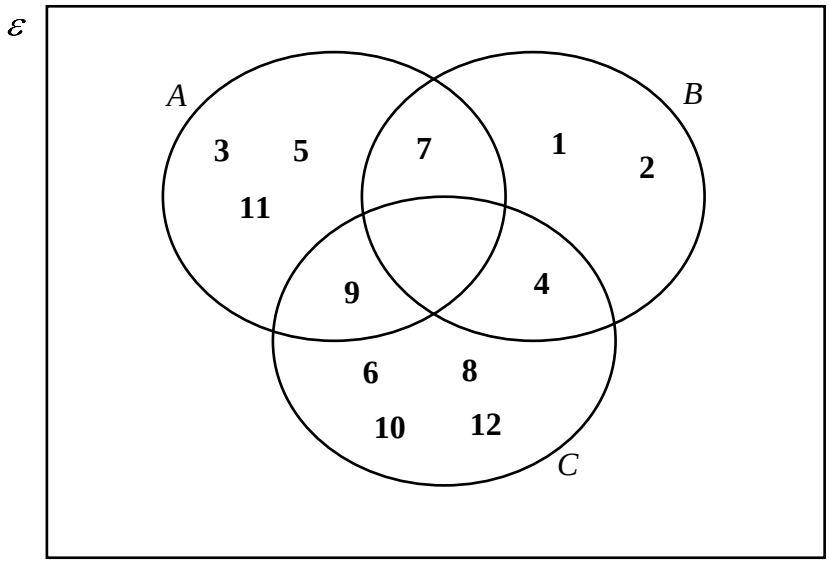
(i) $A \cup B$,

(ii) $B \cap C$,

Answer : **(a)(i)** $A \cup B$ = **{ 1, 2, 3, 4, 5, 7, 9, 11 }** **[B1]** [1]

(ii) $B \cap C$ = **{ 4 }** **[B1]** [1]

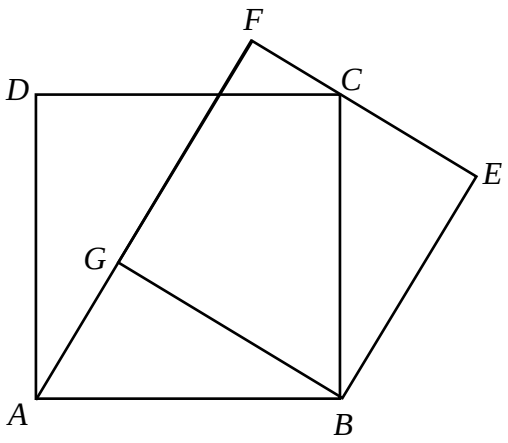
(b) Represent the elements of ε and sets A, B and C in the Venn Diagram below. [1]



[B1 – All must be correct]

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18 In the diagram below, *ABCD* is a square and *BEFG* is a rectangle.
AGF and *FCE* are straight lines.



(a) Show that ΔABG is congruent to ΔCBE.

Answer : **(a)** $\angle FGB = 90°$ ($\angle$s of rectangle *BEFG*)

$\angle AGB = 180° - 90° = 90°$ (Adjacent $\angle$s on a st line) **A**

$\angle BEC = 90°$ ($\angle$s of rectangle *BEFG*)

$\therefore \angle AGB = \angle BEC = 90°$ **[M1 – min 2 criteria given]**

Let $\angle GBC = x°$

$\angle GBA + x° = 90°$ ($\angle$s of square *ABCD*) **A** **[M2 – all 3 criteria given]**

$\angle EBC + x° = 90°$ ($\angle$s of rectangle *BEFG*)

$\therefore \angle GBA = \angle EBC$

AB = *BC* (Sides of square *ABCD*) **S**

$\therefore \Delta ABG \equiv \Delta CBE$ (AAS) **[A1 – Correct TEST]**

[3]

(b) Alice commented that *BEFG* cannot be a square. Do you agree with her, giving your reasons?

Answer : **(b)** **Disagreed.**

Since proven in part (a) $\Delta ABG \equiv \Delta CBE$,

GB = *BE* which are the sides of *BEFG*. **[B1]**

$\therefore$ *BEFG* is a square.

[1]

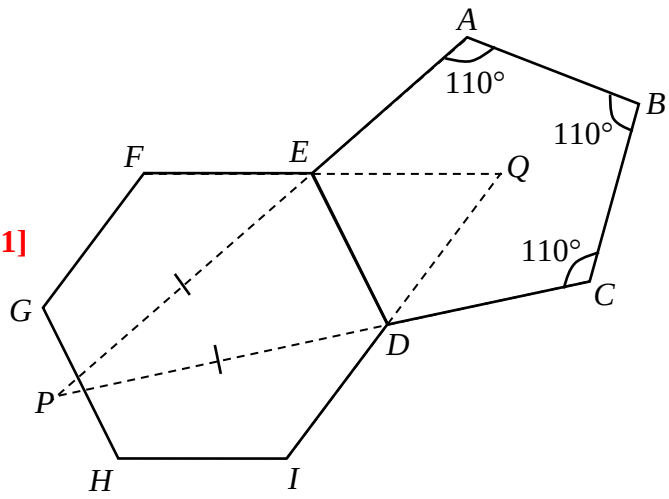
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19 The diagram shows a pentagon *ABCDE* and a regular hexagon *DEFGHI* sharing a common side *DE*. *AE* and *CD* are produced to meet at *P* such that ΔDEP is an isosceles triangle. *FE* and *ID* are produced to meet at *Q*. $\angle EAB = \angle ABC = \angle BCD = 110°$.
Stating your reasons clearly, find

(a) $\angle AED$,

(b) $\angle DPE$,

(c) $\angle PDQ$.



(a) $\angle AED = \dfrac{(5-2)\times 180° - 3\times 110°}{2}$ **[M1]**

$= \dfrac{540° - 330°}{2}$

$= 105°$

(b) $\angle DEP = 180° - 105°$ (Adj. $\angle$s on a st. line) **[M1]**

$= 75°$

$\angle DPE = 180° - 2(75°)$ ($\angle$ sum of Δ)

$= 30°$

(c) $\angle EDI = \dfrac{(6-2)\times 180°}{6}$ **[M1]**

$= 120°$

$\angle EDQ = 180° - 120°$ (Adj. $\angle$s on a st. line)

$= 60°$

$\angle PDQ = 75° + 60°$

$= 135°$

Answer : **(a)** $\angle AED$ = 105 **[A1]** ° [2]

(b) $\angle DPE$ = 30 **[A1]** ° [2]

(c) $\angle PDQ$ = ... **[A1]** ° [2]

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20 A box contains a number of blue, green and red balls. There are 27 blue and green balls altogether. A ball is selected at random from the box. The probability of drawing a green ball is twice the probability of drawing a blue ball. The probability of drawing a red ball is $\frac{4}{7}$.

(a) Show that the probability of drawing a **green** ball is $\frac{2}{7}$. [2]

(a) $P(G) + P(B) + P(R) = 1$

$P(G) + \frac{1}{2}P(G) + \frac{4}{7} = 1$ **[M1]**

$\frac{3}{2}P(G) = \frac{3}{7}$

$P(G) = \frac{3}{7} \times \frac{2}{3}$

$= \frac{2}{7}$ (shown) **[A1]**

(b) Find the number of red balls in the bag.

(b) G : B : R

2 : 1 : 4

No. of red balls $= \frac{27}{3} \times 4$

$= 36$

(2 + 1) units —— 27 balls

4 units —— ? balls

Answer : **(b)** 36 **[B1]** [1]

(c) Two balls are selected at random from the box, **with replacement**.
Find the probability that both balls are blue.

(c) G : B : R

18 : 9 : 36

$P(BB) = \left(\frac{9}{63}\right)\left(\frac{9}{63}\right)$ or $\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)$

$= \frac{1}{49}$

Answer : **(c)** $\frac{1}{49}$ **[A1]** [1]

(d) Two balls are selected at random from the box, **without replacement**.
Find the probability that both balls are different colours.

(d) P(different colours) = 1 – P(GG) – P(BB) – P(RR)

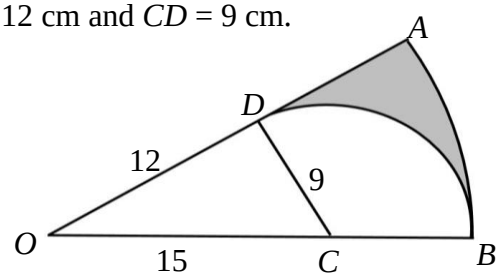
$= 1 - \left(\frac{18}{63}\right)\left(\frac{17}{62}\right) - \left(\frac{9}{63}\right)\left(\frac{8}{62}\right) - \left(\frac{36}{63}\right)\left(\frac{35}{62}\right)$ **[M1]**

$= \frac{18}{31}$

Answer : **(d)** $\frac{18}{31}$ **[A1]** [2]

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21 The figure shows a sector OAB with centre O, and an arc BD of another circle with centre C. It is given that OC = 15 cm, OD = 12 cm and CD = 9 cm.



(a) Prove that ΔODC is a right-angled triangle.

$CD^2 + OD^2 = 9^2 + 12^2 = 225$

$OC^2 = 15^2 = 225$ **[A1]**

Since $CD^2 + OD^2 = OC^2$, by converse of Pythagoras' Theorem,

ΔODC is a right-angled triangle. [1]

(b) Find

(i) $\angle AOB$, in radian,

(ii) the perimeter of the shaded region,

(iii) the area of the shaded region.

(b) (i) $\tan \angle DOC = \frac{9}{12}$

$\angle AOB = \angle DOC = 0.643501$

$= 0.644$ radian (3 s.f.)

(b) (ii) $\angle DCB = \frac{\pi}{2} + 0.643501$ (Ext. $\angle$ of Δ)

$= 2.21429$ radians

Perimeter of shaded region

$= (15 + 9)(0.643501) + (24 - 12) + 9(2.21429)$ **[M1]**

$= 47.37261$

$= 47.4$ cm (3 s.f.)

(b) (iii) Area of shaded region

$= \frac{1}{2}(24)^2(0.643501) - \frac{1}{2}(9)(12) - \frac{1}{2}(9)^2(2.21429)$ **[M1]**

$= 41.649$

$= 41.6$ cm^2 (3 s.f.)

Answer : **(b)(i)** 0.644 **[A1]** radian [1]

(ii) 47.4 **[A1]** cm [2]

(iii) 41.6 **[A1]** cm^2 [2]

~ END OF PAPER ~

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Name: _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Tuesday	**MATHEMATICS**	**4048/02**
14 August 2018	**Paper 2**	2 h 30 min

INSTRUCTIONS TO CANDIDATES

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together with the string provided.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This question paper consists of 12 printed pages.

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Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{\text{r}}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of a triangle = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

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Answer **all** the questions.

1 **(a)** It is given that $V = 2\pi(r^2 - x^2)$.

(i) Evaluate V when $r = 4.5$ and $x = 0.9$. [1]

(ii) Express x in terms of V and r. [3]

(b) Simplify $\dfrac{4x^3 - 25xy^2}{2x^2 + 7xy - 30y^2}$. [3]

(c) **(i)** Solve the equation $\dfrac{1}{3-2x} = \dfrac{x}{9-12x+4x^2}$, $x \neq \dfrac{3}{2}$. [3]

(ii) Give a reason why x cannot be $\dfrac{3}{2}$. [1]

(d) Solve the inequalities $\dfrac{x+7}{3} < \dfrac{4}{5}x + 1 \leq -9 + x$. [3]

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2 The following table shows the performance of three teams in the 2017 – 2018 English Football Premier League season. A draw is defined by a match ending with both teams having scored the same number of goals (e.g. 0 – 0, 2 – 2, etc).

Rank	Team	Wins	Draws	Losses
2nd	**Manchester Utd**	25	6	7
3rd	**Tottenham**	23	8	7
4th	**Liverpool**	21	12	5

(a) Write down a 3×3 matrix P to represent the information in the table above. [1]

A win is worth three points, a draw is worth one point and a loss results in no points scored. The greater the number of points accumulated, the higher the team's rank.

(b) **(i)** Write down a matrix Q, such that the product PQ, gives the total number of points the three teams accumulated respectively, at the end of the season. [1]

(ii) Evaluate the product PQ. [1]

(c) A Manchester Utd football fan made the following remark – "Your team will end the season with a higher rank than another team, as long as they have lost fewer matches than this other team."

Do you agree with his remark? Give a reason for your answer. [1]

The *Golden Boot* is awarded to the player who scored the most number of goals. The following players scored the most number of goals during the 2017 – 2018 season:
M. Salah – 32
H. Kane – 30
S. Aguero – 21

(d) **(i)** Write down a **row** matrix G to represent the above information. [1]

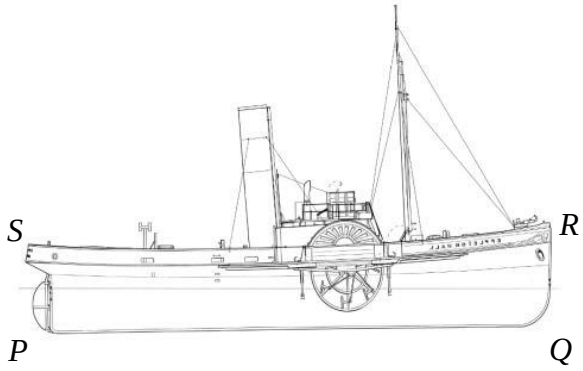
(ii) Given the matrix $T=\begin{pmatrix}1\\1\\1\end{pmatrix}$, evaluate the product GT. [1]

(iii) State what the product GT represents. [1]

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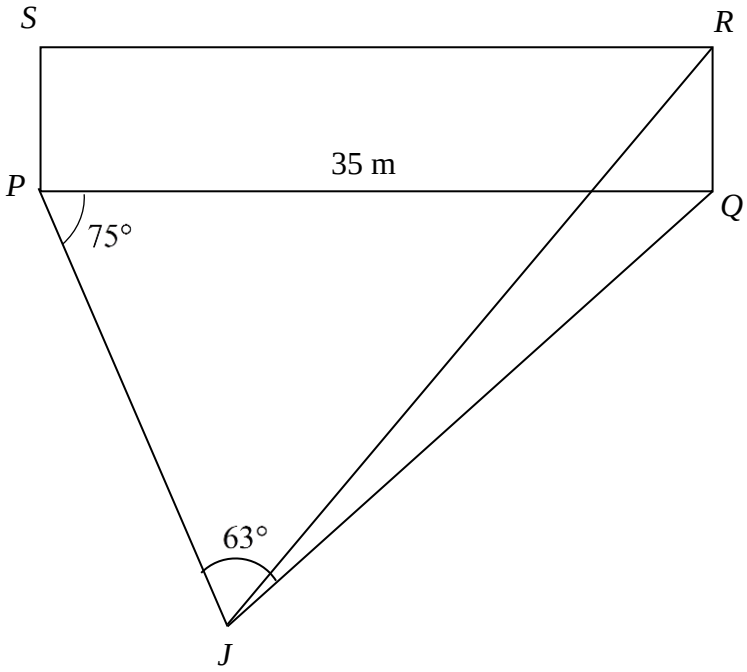
3

Figure 1



Source: www.shipmodell.com

Figure 2 (NOT TO SCALE)



The rectangle *PQRS* in **Figure 2** represents the vertical starboard (right-hand) side of a small fishing vessel shown in **Figure 1**. The bottom of the starboard side, *PQ*, runs from West to East and is at sea level. A jet ski is nearby at *J*.

Angle $QPJ = 75°$, angle $PJQ = 63°$ and $PQ = 35$m.

(a) Showing your reasons clearly, find the bearing of *J* from *Q*. [2]

(b) Show that $JQ = 37.9$m. [2]

(c) Calculate the area of triangle *JPQ*. [2]

(d) Find the shortest distance from the jet ski to the fishing vessel. [2]

(e) The angle of depression of the jet ski from a fisherman standing on the top deck of the fishing vessel at *R* is $18°$.

(i) Calculate the height of the top deck of the fishing vessel above sea level. [2]

(ii) Calculate the greatest possible value of the angle of elevation of the top deck of the fishing vessel when viewed from the jet ski. [2]

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4 **(a)** The cash price of a new washer-dryer is \$2595. It is also available on hire purchase with a deposit of one quarter of the cash price followed by monthly instalments of \$60 for three years.

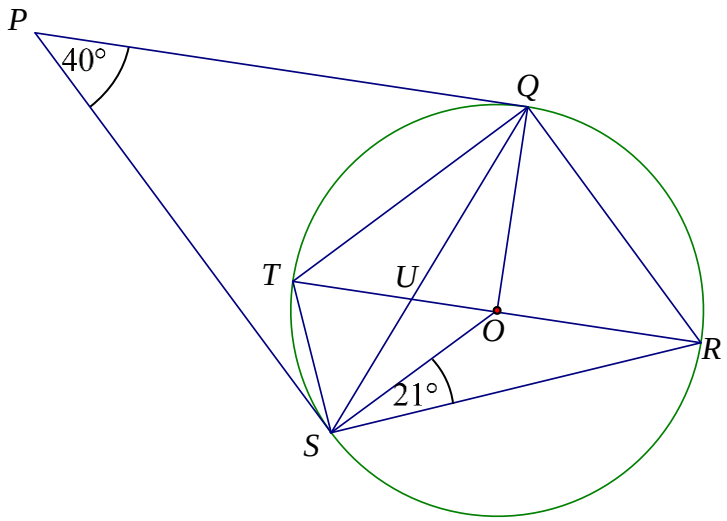
Pavithra buys this washer-dryer on hire purchase.

(i) Calculate the rate of simple interest charged per annum, correct to two decimal places. [4]

(ii) Find the extra cost of buying the washer-dryer on hire purchase as a percentage of the cash price. [2]

(b) A sum of money grows to \$5800.15 in 3 years at a compound interest of 2.75% per annum. Find the sum of money if the interest is compounded quarterly. [2]

5



The diagram shows a circle $QRST$, centre O. U is the point of intersection of QS and diameter RT. PQ and PS are tangents to the circle such that angle $QPS = 40°$. Angle $OSR = 21°$.

Find, with clearly stated reasons,

(a) obtuse angle QOS, [2]

(b) angle QTS, [2]

(c) angle OSU, [1]

(d) angle PQT. [3]

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6 Answer the whole of this question on a sheet of graph paper.

During a Physics experiment conducted on gravity, a rubber ball was thrown from the top of a vertical building.

Its position during its flight is represented by the equation $y = 30 + 4x - x^2$, where y metres is the height of the ball above the ground and x metres is its horizontal distance from the foot of the building.

Some corresponding values of x and y are given in the table below.

x	0	1	2	3	4	5	6	7
y	30	33	34	33	30	25	p	9

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 metre, draw a horizontal axis for $0 \le x \le 7$.
Using a scale of 2 cm to represent 5 metres, draw a vertical axis for $0 \le y \le 40$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the greatest height reached by the ball. [1]

(d) Find the estimated horizontal distances of the ball from the building when its height is 32 m above the ground. [2]

(e) (i) By drawing a tangent, find the gradient of the graph at the point (5, 25). [2]

(ii) Use your answer to **(e)(i)** to explain what was happening to the ball at $x = 5$. [1]

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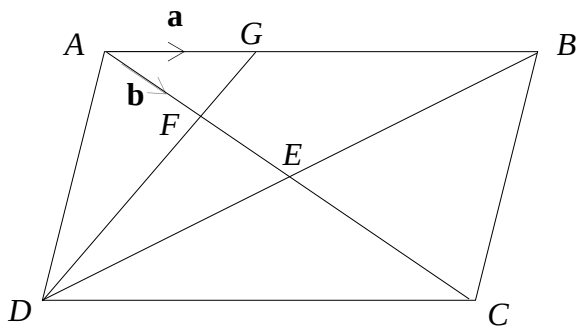
7 **(a)** X is the point (2, 3) and Y is the point (– 6, 10).

(i) Find the vector $\overrightarrow{XY}$. [1]

(ii) Find $|\overrightarrow{XY}|$. [1]

(iii) Given that $\overrightarrow{XY} = 2\overrightarrow{YZ}$, find the coordinates of the point Z. [2]

(b)



In the diagram above, $ABCD$ is a parallelogram. The diagonals AC and BD intersect at E and the point F is the mid-point of AE. G is a point on AB such that $2AG = GB$. $\overrightarrow{AG} = \mathbf{a}$ and $\overrightarrow{AF} = \mathbf{b}$.

(i) Express the following in terms of **a** and/or **b**.

(a) $\overrightarrow{FG}$, [1]

(b) $\overrightarrow{DF}$, [1]

(c) $\overrightarrow{GC}$. [2]

(ii) Show that D, F and G are collinear points. [1]

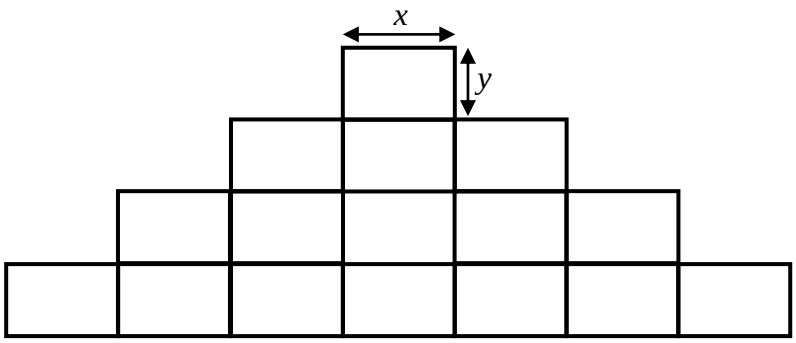
(iii) Find the ratio of

(a) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta CFD}$, [1]

(b) $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC}$. [2]

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8 The diagram shows an exhibition area which has been divided into 16 identical rectangular plots with dimensions x m by y m, where $x > y$, which will be leased out as booth spaces.



345 m of tape was used by the planning committee to mark out the exhibition area.

(a) Write down an equation connecting x and y. [1]

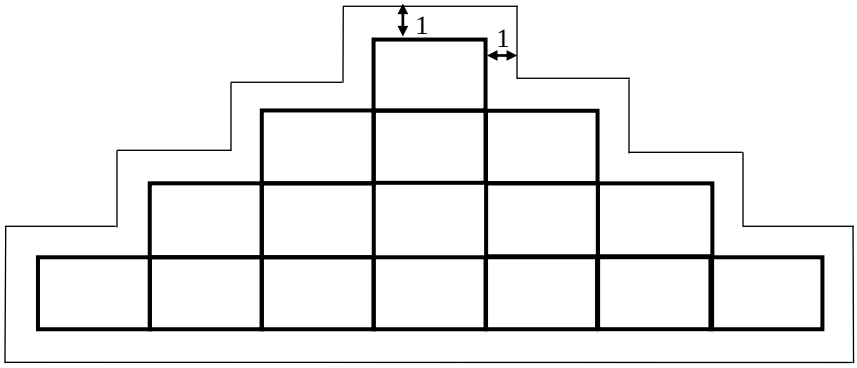
The total area enclosed by the tape is 644 m^2.

(b) Form an equation in x to represent this information and show that it reduces to
$$x^2 - 15x + 35 = 0.$$ [3]

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(c) Solve the equation $x^2 - 15x + 35 = 0$. [3]

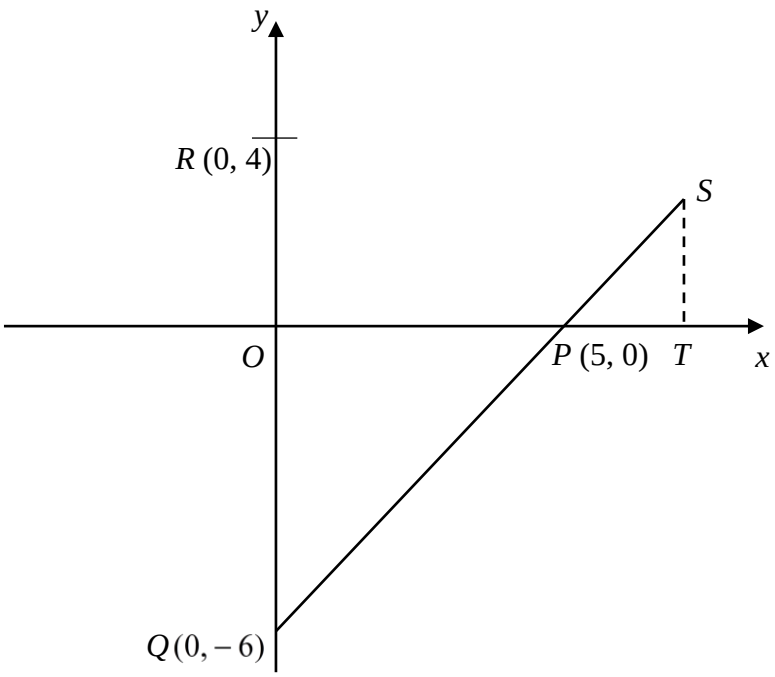
(d) On the night before the commencement of the exhibition, the planning committee plans to cordon off a 1 metre–wide security barrier along the perimeter of the exhibition area.



Calculate the total area of the security barrier. [3]

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9



In the diagram, P, Q and R are the points (5, 0), $(0, -6)$ and (0, 4) respectively.

(a) Find the equation of the line PQ. [1]

S is a point on QP produced such that the x-coordinate of S is k and T lies vertically below S on the x-axis.

(b) Given that the area of triangle RQS is 40 units^2, show that $k = 8$. [2]

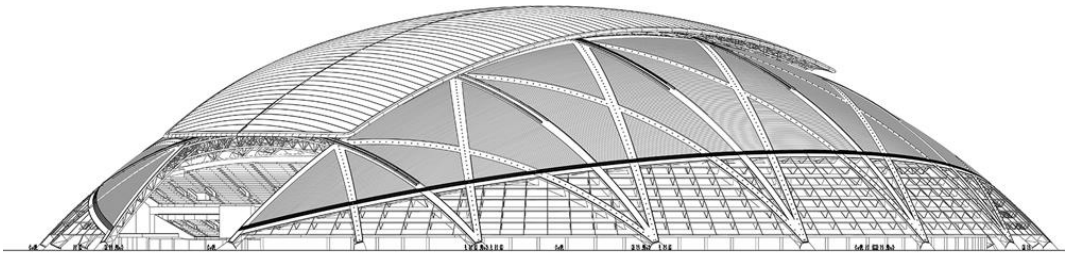
(c) **(i)** Calculate the lengths of PQ and PS respectively. [3]

(ii) Hence, using your answers in **(c)(i)** and stating your reasons clearly, show that triangle PQO is similar to triangle PST. [3]

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10 **(a)** The National Day Parade will be held at the National Stadium in August this year. The stadium dome shown below, may be modelled as a spherical cap (part of a hemisphere). Its volume is approximately 17.5% of the volume of a hemisphere of radius 192 metres and its surface area is approximately 33.9% of the curved surface area of the same hemisphere. The stadium is equipped with a state-of-the-art retractable roof which has two panels, each of area 10,000 m^2.

Source: www.static.dezeen.com



Calculate

(i) the estimated volume of the dome, [2]

(ii) the surface area of the dome when its retractable roof is **fully** open. [2]

(b) The organizers of the 2018 National Day Parade are deciding between two companies' drinking water for the expected 55,000 maximum-capacity spectators.

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The table below summarizes some information for the two brands.

	Brand A	Brand B
Packaging type	Tetra pak rectangular prism with dimensions 5.7cm×5.3cm×10cm	Cylindrical with radius 2.75 cm and height 12.8 cm
Material	Recycled paperboard	Plastic
Fill volume	Nearest 10 ml of pack volume	Nearest 10 ml of bottle volume
Unit cost per package	60 cents	55 cents
Bulk refund for empty packages	$11000	$5200

Additional information:

1) Each spectator is projected to require an average of 600 ml of water during the Parade.
2) The chosen package is to be environmentally-friendly where possible.
3) Cost effectiveness to be taken into consideration.

Determine, with clear calculations, which brand of water should be awarded the contract. State an assumption you made to support your decision. [6]

~~ End of Paper 2 ~~ ☺

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2018 Mathematics Prelim P2 Marking Scheme

1. (a) (i) $V = 2\pi(r^2 - x^2)$

$V = 2\pi[(4.5)^2 - (0.9)^2]$

$= 122$

(ii) $V = 2\pi r^2 - 2\pi x^2$

$2\pi x^2 = 2\pi r^2 - V$

$x^2 = \dfrac{2\pi r^2 - V}{2\pi}$

$\therefore x = \pm\sqrt{\dfrac{2\pi r^2 - V}{2\pi}}$

(b) $\dfrac{4x^3 - 25xy^2}{2x^2 + 7xy - 30y^2} = \dfrac{x(2x+5y)(2x-5y)}{(2x-5y)(x+6y)}$

$= \dfrac{x(2x+5y)}{x+6y}$

(c) (i) $\dfrac{1}{3-2x} = \dfrac{x}{9-12x+4x^2}$

$\dfrac{1}{3-2x} = \dfrac{x}{(3-2x)^2}$

$3 - 2x = x$

$\therefore x = 1$

Or $9 - 12x + 4x^2 = 3x - 2x^2$

$6x^2 - 15x + 9 = 0$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$\therefore x = 1$ or $x = \dfrac{3}{2}$ (N.A.)

(ii) If $x = \dfrac{3}{2}$, the denominator/s will be zero; fraction will be undefined.

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(d) $\frac{x+7}{3} < \frac{4}{5}x+1 \le -9+x.$

$\frac{x+7}{3} < \frac{4}{5}x+1$ $\qquad$ $\frac{4}{5}x+1 \le -9+x.$

$5x+35 < 12x+15$ $\qquad$ $10 \le \frac{1}{5}x$

$20 < 7x$

$\therefore x > \frac{20}{7}$ $\qquad$ $\therefore x \ge 50$

$\therefore x \ge 50$

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2. **(a)** $P = \begin{pmatrix} 25 & 6 & 7 \\ 23 & 8 & 7 \\ 21 & 12 & 5 \end{pmatrix}$

(b) **(i)** $Q = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

(ii) $PQ = \begin{pmatrix} 25 & 6 & 7 \\ 23 & 8 & 7 \\ 21 & 12 & 5 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 81 \\ 77 \\ 75 \end{pmatrix}$

(c) Disagree.

Liverpool had fewer losses than the other two teams but still ended up ranked behind them.

(d) **(i)** $G = (32 \quad 30 \quad 21)$

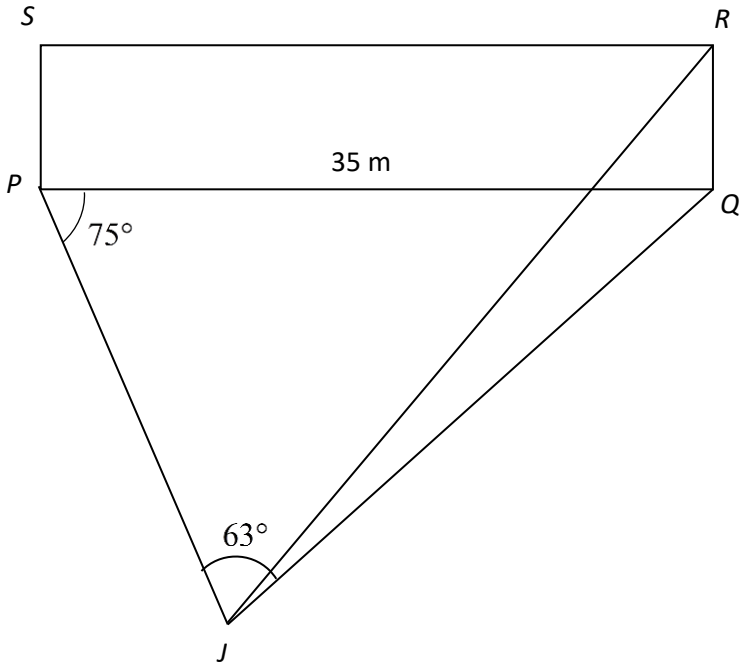
(ii) $G = (32 \quad 30 \quad 21)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= (83)$

(ii) The element represents the total number of goals scored by the top three scorers.

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3.



(a) $\angle PQJ = 180° - 75° - 63°$ (angles in a triangle)

$= 42°$

$\therefore$ bearing of J from $Q = 360° - 90° - 42°$ (angles at a point)

$= 228°$

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(b) $\dfrac{JQ}{\sin 75°} = \dfrac{35}{\sin 63°}$

$\therefore JQ = \dfrac{35 \sin 75°}{\sin 63°}$

$= 37.9429$

$= 37.9$ m (3 s.f.)

(c) Area of triangle $JPQ = \dfrac{1}{2}(35)(37.9429)\sin 42°$

$= 444.303$

$= 444\,\text{m}^2$ (3 s.f.)

(d) Let shortest distance be d m .

$\sin 42° = \dfrac{d}{37.9429}$

$\therefore$ shortest distance $= 37.9429 \sin 42°$

$= 25.3888$

$= 25.4$ m (3 s.f.)

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(e) **(i)** Let the height be h m.

$$\tan 18° = \frac{h}{37.9429}$$

$$\therefore \text{ height } = 12.3284$$

$$= 12.3 \text{ m (3 s.f.)}$$

(ii) Let the angle be θ .

$$\tan\theta = \frac{12.3284}{25.3888}$$

$\therefore$ greatest angle of elevation is $25.9°$ (1 d.p.)

---

4. **(a)** **(i)** Balance after deposit $= \frac{75}{100}(2595)$

$$= \$1946.25$$

$$\text{Total interest paid } = \frac{25}{100}(2595) + 60(12)(3) - 2595$$

$$= \$213.75$$

Let r be the rate of interest.

$$213.75 = 1946.25(\frac{r}{100})(3)$$

$$\therefore \text{ rate } = \frac{213.75(100)}{1946.25(3)}$$

$$= 3.66\% \quad (2 \text{ d.p.})$$

(ii) $\therefore$ percentage $= \frac{213.75}{2595} \times 100\%$

$$= 8.24\% \text{ (3 s.f.) or } 8\frac{41}{173}\%$$

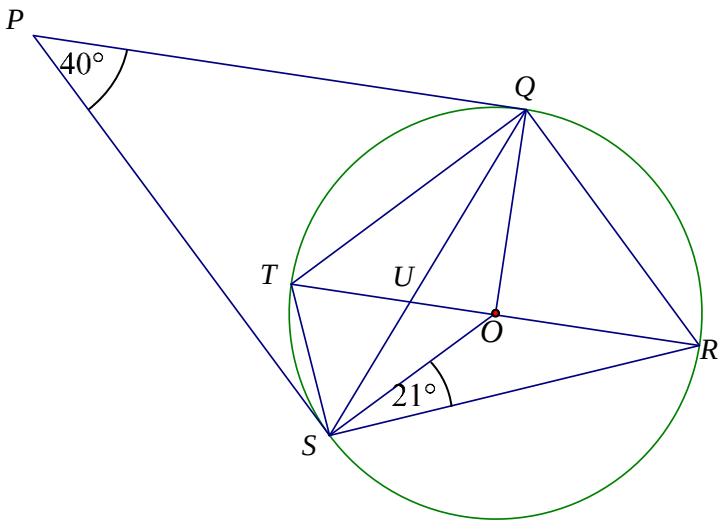
(b) Let the sum of money be x.

$$5800.15 = x\left[1 + \frac{2.75}{4(100)}\right]^{3(4)}$$

$\therefore$ the sum of money $= \$5342.35$ (2 d.p.)

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5.



(a) Angle PQO = angle PSO = $90°$ (tangent perpendicular to radius)

$\therefore$ obtuse angle QOS = $360° - 90° - 90° - 40°$ (angles in a quadrilateral)

$= 140°$

(b) Reflex angle QOS = $360° - 140°$ (angles at a point)

$= 220°$

$\therefore$ angle QTS = $\dfrac{220°}{2}$ (angle at centre = 2 angle at circumference)

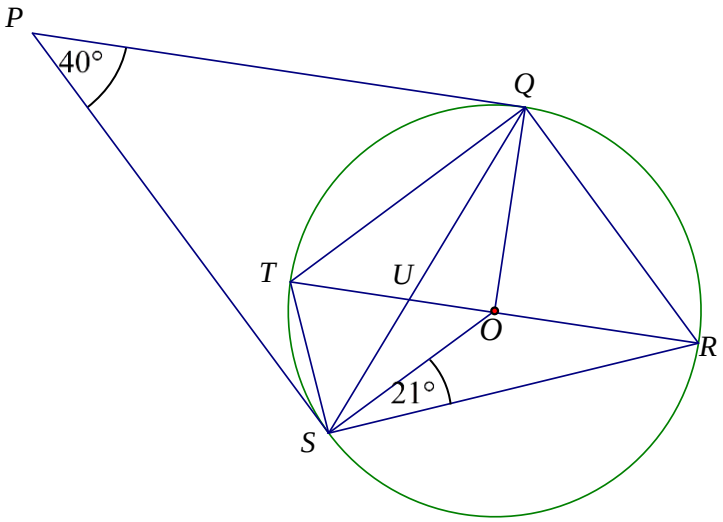
$= 110°$

(c) Angle OSU = $\dfrac{180° - 140°}{2}$ (base angles of isosceles triangle)

$= 20°$

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(d) Method 1

Angle $ORS = 21°$ (base angles of isosceles triangle)

$\therefore$ angle $TQU = 21°$ (angles in the same segment)

Angle $OQU = 20°$ (base angles of isosceles triangle)

$\therefore$ angle $PQT = 90° - 21° - 20°$ (tangent perpendicular to radius)

$= 49°$

Method 2

$PQ = PS$ (tangents from external point)

$\therefore$ triangle PQS is isosceles. Angle $PQS = \dfrac{180° - 40°}{2}$

$= 70°$

Angle $ORS = 21°$ (base angles of isosceles triangle)

$\therefore$ angle $TQU = 21°$ (angles in the same segment)

$\therefore$ angle $PQT = 70° - 21° = 49°$

Method 3

Angle $TSU = 90° - 20° - 21°$ (angle in semicircle)

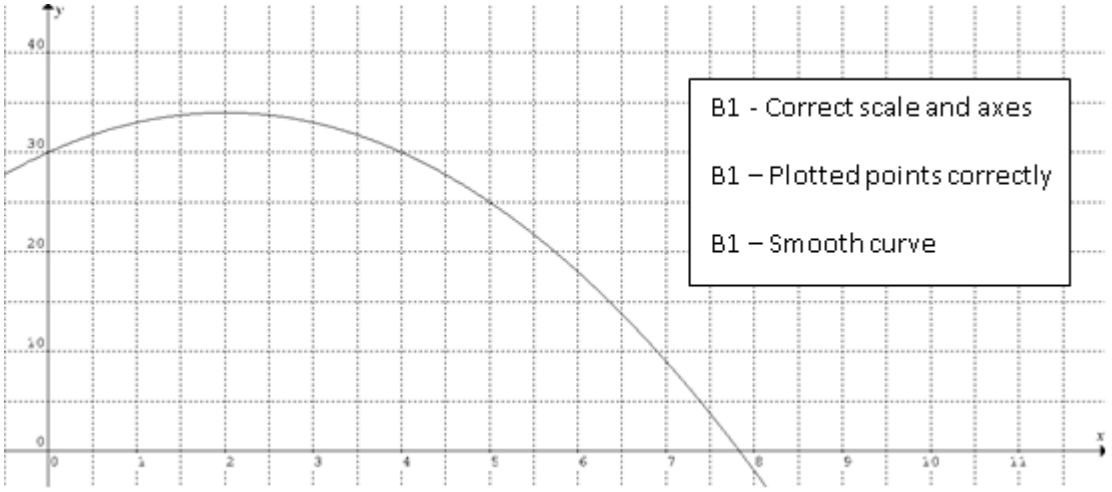
$= 49°$

$\therefore$ angle $PQT = 49°$ (angles in alternate segments/alternate segment theorem)

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6. **(a)** $p = 18$

(b)



(c) The greatest height is 34 m.

(d) 3.41 m (3 s.f.) accept $3.35 \le x \le 3.45$

0.586 (3 s.f.) m accept $0.535 \le x \le 0.635$

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(e) **(i)** tangent constructed

Gradient $= -6$ accept $-6.5 \le m \le -5.5$

(ii) The height of the ball above the ground was decreasing at a rate of 6 m for every 1 m it travels horizontally / falls.

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7. (a) (i) $\overrightarrow{XY} = \begin{pmatrix} -6 \\ 10 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ 7 \end{pmatrix}$

(ii) $|\overrightarrow{XY}| = \sqrt{(-8)^2 + 7^2}$

= 10.6 u (3 s.f.)

(iii) $\overrightarrow{YZ} = \frac{1}{2}\overrightarrow{XY}$

$= \frac{1}{2}\begin{pmatrix} -8 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 3.5 \end{pmatrix}$

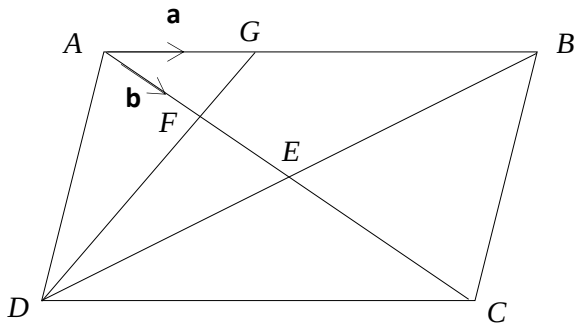
$\therefore \overrightarrow{OZ} = \begin{pmatrix} -6 \\ 10 \end{pmatrix} + \begin{pmatrix} -4 \\ 3.5 \end{pmatrix}$

$= \begin{pmatrix} -10 \\ 13.5 \end{pmatrix}$

$\therefore Z = (-10, 13.5)$

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(b)



(i) **(a)** $\overrightarrow{FG} = \mathbf{a} - \mathbf{b}$

(b) $\overrightarrow{DF} = \mathbf{3a} - \mathbf{3b}$

(c) $\overrightarrow{BC} = \mathbf{4b} - \mathbf{3a}$

$\therefore \overrightarrow{GC} = \mathbf{2a} + \mathbf{4b} - \mathbf{3a}$

$= \mathbf{4b} - \mathbf{a}$

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(ii) Since $\overrightarrow{DF} = 3\overrightarrow{FG}$ and F is a common point, D, F and G are collinear.

(iii) **(a)** $\dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta CFD} = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

(b) $\dfrac{\text{the area of } \Delta DEF}{\text{the area of } \Delta DEC} = \dfrac{1}{2} = \dfrac{3}{6}$

$\therefore \dfrac{\text{the area of } \Delta AFG}{\text{the area of } \Delta DEC} = \dfrac{1}{6}$

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8. **(a)** $23x + 20y = 345$

(b) $16x\left(\dfrac{345 - 23x}{20}\right) = 644$

$345x - 23x^2 = 805$

$23x^2 - 345x + 805 = 0$

$x^2 - 15x + 35 = 0$

(c) $x = \dfrac{15 \pm \sqrt{15^2 - 4(35)}}{2}$

$x = 2.89022$ or $x = 12.1097$

$= 2.89$ (3 s.f.) or $= 12.1$ (3 s.f.)

(d) Since $x > y$, $x = 12.1097$ and $y = 3.32376$

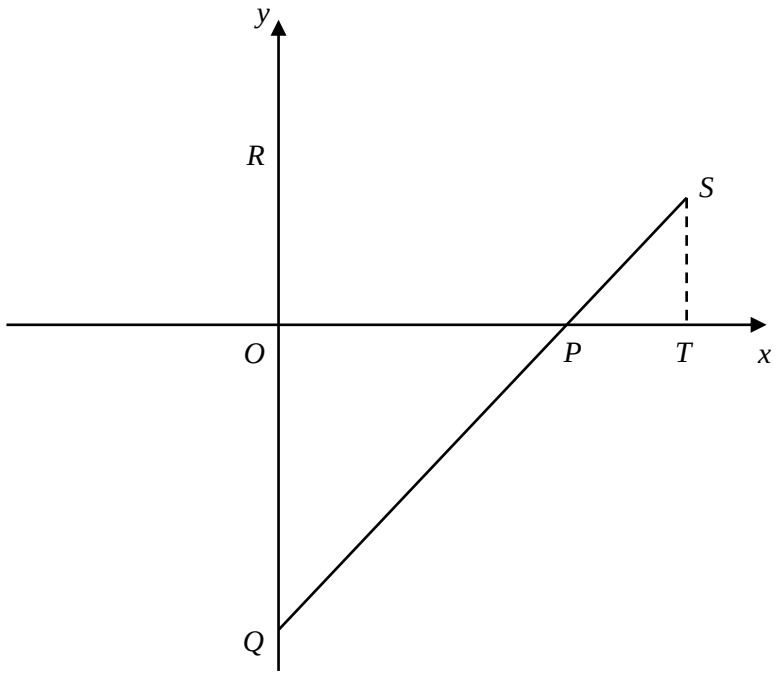
Area of path

$= [7(12.1097) + 2][4(3.32376) + 2] - 644 - 12(12.097)(3.32376)$

$= 200$ m^2 (3 s.f.)

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9.



(a) Gradient of $PQ = \dfrac{0-(-6)}{5-0}$

$= \dfrac{6}{5}$

$\therefore PQ: y = \dfrac{6}{5}x - 6$ or $5y = 6x - 30$

(b) $\dfrac{1}{2}(10)(k) = 40$

$\therefore k = 8$ (shown)

(c) **(i)** Length of $PQ = \sqrt{(5-0)^2 + (0+6)^2}$

$= \sqrt{61}$

$= 7.81$ units (3 s.f.)

$y = \dfrac{6}{5}(8) - 6 = 3.6$

$\therefore S = (8, 3.6)$

Length of $PS = \sqrt{(5-8)^2 + (0-3.6)^2} = 4.69$ units (3 s.f.)

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(ii) $\frac{PQ}{PS} = \frac{\sqrt{61}}{\sqrt{21.96}}$

$= \frac{5}{3}$ (exact) ~~ [1]

$\angle OPQ = \angle TPS$ (vertically opposite angles)

$\frac{PO}{PT} = \frac{5}{3}$ ~~ [2]

$\therefore \Delta PQO$ is similar to ΔPST (SAS similarity) test name optional

Or [1], [2] and

$\frac{OQ}{TS} = \frac{6}{3.6}$

$= \frac{5}{3}$

Since $\frac{PQ}{PS} = \frac{PO}{PT} = \frac{OQ}{TS} = \frac{5}{3}$,

$\therefore \Delta PQO$ is similar to ΔPST (SSS similarity)

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10. (a) (i) $\text{Volume} = \frac{17.5}{100}\left(\frac{2}{3}\pi(192)^3\right)$

$= 2,590,000\,\text{m}^3$ (3 s.f.)

(ii) $\text{Surface area} = \frac{33.9}{100}[2\pi(192)^2] - 2(10000)$

$= 58,500\ \text{m}^2$ (3 s.f.)

(b) Fill volume of Brand A $= 5.7(5.3)(10)$

$= 302.1$

$= 300$ ml (nearest 10 ml)

Fill volume of Brand B $= \pi(2.75)^2(12.8)$

$= 304.106$

$= 300$ ml (nearest 10 ml)

Since both brands' packages are 300 ml, each spectator requires 2 bottles/packs – no difference.

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Total cost for Brand A $= 0.6(110000)$

$= \$66000$

Less refund \$11000 Brand A: \$55 000

Total cost for Brand B $= 0.55(110000)$

$= \$60500$

Less refund \$5200 Brand B: \$55 300

Decision.

Possible reasoning:

Brand A should be chosen. Despite being slightly more expensive, Brand A has the advantage of using more environmentally-friendly material.

Assumption: Every pak/bottle is emptied and recycled.

Assumption: Total number of paks/bottles is 110000 (2/spectator); no extras.

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